22.101 Fall 2003 Professor Sow-Hsin Chen Due: Dec. 3, 2003.

Problem Set No.6

1. Show that the non-relativistic Bethe-Block formula can be written in a more practical form as

$$-\frac{dE(MeV)}{d\xi(mg/cm^2)} = \frac{0.144}{E(MeV)}\left(\frac{Z}{A}\right)\ln\left(\frac{E(eV)}{459\bar{I}(eV)}\right) \tag{1}$$

for the case of an incident proton beam.

Given the following table of data:

Element	A	Z	$\bar{I}$ (eV)
C	12.00	6	76.4
A1	26.98	13	150
Fe	55.85	26	241
Cu	63.54	29	276
Pb	207.19	82	705
U	238.03	92	811
Air	15.56	7.22	80.5
Si	28.09	14	169
Ge	72.59	32	346

compute the formula (1) numerically for aluminum and compare your results with graph **4**-63 in the book by Marmier and Sheldon. Next, modify the formula for the case of an $\alpha-$particle and plot the curve for silicon and air over the energy range from 1 to 10 MeV.

2. As a means of degrading the energy of a proton beam from 6 MeV to half this value, the beam was passed through two adjacent metal foils having the same thickness but different composition. The requisite slowing down was obtained when the foils were so arranged that the beam first passed through copper and then through gold. Would it have made any difference if the foils had been interchanged, and if so, what would have been the mean energy of the emergent protons?

3. The energy loss of a heavy charged particle (charge ze) of speed V in a material with n atoms per unit volume (atomic number Z) is,

$$-\frac{dE}{dx} = \frac{4\pi e^4 z^2}{m_e V^2}(nZ)\left[\ln\left(\frac{2m_e V^2}{\bar{I}}\right) - \ln\left(1-\frac{V^2}{c^2}\right) - \frac{V^2}{c^2}\right] \tag{2}$$

Show that this expression passes through a minimum as V is varied and find the approximate kinetic energy of the particle at that speed.

4. The energy loss formula for heavy charged particles in a monoatomic substanceis is often written as

$$-\frac{dE}{dx}=\frac{4\pi e^4 z^2}{m_e V^2}(nZ)B_e \tag{3}$$

where B_e is called the atomic stopping number per electron. Suppose an absorbing material consists of a fraction f_1 (by number) of atoms of kind 1 (Z_1, A_1) and a fraction f_2 of atoms of kind 2 (Z_2,.A_2). (a) Derive an equation for the energy loss of heavy charged particles in this material in terms of B_{e1} and $B_{e2.}$ Call the mass density of the material ρ. (b) For 8 MeV alpha-particles, observed values of B_e are 5.6 for hydrogen and 4.0 for nitrogen. Compute the energy loss of 8 MeV alpha particles in ammonia (NH_3) gas at NTP.

5. An empirical formula for the mean range (in g/cm^2) of electrons as a function of kinetic energy E (in MeV) have been given by Katz and Penfold:

$$\begin{aligned}\overline{R}&=0.412E^n \quad \text{with} \quad n=1.265-0.0954\ln E \quad (0.01<E<3 \text{ MeV})\\ \overline{R}&=0.530E-0.106 \quad (2.5<E<20 \text{ MeV})\end{aligned} \tag{4}$$

Plot a graph of E versus $\overline{R}$ in log-log scale. Give a table of ranges (in cm) in air and in aluminum at electron energies of 10 KeV, 100 KeV, 1.0 MeV and 10 MeV.

6. Given that the differential cross section fo the Compton scattering of a polarized high energy photon of a frequency ν ($h\nu > m_e c^2$) with a polarization vector $\hat{\varepsilon}$ to a lower frequency ν' with a polarization vector $\hat{\varepsilon}'$, by a free electron, at a scattering angle θ_γ, is

$$\left(\frac{d\sigma}{d\Omega}\right)_{pol}=\frac{1}{4}r_e^2\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu}{\nu'}+\frac{\nu'}{\nu}+4\cos^2\Theta-2\right) \tag{5}$$

where $\hat{\varepsilon}\cdot\hat{\varepsilon}'=\cos\Theta$, show that the cross section for the unpolarized photon is given by

$$\left(\frac{d\sigma}{d\Omega}\right)_{unpol}=\frac{1}{2}r_e^2\left(\frac{\nu'}{\nu}\right)^2\left(\frac{\nu}{\nu'}+\frac{\nu'}{\nu}-\sin^2\theta_\gamma\right). \tag{6}$$

For the proof, read page 680 of the reference book by R.D. Evans, The Atomic Nucleus (McGraw-Hill).

Furthermore, by using the Compton relation, show that the above equation can be re-written as

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}} = r_e^2\left(\frac{1+\cos^2\theta_\gamma}{2}\right)\frac{1}{\left[1+\varepsilon(1-\cos\theta_\gamma)\right]^2}\left\{1+\frac{\varepsilon^2\left(1-\cos\theta_\gamma\right)^2}{\left(1+\cos^2\theta_\gamma\right)\left[1+\varepsilon\left(1-\cos\theta_\gamma\right)\right]}\right\}$$

where $\varepsilon = h\nu/m_e c^2$.