

Due November 18, 2009 by 12:00 pm

## TAKE HOME

## QUIZ 2 (SOLUTION)

**Problem 1 (70%) – Temperature distribution in pebble fuel of advanced design**

i)

The maximum temperature,  $T_{\max}$ , within the pebble occurs at  $R_1$  (note that the temperature is constant and equal to  $T_{\max}$  for  $r < R_1$ ). To find the max allowable power,  $\dot{q}$ , given a fixed  $T_{\max} - T_{\infty}$ , one needs to develop expressions for the temperature difference in each region of the pebble. For the temperature at  $r = R_3$ , Newton's law of cooling immediately provides the following equation:

$$T_3 - T_{\infty} = \frac{\dot{q}}{4\pi R_3^2 h} \quad (1)$$

Where  $h$  is the heat transfer coefficient. For  $r < R_3$ , the heat conduction equation in spherical coordinates has to be solved:

$$\frac{1}{r^2} \frac{d}{dr} \left[ r^2 k \frac{dT}{dr} \right] = -q''' \quad (2)$$

Integrating for each zone (outer graphite and fueled zone) and imposing the boundary conditions ( $T = T_3$  at  $r = R_3$  and  $dT/dr = 0$  at  $r = R_1$ ), one gets:

$$T_2 - T_3 = \frac{\dot{q}}{4\pi k_C} \left( \frac{1}{R_2} - \frac{1}{R_3} \right) \quad (3)$$

$$T_{\max} - T_2 = \frac{\dot{q}}{4\pi k_F (R_2^3 - R_1^3)} \left[ \frac{1}{2} (R_2^2 - R_1^2) - R_1^3 \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \right] \quad (4)$$

Where  $k_C$  and  $k_F$  are the thermal conductivity of the graphite and fueled zones, respectively. Adding Eqs. (1), (3) and (4), and solving for  $\dot{q}$ , one gets:

$$\dot{q}_{\max} = (T_{\max} - T_{\infty}) / \left\{ \frac{1}{4\pi R_3^2 h} + \frac{1}{4\pi k_C} \left( \frac{1}{R_2} - \frac{1}{R_3} \right) + \frac{1}{4\pi k_F (R_2^3 - R_1^3)} \left[ \frac{1}{2} (R_2^2 - R_1^2) - R_1^3 \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \right] \right\}$$

$\approx 2330 \text{ W}$

ii)

Same equations as in 'i' for  $r > R_2$ . However, for  $r < R_2$ , one has to solve the heat conduction equation (Eq. 2) anew. Integrating and imposing the boundary conditions ( $dT/dr=0$  for  $r=0$ ), one gets:

$$T_{\max} - T_2 = \frac{\dot{q}}{8\pi k_F R_2}$$

Thus:

$$\dot{q}_{\max} = (T_{\max} - T_{\infty}) \left\{ \frac{1}{4\pi R_3^2 h} + \frac{1}{4\pi k_C} \left( \frac{1}{R_2} - \frac{1}{R_3} \right) + \frac{1}{8\pi k_F R_2} \right\} \approx 1414 \text{ W}$$

iii)

As shown by the calculations in 'i' and 'ii' above, the shell design allows for a higher power operation wrt the solid design. This is a major economic advantage, countered by the shell design's higher fabrication costs and lower heavy metal loading. Without quantitatively assessing this economic tradeoff, it is not possible to pick one design vs the other. FYI, the University of California at Berkeley, which is developing a molten-salt-cooled, pebble-fueled reactor concept, has selected the shell design.

iv)

The conservation of energy for the pebble, after the coolant ceases suddenly and completely, is:

$$MC \frac{dT}{dt} = \dot{q}_{dec}(t) \tag{5}$$

Where MC is the total heat capacity of the pebble, calculated as follows:

$$MC = \rho_F c_F \frac{4}{3} \pi (R_2^3 - R_1^3) + \rho_C c_C \frac{4}{3} \pi [(R_3^3 - R_2^3) + R_1^3] \approx 22.8 \text{ J/K}$$

$\rho_F$ ,  $c_F$ ,  $\rho_C$ ,  $c_C$  are the fuel and graphite density and specific heat, respectively.  $T(t)$  is the pebble temperature (same everywhere within the pebble, per the problem statement) and

$\dot{q}_{dec}(t) = \dot{q}_0 0.066t^{-0.2}$  is the decay power associated with a nominal power  $\dot{q}_0 = 700 \text{ W}$ .

Integrating Eq. (5) from  $t=0$  to the time  $t_f$ , at which  $T_f=1600^\circ\text{C}$ , one gets:

$$T_f = T_0 + \frac{\dot{q}_0 0.066}{0.8MC} t_f^{0.8} \tag{6}$$

where  $T_0=800^\circ\text{C}$ . Solving Eq. (6) for  $t_f$ , one gets  $t_f \approx 1332 \text{ s}$ .

## Problem 2 (30%) – Emergency core cooling system for sodium-cooled reactor

From the conservation of energy, the temperature rise in the core,  $\Delta T$  is:

$$\Delta T = \frac{\dot{Q}}{\dot{m}c} \quad (7)$$

Where  $c$  is the sodium specific heat,  $\dot{m}$  is the sodium flow rate in the core, and  $\dot{Q}=30$  MW is the decay power. To find  $\dot{m}$ , we need to analyze the pump. The pumping power  $\dot{W}$  (=58 kW) can be expressed as:

$$\dot{W} = \dot{m} \frac{\Delta P_{core}}{\rho \eta_p} \quad (8)$$

Where  $\rho$  is the sodium density,  $\eta_p=0.8$  is the isentropic efficiency of the pump, and  $\Delta P_{core}$  is the pressure drop in the core (assumed to be only due to friction, per the problem statement):

$$\Delta P_{core} = f \frac{L}{D_e} \frac{G^2}{2\rho} \quad (9)$$

$L$  is the fuel assembly length,  $G = \frac{\dot{m}}{NA}$  is the mass flux, with  $A$  being the flow area of each fuel assembly and  $N$  the total number of fuel assemblies. Assuming that the flow in the core is turbulent and  $Re < 30000$ , the friction factor is:

$$f = \frac{0.316}{Re^{0.25}} = \frac{0.316}{(GD_e / \mu)^{0.25}} \quad (10)$$

From Eq. (8), (9) and (10) one gets:

$$G = \left[ \frac{2\dot{W}\rho^2 D_e^{1.25}}{0.316LAN\mu^{0.25}} \right]^{1/2.75} \approx 1274 \text{ kg/m}^2\text{s}$$

And  $\dot{m} \approx 1154$  kg/s. Then from Eq. (7), one gets  $\Delta T \approx 20^\circ\text{C}$ .

Note that  $Re \approx 22500$ , so the assumption of turbulent flow was accurate.

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