1 Problem Set 4 Solutions

1.a) (20pts)

$$kcot(kb+\delta) = KcotKb$$

$$k(\frac{(coskb)(cos\delta) - (sinkb)(sin\delta)}{(sinkb)(cos\delta) + (coskb)(sin\delta)}) = KcotKb$$

$$tan\delta = \frac{(KcotKb)(tankb) - K}{-(ktankb) - (KcotKb)}$$

$$cot\delta = \frac{(KcotKb) + (ktankb)}{k - (KcotKb)(tankb)}$$

$$kcot\delta = \frac{(KcotKb) + (ktankb)}{1 - (K/k)(cotKb)(tankb)}$$

b.) We know that,

$$kcot\delta = -(1/a)$$

as k goes to zero. Therefore,

$$-(1/a) = \frac{KcotKb + ktankb}{1 - (K/k)(tankb)(cotKb)}$$

In the same limit,

$$tanx = x$$

$$K = K_0$$

$$-(1/a) = \frac{K_0 cotK_0 b}{1 - K_0 bcotK_0 b}$$

$$K_0 cotK_0 b(b-a) = 1$$

$$K_0 cotK_0 b = \frac{1}{b-a}$$

c.) In the small k limit, the expansion dictates,

$$K_0 = \sqrt{\kappa^2 + \alpha^2}$$

$$= \kappa(1 + (1/2)\frac{\alpha^2}{\kappa^2})$$

$$= \kappa + \frac{\alpha^2}{2\kappa}$$

The Taylor expansion of f(x) says,

$$f(x) = f(x_0) + f'(x_0)(x - x_0)$$

If, $x_0 = \kappa$ then Taylor expansion reads,

$$= \kappa cot\kappa b + (cot\kappa b + \frac{\kappa b}{sin^2\kappa b})(K_0 - \kappa)$$

$$(K_0 - \kappa) = \frac{\alpha^2}{2\kappa}$$

$$K_0 cot K_0 b = \kappa cot\kappa b + \frac{\alpha^2}{2\kappa}(cot\kappa b - \kappa b csc^2\kappa b)$$

$$= -\alpha + \frac{\alpha^2}{2\kappa^2}(\kappa cot\kappa b - b\kappa^2(1 + cot^2\kappa b))$$

$$= -\alpha + \frac{\alpha^2}{2\kappa^2}(-\alpha - b(\kappa^2 + \alpha^2))$$

Terms in higher order α are negligible,

$$= -\alpha - \frac{b\alpha^2}{2}$$

d.) We have,

$$\frac{1}{b-a} = -\alpha - \frac{\alpha^2 b}{2}$$

From a binomial expansion,

$$a = b + (1/\alpha) + \alpha^{-2}(\frac{\alpha^2 b}{2})(-1)$$

$$a = b + (1/\alpha)(1 - \frac{\alpha b}{2})$$

e.) Using the same technique as in part c, we have,

$$KcotKb = K_0cotK_0b + \frac{k^2}{2K_0^2}(K_0cotK_0b - K_0^2b(1+cot^2K_0b))$$

$$= \frac{1}{b-a} + \frac{k^2}{2K_0^2}(\frac{1}{b-a} - b(K_0^2 + \frac{1}{(b-a^2)}))$$

$$= \frac{1}{b-a} - (1/2)bk^2 - \frac{ak^2}{2K_0(b-a)^2}$$

f.) Combine the two and remember,

$$kcot\delta = \frac{KcotKb}{1-bKcotKb}$$

2.) (20 pts.) In general,

$$\sigma(k) = \frac{4\pi}{k^2 + k^2cot^2\delta(k)}$$

Let k go to zero and sub in $kcot\delta$,

$$= \frac{4\pi}{(\frac{K_0cotK_0b}{1-bK_0cotK_0b})^2}$$

$$= 4\pi b^2(1 - \frac{tanK_0b}{K_0b})^2$$

3.) (20 pts.) We know from previous problems that if $R(r) = u(r)/r$ we get,

$$\frac{d^2u}{dr^2} - K^2u = 0 r > b$$

$$\frac{d^2u}{dr^2} + k^2u = 0 r < b$$

Solutions to these two equations are,

$$u(r) = Ae^{-Kr} + Be^{Kr} r < b$$

$$u(r) = Csin(kr+\delta) r > b$$

We must now impose boundary conditions,

$$u(0) = 0$$

$$u(b^-) = u(b^+)$$

$$u'(b^-) = u'(b^+)$$

In order to avoid singularities at the origin, $A = -B$, and the rest follow,

$$A(e^{-Kb} - e^{Kb}) = Csin(kb + \delta)$$

$$-KA(e^{-Kb} - e^{Kb}) = kCcos(kb + \delta)$$

Divide one by the other,

$$kcot(kb + \delta) = K\frac{e^{Kb} + e^{-Kb}}{e^{Kb} - e^{-Kb}} = Kcoth(Kb)$$

Expand the left side to get,

$$kcot\delta = \frac{Kcoth(Kb) + ktankb}{1 - (K/k)tankb(cothKb)}$$

For S-Wave scattering, in the limit k goes to zero,

$$\sigma = 4\pi b^2 (1 - \frac{1}{K_0 bcoth(K_0 b)})^2$$

And as V_0 and K_0 go to infinity, we have $coth(K_0 b)$ going to one. Therefore,

$$\sigma = 4\pi b^2$$

4.) (20 pts.) Incident flux,

$$\phi = \frac{I}{eA} = 9.95E18/m^2 - s$$

Obtain the number density of H_2 using ideal gas law,

$$n/V = \frac{2P}{k_b T} = 7.06E21 protons/m^3$$

The detector counting rate is,

$$R = \phi(\frac{d\sigma}{d\Omega}\Delta\Omega) n_P V = 955/s \tag{1}$$

5.) (20 pts)

$$n_{238}\sigma_{238}^f + n_{235}\sigma_{235}^f = (n_{238} + n_{235})\sigma_n^f$$

$$\sigma_{238}^f = 0$$

$$\sigma_{235}^f = \frac{n_{238} + n_{235}}{n_{235}}\sigma_n^f = 4.22/.72 = 586 barns$$

$$R = \phi\sigma_n^f nV = 1.99E7A$$