Exercise 3. Solving 2-point ODEs.

1. Write a code to solve, using matrix inversion, a 2-point ODE of the form

$$\frac{d^2y}{dx^2} = f(x)$$

on the x-domain $[0, 1]$, spanned by an equally spaced mesh of N nodes. With Dirichlet boundary conditions $y(0) = y_0$, $y(1) = y_1$.

When you have got it working, obtain your personal expressions for $f(x)$, N, y_0, and y_1 from one of the three supplied sets. And solve the differential equation so posed.
Plot the solution.

Submit the following as your solution:

a. Your code in a computer format that is capable of being executed.

b. The expressions of your problem $f(x)$, N, y_0, and y_1

c. The numeric values of your solution y_j.

d. Your plot.

e. Brief commentary ($<$ 300 words) on what problems you faced and how you solved them.

2. Save your code and make a copy with a new name. Edit the new code so that it solves the ODE

$$\frac{d^2y}{dx^2} + k^2y = f(x)$$

on the same domain and with the same boundary conditions, but with the extra parameter k^2. Verify that your new code works correctly for small values of k^2, yielding results close to those of the previous problem.

Investigate what happens to the solution in the vicinity of $k = \pi$.

Describe what the cause of any interesting behavior is.

Submit the following as your solution:

a. Your code in a computer format that is capable of being executed.

b. The expressions of your problem $f(x)$, N, y_0, and y_1

c. Brief description ($<$ 300 words) of the results of your investigation and your explanation.

d. Back up the explanation with plots if you like.

22.15 Essential Numerical Methods

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