22.312 ENGINEERING OF NUCLEAR REACTORS

Thursday, November 13th, 2014, 1:00 – 2:30 p.m.

OPEN BOOK QUIZ 2 (SOLUTIONS)

Problem 1 (50%) – Gas lift pump for a lead-cooled reactor

i)

The steady-state momentum equation for the loop is:

$$(\rho_p - \rho_{ch})gH = f\frac{L}{D_c}\cdot\frac{G^2}{2\rho_p} \qquad (1)$$

where ρ_p=10,400 kg/m^3 is the liquid lead density, H=4.1 m is the chimney length, L=3.5 m is the core length, D_c=4.7 mm is the core equivalent diameter, $G=\frac{\dot{m}_p}{A_c}\approx$10,000 kg/m^{2}s is the coolant mass flux in the core, $\dot{m}_p$=2,500 kg/s is the desired coolant mass flow rate in the core, A_c=0.25 m^2 is the core flow area, f is the friction factor in the core, which can be calculated using the Blasius correlation for turbulent fully-developed flow:

$$f=\frac{0.316}{\text{Re}^{0.25}}\approx 0.0252$$

where $\text{Re}=\frac{GD_c}{\mu_p}\approx$24,740 and μ_p=1.9×10^{-3} is the liquid lead viscosity. The density in the chimney is:

$$\rho_{ch}=\rho_p(1-\alpha)+\rho_{He}\alpha \qquad (2)$$

where ρ_{He}=0.3 kg/m^3 is the helium density and α is the void fraction in the chimney. Substituting Eq. 2 into Eq. 1 and solving for α, we get:

$$\alpha = f\frac{L}{D_c}\cdot\frac{G^2}{2\rho_p}\cdot\frac{1}{(\rho_p-\rho_{He})gH}\approx 0.217$$

ii)

The flow quality in the chimney, x, can be found from the fundamental relation of two-phase flow:

$$x=\frac{1}{1+\frac{1-\alpha}{\alpha}\frac{\rho_p}{\rho_{He}}\frac{1}{S}}\approx 1.2\times 10^{-5}$$

where S=1.5 is the slip ratio, given in the problem statement. Then the mass flow rate of helium in the chimney is easily found:

$$\dot{m}_{He} = \dot{m}_p \frac{x}{1-x} \approx 0.03 \text{ kg/s}$$

iii)
At a void fraction of ~0.22 the flow regime is likely bubbly flow, since significant bubble coalescence does not occur at void fractions below 0.25-0.30, according to Taitel's criterion (Section 11.2.2.1 in the textbook). A more accurate determination could be made by building a flow regime map, again using the methodology developed by Taitel et al.

Problem 2 (50%) – Lowering the temperature of the fuel in a PWR

i)
The average linear power in the core $\bar{q}' = \dot{Q}/(N_{FA} N_{pin} L) \approx 16.8$ kW/m, where $\dot{Q}$=4500 MW, N_{FA}=241, N_{pin}=264 and L=4.2 m. Therefore, the maximum linear power in the core is $q'_{\max} = \bar{q}' P_{rad} P_{loc} P_{ax} \approx 41.7$ kW/m, where P_{rad} = 1.40, P_{loc} = 1.18 and P_{ax}=1.50.

ii)
First, let us calculate the temperatures in the cladding, which are the same for all cases.

$$T_{co} = T_b + \frac{q'_{\max}}{\pi d_{co} h} \approx 346.8°\text{C}$$

$$T_{ci} = T_{co} + \frac{q'_{\max}}{2\pi k_c} \ln \frac{d_{co}}{d_{ci}} \approx 391.6°\text{C}$$

where d_{co}=9.5 mm, d_{ci}=8.3 mm, k_c=20 W/m°C and h=38 kW/m^2°C. For all cases except (b), the temperature on the outer surface of the fuel pellet can be found from the gap conductance given in the problem statement:

$$T_{fo} = T_{ci} + \frac{q'_{\max}}{\pi d_g h_g} \approx 553.6°\text{C} \qquad \text{(for Case (a) and (c))}$$

where d_g=8.2 mm and h_g=10 kW/m^2°C. For case (b), the gap is occupied by stagnant molten tin, thus there is only conduction; as such, the temperature on the outer surface of the fuel pellet is found from an expression similar to that in the cladding:

$$T_{fo} = T_{ci} + \frac{q'_{\max}}{2\pi k_{Sn}} \ln \frac{d_{ci}}{d_f} \approx 394.1°\text{C} \qquad \text{(for Case (b))}$$

where d_f=8.1 mm and k_{Sn}=65 W/m°C.

Now let us look at the temperatures in the fuel pellet. Note that the *average* temperature in the fuel is formally defined as follows:

$$\bar{T}_f = T_{fo} + \frac{\int_0^{R_f} T(r) 2\pi r dr}{\pi R_f^2} \quad (1)$$

where R_f=4.05 mm is the radius of the fuel pellet. Therefore, one needs to first find the temperature distribution in the fuel pellet, *T(r)*, and then perform the integration in Eq. 1. For the reference UO_2 case, the volumetric heat generation rate is radially uniform within the pellet, thus the temperature distribution is parabolic, as derived in class, and the average fuel temperature is found from Eq. 1 to be $\bar{T}_{f,ref} = T_{fo} + \frac{q'_{\max}}{8\pi k_{UO2}} \approx 1147°C$, where k_{UO2}=2.8 W/m°C. Similarly for Case (a) $\bar{T}_{f,UZr} = T_{fo} + \frac{q'_{\max}}{8\pi k_{UZr}} \approx 620°C$, where k_{UZr}=25 W/m°C, and for Case (b) $\bar{T}_{f,Sn} = T_{fo} + \frac{q'_{\max}}{8\pi k_{UO2}} \approx 987°C$. Both temperatures are lower than the reference case, as expected.

For Case (c), in which the volumetric heat generation rate is not radially uniform, we first need to actually solve the heat equation to find the temperature distribution in the pellet:

$$k_{UO2} \frac{1}{r}\frac{d}{dr}[r\frac{dT}{dr}] + q'''(r) = 0 \quad \Rightarrow \quad \frac{d}{dr}[r\frac{dT}{dr}] = -\frac{r q_0'''}{k_{UO2}}[1 + (r/R)] \quad (2)$$

The boundary conditions are as follows:

$T=T_{fo}$ at $r=R$

$\frac{dT}{dr} = 0$ at r=0

Integrate Eq 2 from r=0 to a generic location r, to get:

$$r\frac{dT}{dr} - r\frac{dT}{dr}\bigg|_{r=0} = -\frac{q_0'''}{k_{UO2}}[\frac{r^2}{2} + \frac{r^3}{3R_f}] \Rightarrow \quad \frac{dT}{dr} = -\frac{q_0'''}{k_{UO2}}[\frac{r}{2} + \frac{r^2}{3R_f}] \quad (3)$$

where the second boundary condition was imposed. Integrating Eq 3 from $r=R_f$ to a generic location r, we get:

$$T(r) - T_{fo} = -\frac{q_0'''}{k_{UO2}}[\frac{r^2 - R_f^2}{4} + \frac{r^3 - R_f^3}{9R_f}] \quad (4)$$

where the first boundary condition was imposed. Introducing Eq. 4 into Eq. 1 and performing the integration we get:

$$\bar{T}_f = T_{fo} + \frac{23}{120}\frac{q_0'''R_f^2}{k_{UO2}} \qquad (5)$$

Finally note that we have:

$$q'_{\max} = \int_0^{R_f} q'''(r)2\pi r dr = \int_0^{R} q_0'''[1+(r/R)]2\pi r dr = \therefore q_0'''\pi R^2 \frac{5}{3} \qquad (6)$$

Eliminating q_0''' from Eqs. 5 and 6, we finally get an expression for the average temperature in the fuel pellet as a function of $q'_{\max}$:

$\bar{T}_f = T_{fo} + \frac{23}{200}\frac{q'_{\max}}{\pi k_{UO2}} \approx 1099°C$, which is lower than the reference case, as expected because now more power is generated close to the periphery of the pellet.

iii)

The use of highly conductive metallic fuel (Case (a)) yields a large reduction (~530°C) in average fuel temperature. It also increases the heavy metal loading, thus allowing for a reduction in enrichment, for the same burnup. However, there are some drawbacks with the use of U-Zr, including a much lower melting point than UO_2, and oxidation by water, should the cladding be breached. The latter issue is potentially a showstopper, because of the large amounts of hydrogen that would be generated during severe accidents.

Case (b) results in a significant reduction (~160°C) of the average temperature fuel, with a modest increase in fuel fabrication cost. Compatibility of tin and zirconium at the temperature and irradiation conditions of interest would have to be evaluated carefully.

Varying the enrichment radially (Case (c)) provides a modest reduction (~50°C) in the average fuel temperature, at the expense of much greater manufacturing complexity and thus cost.

In summary, Case (b) probably provides the best compromise of improved safety, cost and ease of fuel fabrication. Note that the use of a molten metal gap for LWR fuel indeed has been proposed, e.g. R. Wright et al., "Thermal bonding of light water reactor fuel using nonalkaline liquid-metal alloy", *Nuclear Technology*, 115(3), pp. 281-292, 1996.