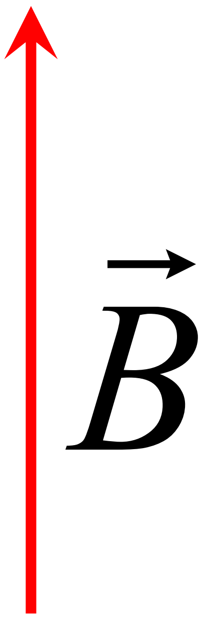
Massachusetts Institute of Technology
22.68J/2.64J
Superconducting Magnets
⇓

February 6, 2003

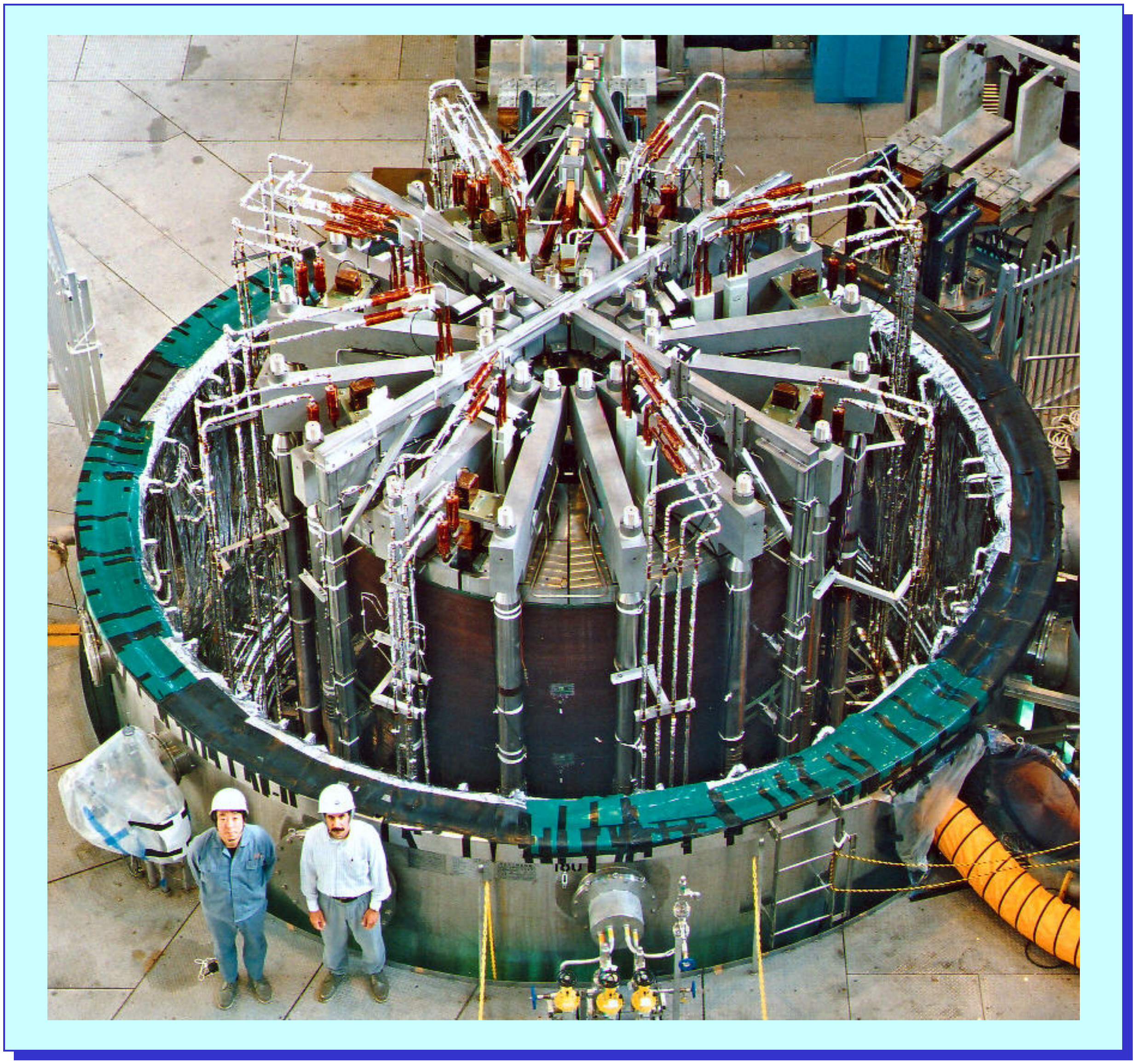
- Course Information
- Lecture #1 – Introduction
  - Superconductivity and Applications
  - Prospects and Challenges

Magnetic Field – Two Distinct Views

Users' (Physicists, Doctors, etc.)

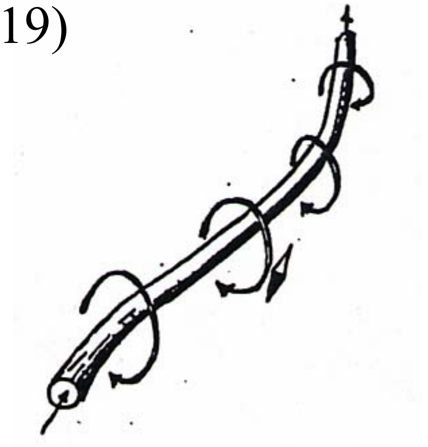


Magnet Engineers Perspective

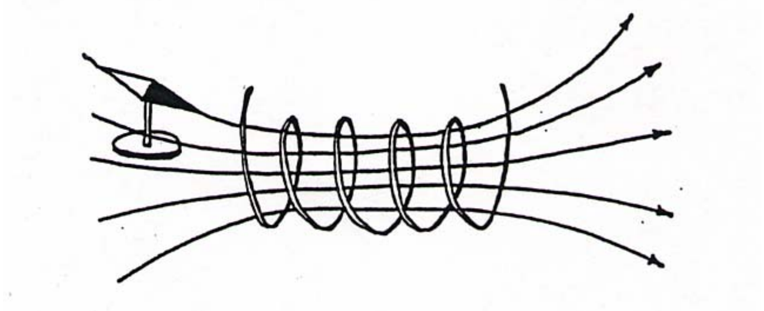


Electromagnets

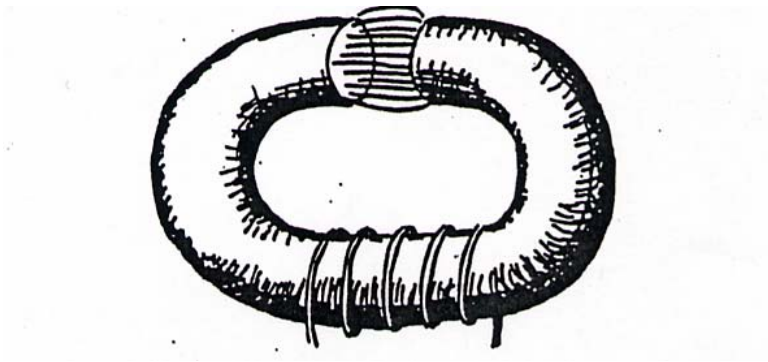
Current Carrying Wire: generates lines of magnetic flux (H.C. Oersted, 1819)



Coil: Produces a bundle of magnetic flux



Iron Electromagnet: The flux can be use to align magnetic domains in iron, producing ~1000 times as much flux. The iron will be saturated, limiting the maximum flux to ~2 Tesla.



High-Field Magnets: High Field (>2T) magnets are ironless electromagnets. There are basically three approaches for high-field electromagnets: 1) nonsuperconducting; 2) superconducting; and 3) hybrid-combination of 1) and 2).

Nonsuperconducting

- RT copper magnets, generally water-cooled.
    - No inherent upper-field limit – only more power (& cooling) and stronger materials required.
    - Current record: 33T (~35MW) at NHMFL.
- Cryogenic Cu or Al magnets, LN2-, LNe-, or LH2-cooled.
    - For special applications only – generally pulsed fields

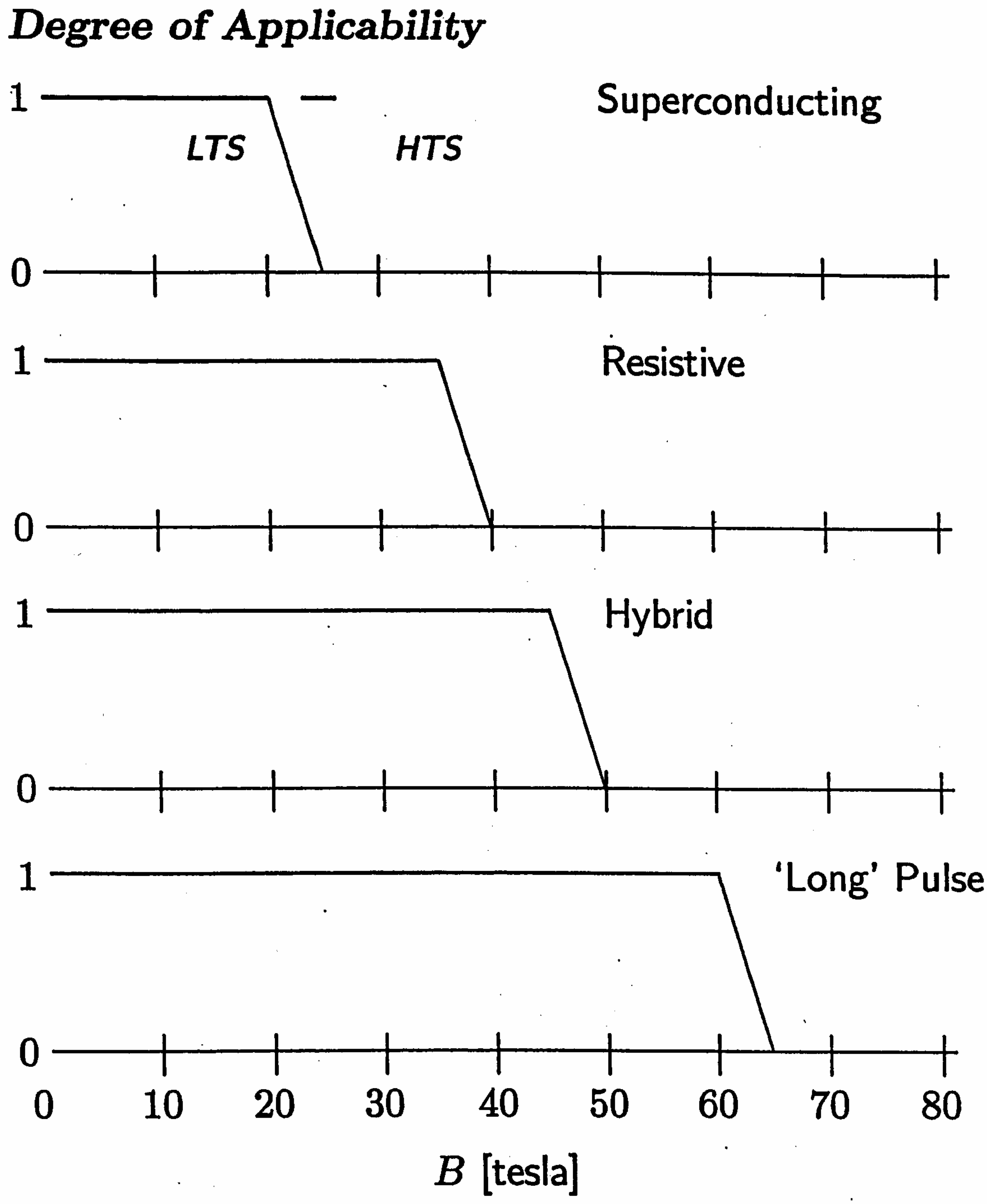
Superconducting

- LTS Magnets, LHe-cooled or cryocooler-cooled.
- HTS magnets. LHe-, cryocooler-, LN2-cooled.
- Superconductor performance a key limitation.

Hybrid

- A copper magnet (inner section) combined with a superconducting magnet (outer section).
- Current record: 45 (30Cu/15SC)T, at NHMFL.

High-Field Magnets



Types of Superconducting Magnet

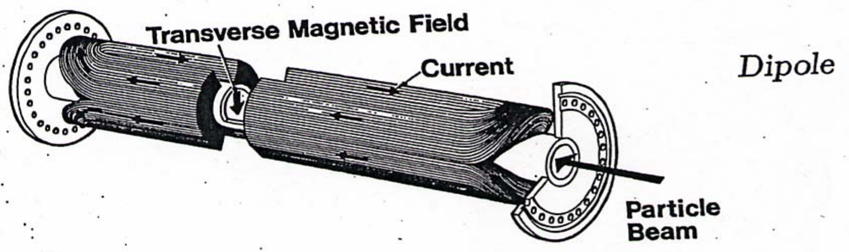
Solenoid: Cylindrical helices; most widely used type.

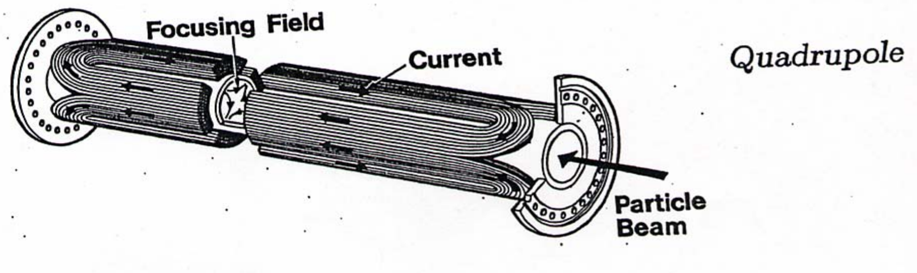
Dipole: Generates a uniform field transverse to its long axis; deflects charged particles in accelerators and MHD.

Quadrupole: Generates a linear gradient field transverse to its axis over the central region of its bore; focuses particles in particle accelerators.

Racetrack: Resembles a racetrack; wound in a plane with each turn consisting of two parallel sides and two semi-circles at each end; a pair may be assembled to approximate the field of a dipole; used in motors and Maglev.

Toroid: Generates a field in the azimuthal direction; it confines hot plasma in a Tokamak; also used for SMES.





Magnet Types, Maxiumum Fields, Applications

Type	B_{max} *[T]*	*Application* **(partial list)**
Solenoid	45[a] 23.5[b] 5[c] ~1.5+[d]	High-field research NMR MRI Magnetic Separation
Dipole	~15[e]	High-energy physics (HEP)
Quadrupole	0[f]	HEP
Racetrack	4-5[g]	Power Electric Devices
Toroid	~16[h] 5-10[i]	Fusion SMES

a) Hybrid magnet (NHMFL).
b) Future Target (1-GHz system); current record: 21.14 (900MHz).
c) Or higher; more widely and universally used systems: 0.5-1.5T.
d) HTS version.
e) Future target:recent prototype (LBNL); current range: 4.8-5 (Large Hadron Collider-CERN).
f) On-axis; peak field at the winding may reach ~8T for current systems.
g) HTS motors and generators.
h) Future target for power-generating systems. Present value 13T (ITER).
i) Future range for HTS systems.

Superconductivity

Heike Kamerlingh Onnes (1853-1926)

“Door meten tot weten”
(“Through measurement to knowledge”)

Superconductivity

Facts on Superconductors

- “Zero electrical resistance (R=0), under DC conditions.
  - Discovered by Kammerlingh Onnes (1911).

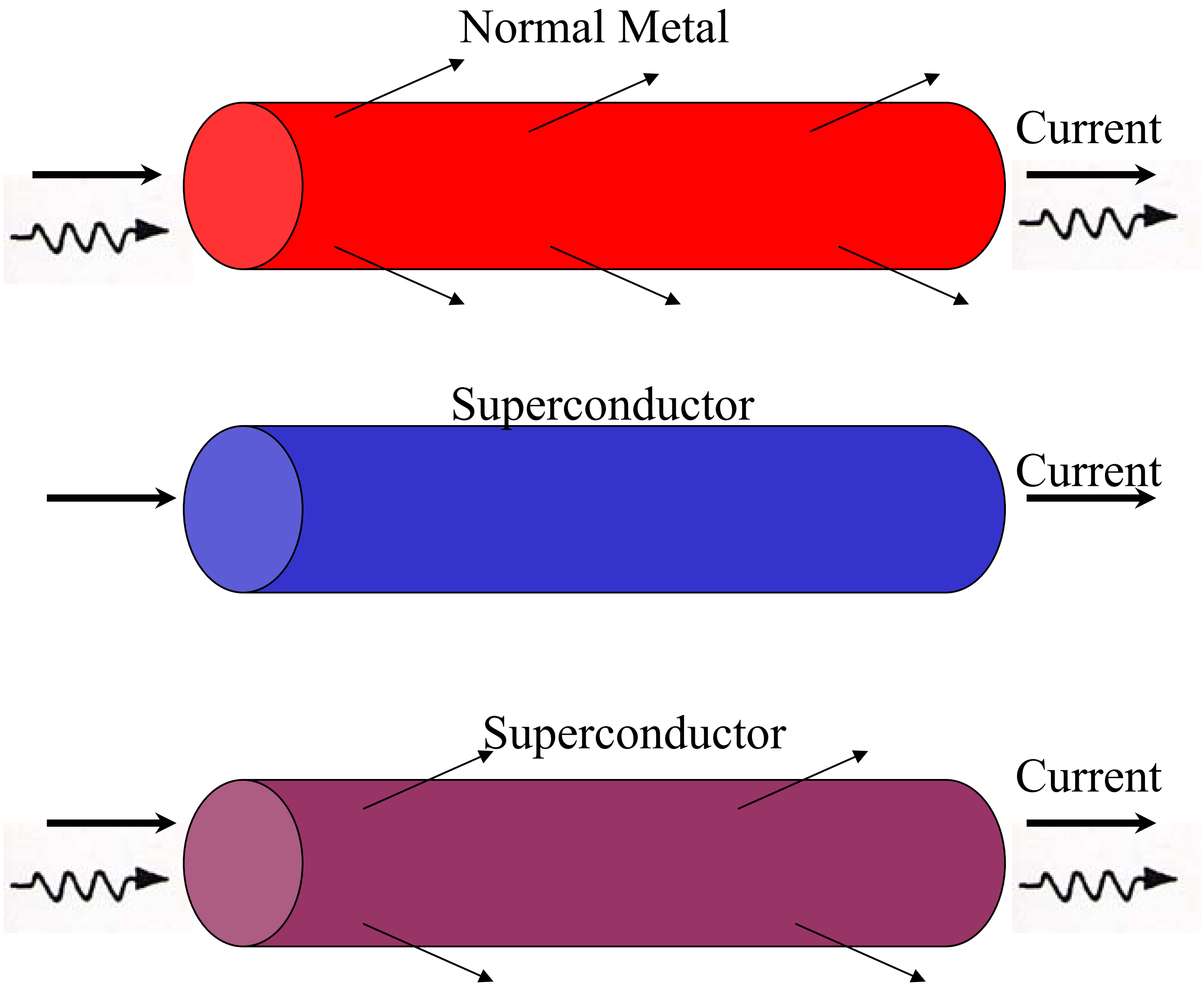
- Some are “perfect” diamagnets (B=0) – Type I.
  - The Mesissener effect (W. Meissner and R. Oschenfeld, 1933).
  - Others are mostly diamagnets and R≃0 – Type II.

- There are two(?) types of superconductors.
  - Low-temperature superconductors (LTS)
  - High-temperature superconductors (HTS)
  - Medium-temperature superconductors (MTS)?

Superconductivity

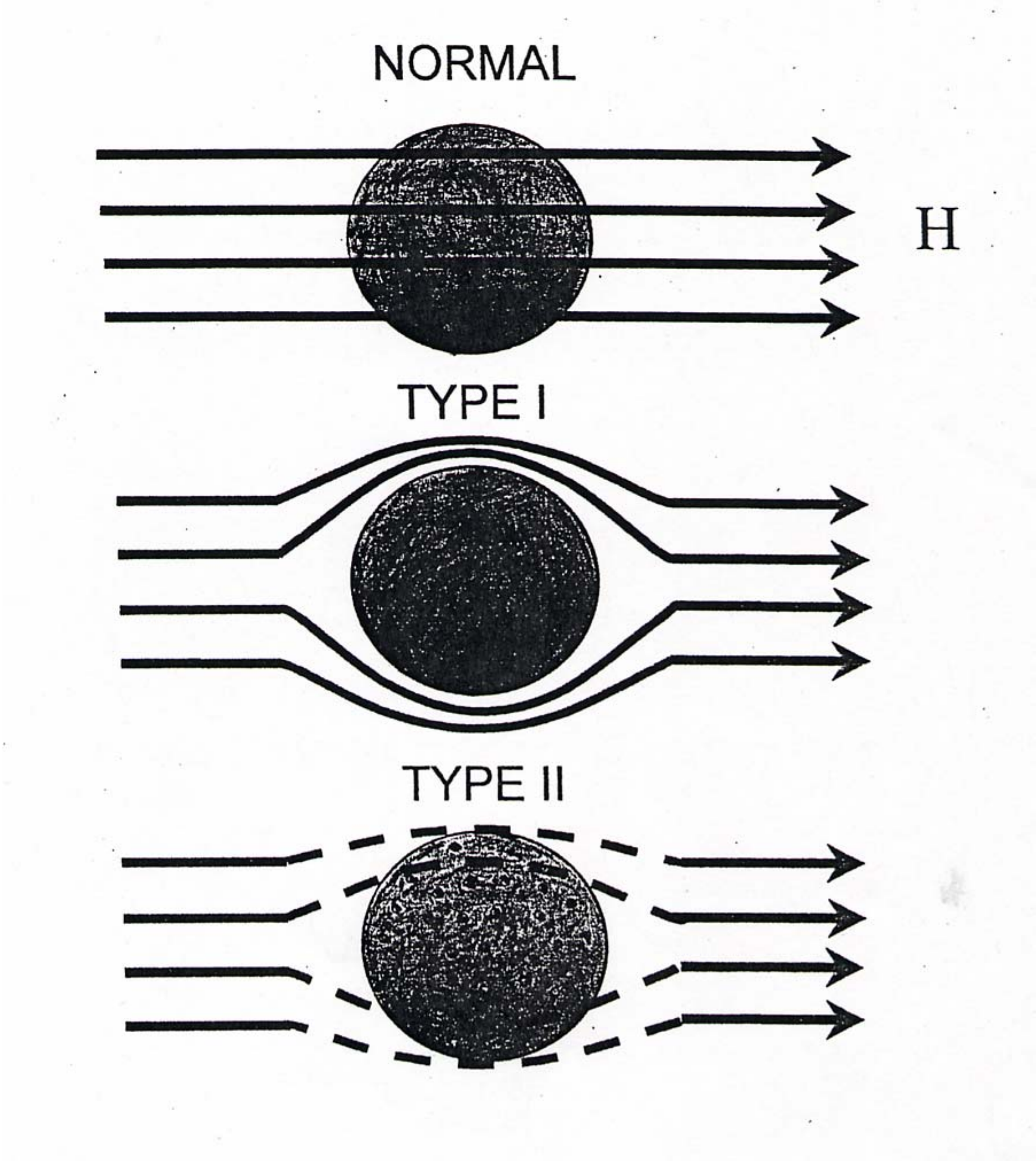
Discovered by Kamerlingh Onnes (1911)

- “Zero electrical resistance (R=0), under DC conditions.
- Dissipative under AC conditions.



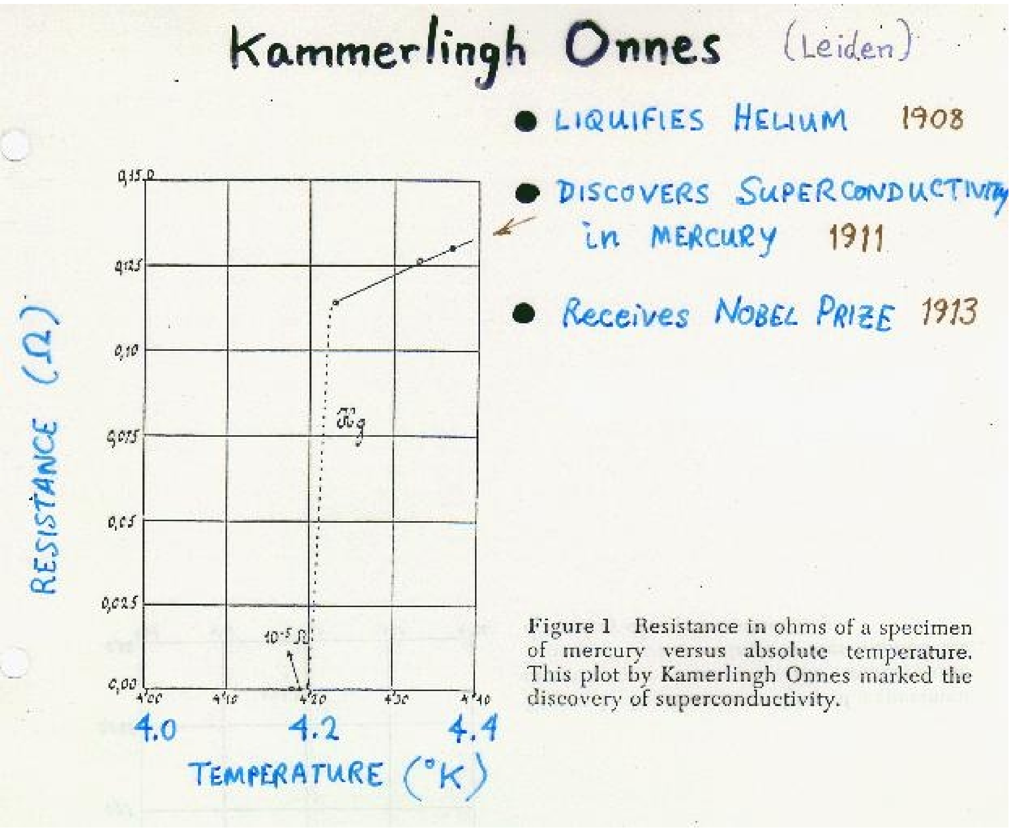
Mesissner Effect
(W. Meissner & R. Oschenfeld, 1933)

- Some are “perfect” diamagnets (B=0) – Type I.
    - The Mesissener effect (W. Meissner and R. Oschenfeld, 1933).
    - Others are mostly diamagnets and R~0 – Type II.



Resistance vs Temperature Plots

- Mercury (1911)

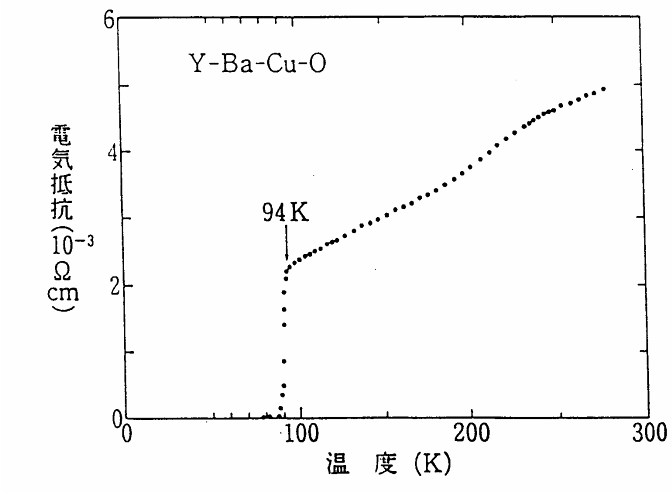


Kammerlingh Onnes (Leiden)

- LIQUIFIES HELIUM 1908
- DISCOVERS SUPERCONDUCTIVITY in MERCURY 1911
- Receives NOBEL PRIZE 1913

Figure 1 Resistance in ohms of a specimen of mercury versus absolute temperature. This plot by Kamerlingh Onnes marked the discovery of superconductivity.

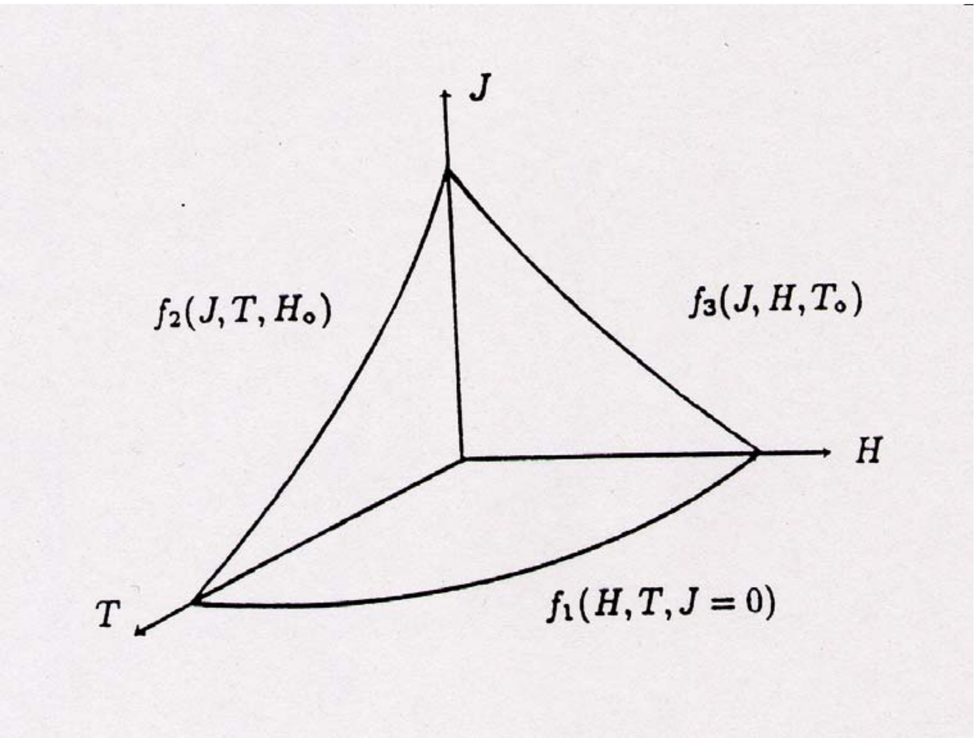
- Y-Ba_Cu-O (c. 1987)



Facts on Superconductors (continued)

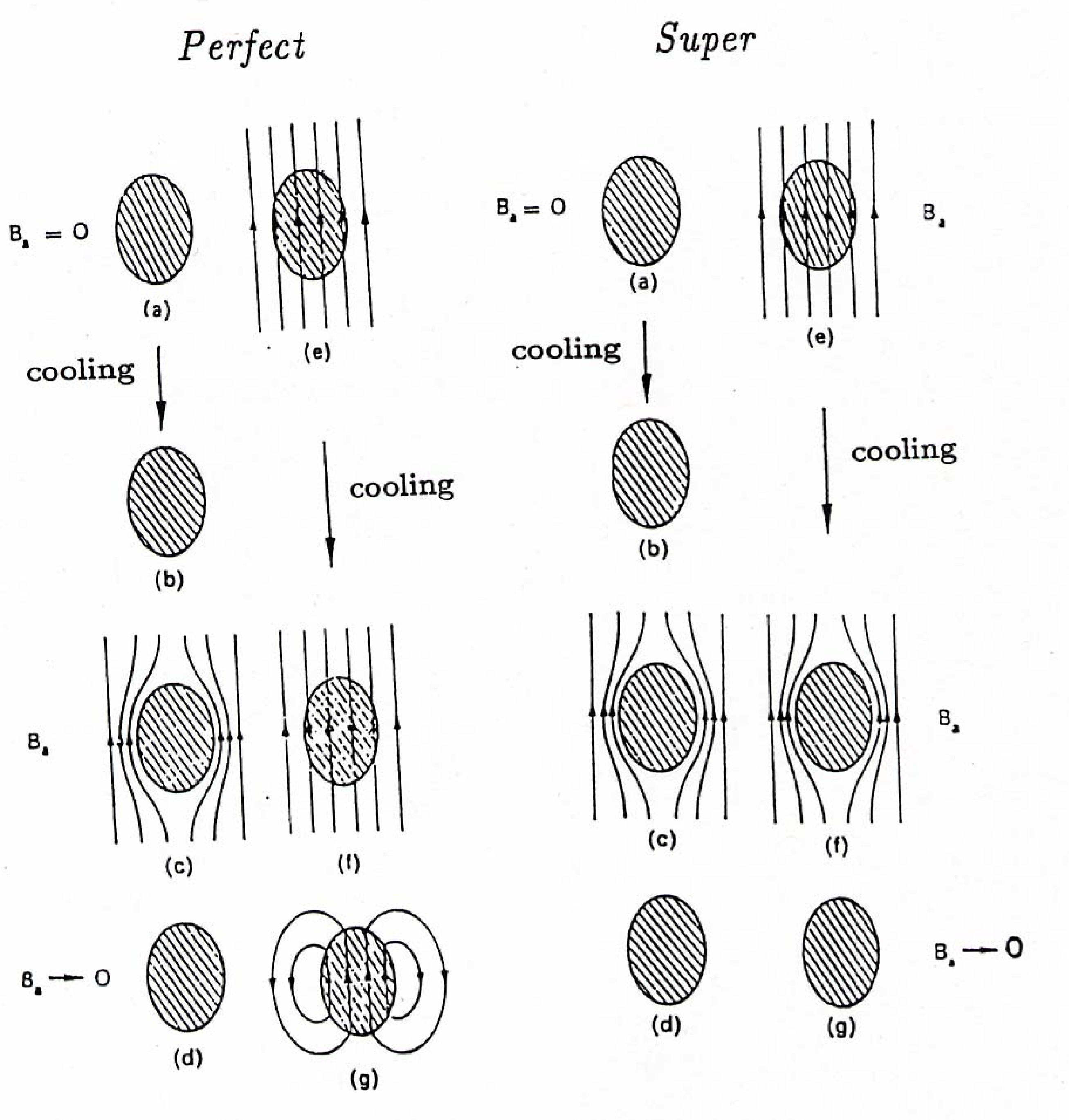
- Superconductivity has three critical parameters:
    - o Critical temperature, T_c
    - o Critical magnetic Field, H_c
    - o Critical current density, J_c.

Critical Surface of a Superconductor



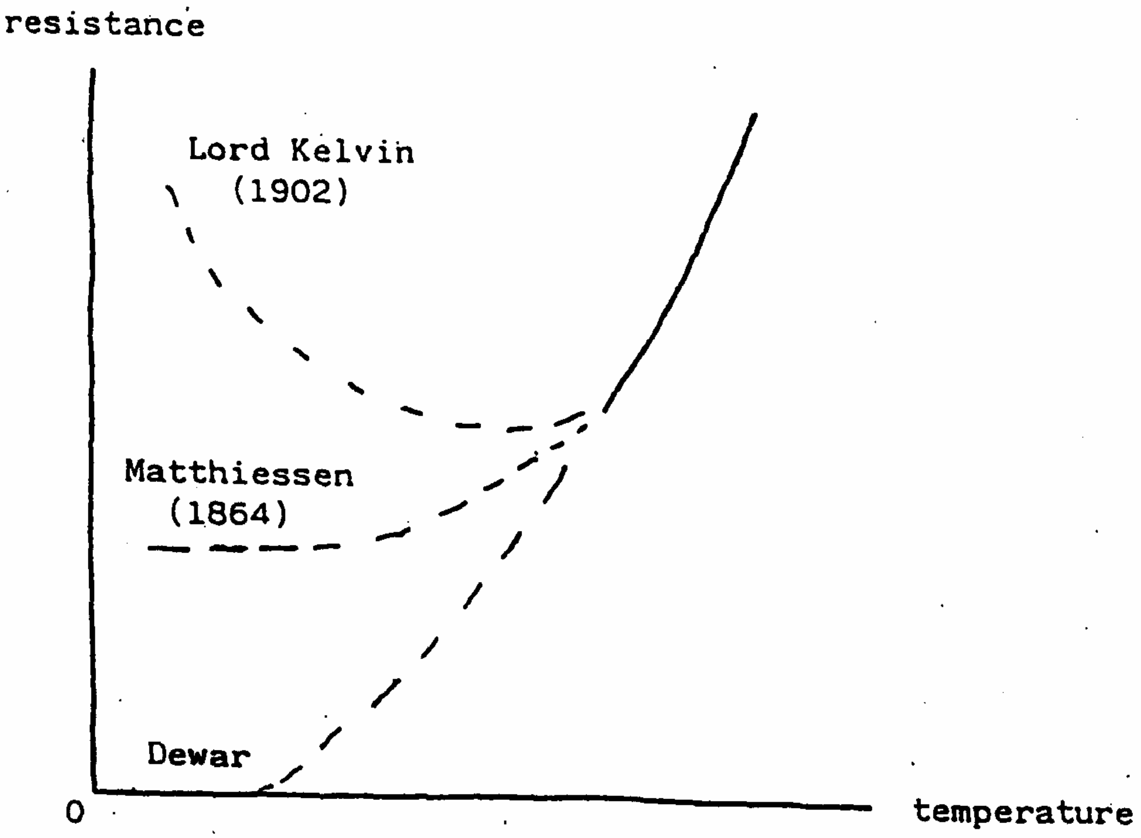
Perfect Conductor vs Superconductor

- Perfect conductor: $\rho = 0 \rightarrow dB/dt = 0$.
- Perfect Superconductor: $\rho = 0$ and $B = 0$.



Why Superconductivity Discovered?

- As a result of solid state physics research in the early 1910s.
  - In 1911 Kamerlingh Onnes of U. Leiden discovered Hg (T_c = 4.18 K) as a superconductor.
  - Discovered (1911) the existence of J_c with Hg.
  - The first superconducting (Pb wire) magnet failed (1913).
  - Received (1913) the Nobel Prize in physics for the discovery of superconductivity and the liquifaction of helium.
  - Discovered (1914) the existence of H_c, with Pb and Sn.

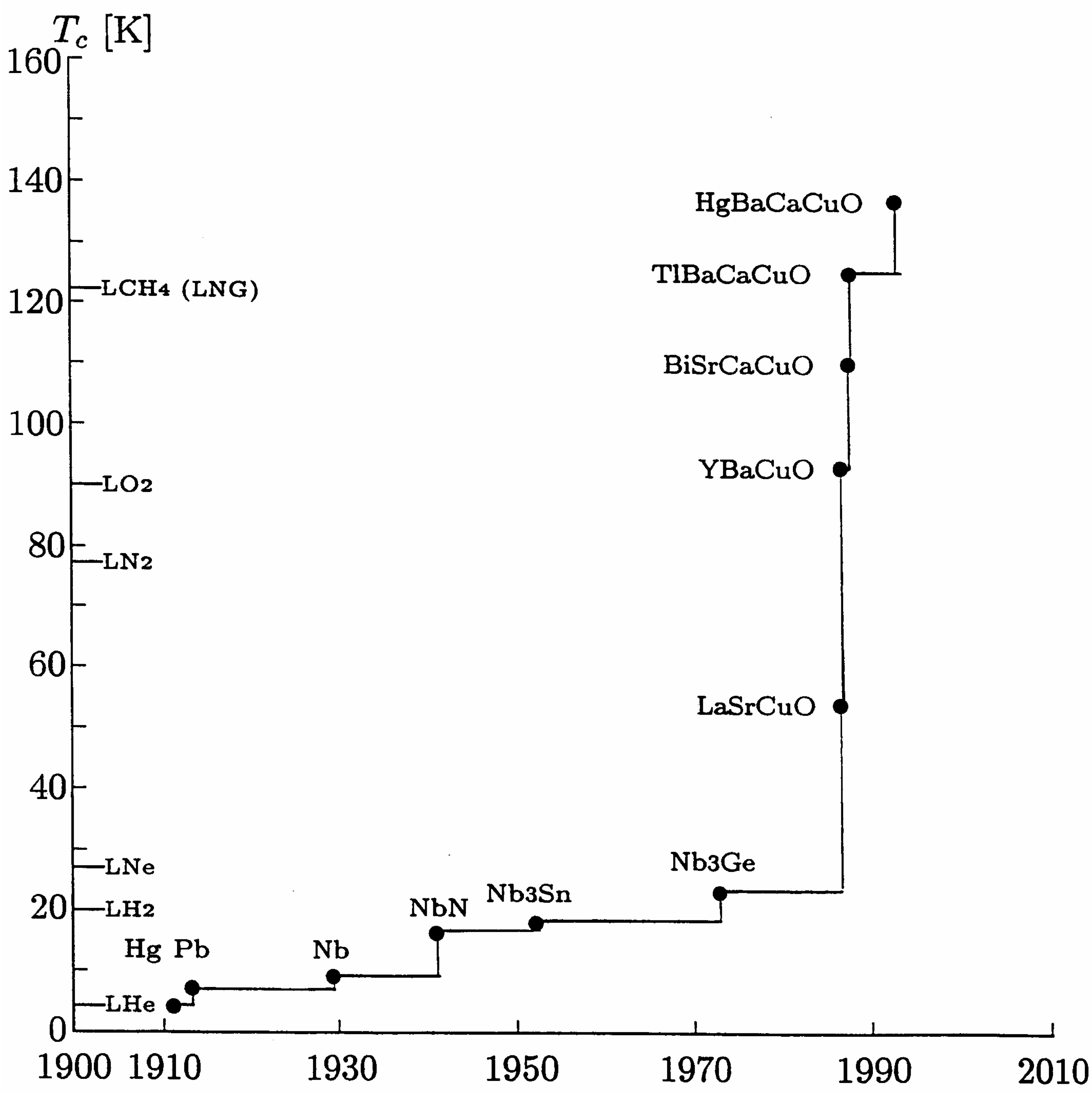


Why Superconductivity Discovered?

(continued)

- As a result of a long-sought desire to push T_c beyond 23.2 K, a stagnant limit since the 1970's, and even reach 77 K, the boiling point of liquid nitrogen.
  - In April 1986, J.G. Bednorz and K.A. Muller of IBM (Zurich) discovered La-Ba-Cu-O, a layered copper oxide perovskite, a superconductor with $T_c = 35$ K.
  - In 9187, P.W. Chu and others at U. of Houston and U. of Alabama discovered YBaCuO (Y-123) or YBCO), $T_c = 93$K, also a copper oxide perovskite.
  - In January 1988, H. Maeda, of the National Institute for Metals ("Kinzai-Ken"), Tsukuba, discovered BiSrCaCuO (BSCCO); now in two forms: Bi-2212 (Tc = 85 K); and Bi-2223 ($T_c =$ 110K).
  - In February 1988, Z.Z. Sheng and A.M. Hermann at U. of Arkansas discovered TlBaCaCuO (Tl-2223), $T_c \simeq 125$K.
  - In 1993, Chu discovered HgBaCaCuO (Hg-1223), $T_c \simeq 135$K (164 K under a pressure of 300 atm).
  - Since 1986 more than a hundred compounds of HTS have been discovered as well as USOs – Unidentified Superconducting Objects – "sighted".

Progress of T_c



Two “Flavors” of Superconductors

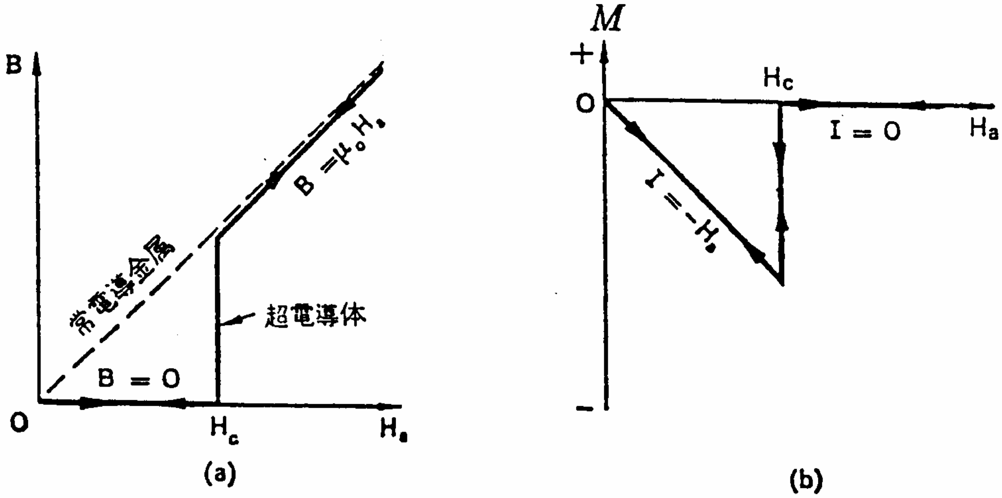
Type I

- Exhibits the Meissner effect; B= 0 beyond “penetration depth,”, δ (F. and H. London, 1935)

Selected Type I Superconductors

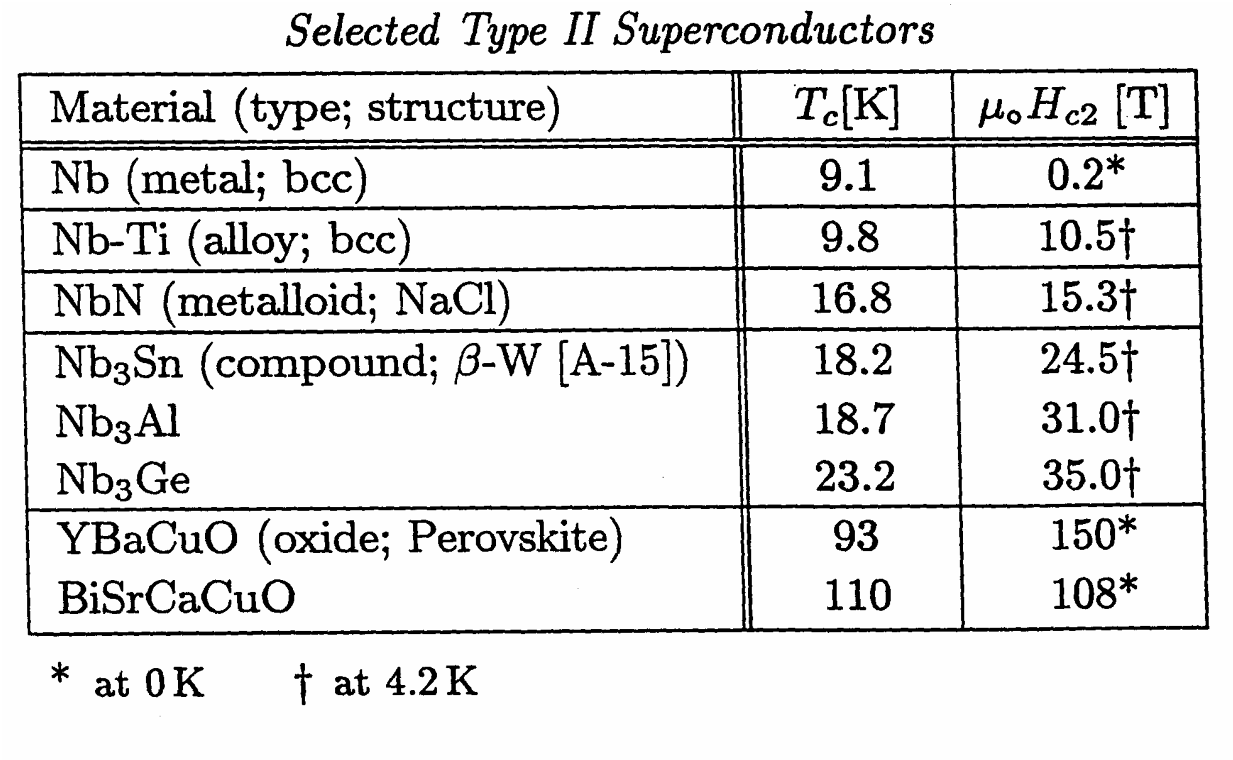
Material	T_c [K]	$\mu_o H_c$ [gauss]
Zn	0.9	53
Al	1.2	99
In	3.4	276
Sn	3.7	306
Hg	4.2	413
Ta	4.5	830
Pb	7.2	803

B vs H_a and M vs H_a Plots



Type II

- Exhibits the "mixed" magnetic state.
  - o Normal "islands" ("vortex") of size "coherence ξ length" in a sea of superconductivity: $R \simeq 0$.
  - o Each votex contains one quantum of magnetic flux, Φ_o, the collection of which is known as the Abrikosov vortex lattice. ($\Phi_o \equiv h/2e \simeq 2.0 \times 10^{-15}$ Tm^2.)
  - o $H_{c2} >> H_c$ and $\mu_o Hc2 \sim \Phi_o / \xi^2$.
- All high-temperature superconductors are Type II.

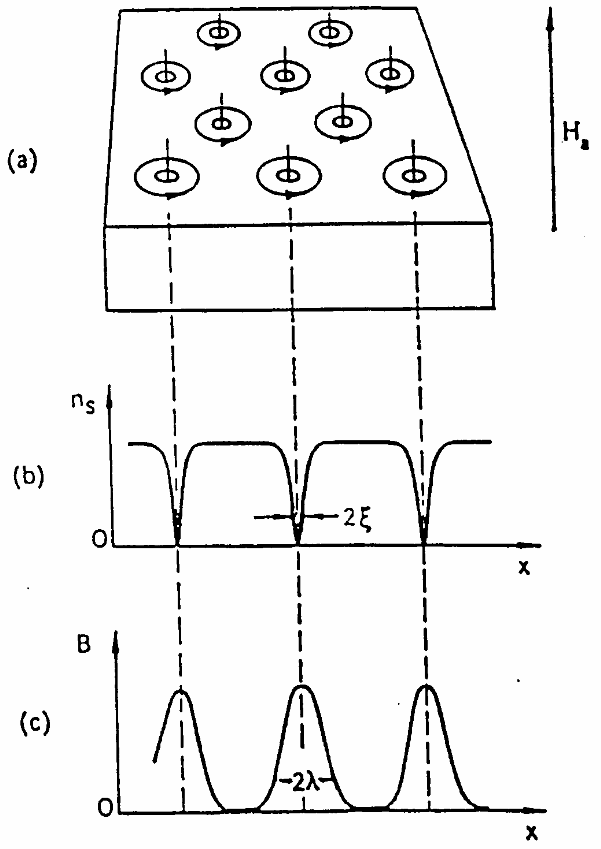


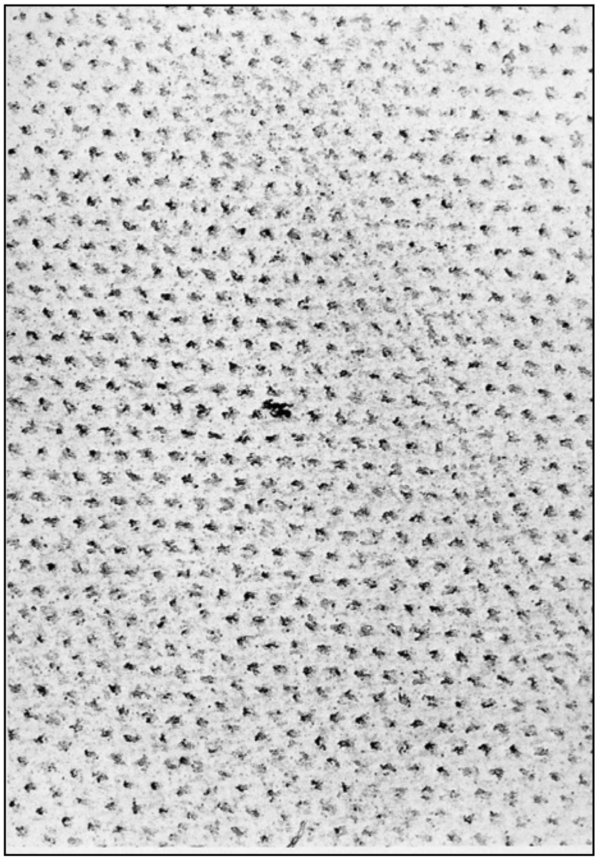
Selected Type II Superconductors

Material (type; structure)	T_c[K]	$\mu_o H_{c2}$ [T]
Nb (metal; bcc)	9.1	0.2*
Nb-Ti (alloy; bcc)	9.8	10.5†
NbN (metalloid; NaCl)	16.8	15.3†
Nb_3Sn (compound; β-W [A-15])	18.2	24.5†
Nb_3Al	18.7	31.0†
Nb_3Ge	23.2	35.0†
YBaCuO (oxide; Perovskite)	93	150*
BiSrCaCuO	110	108*

* at 0 K † at 4.2 K

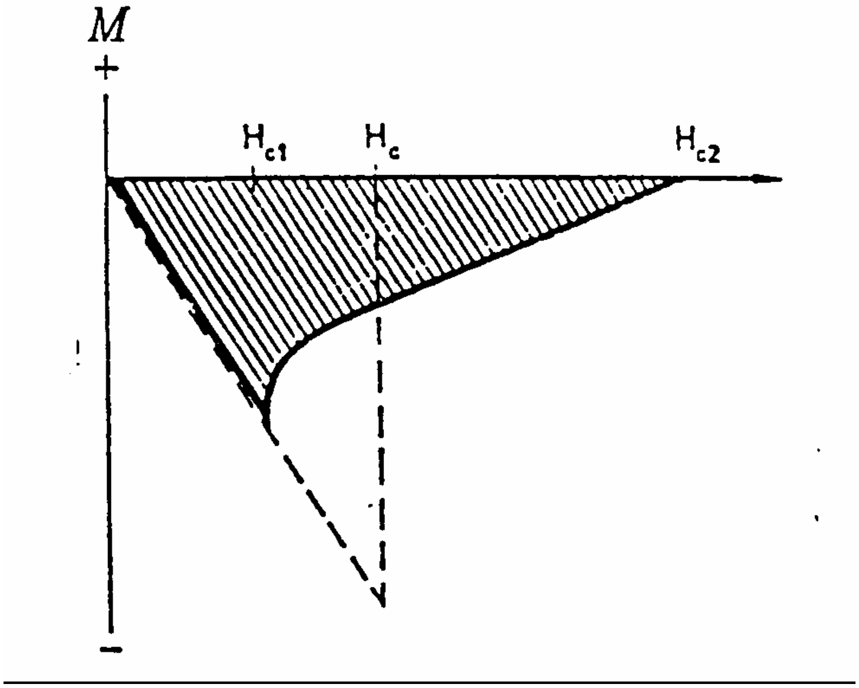
Schematics of Mixed State



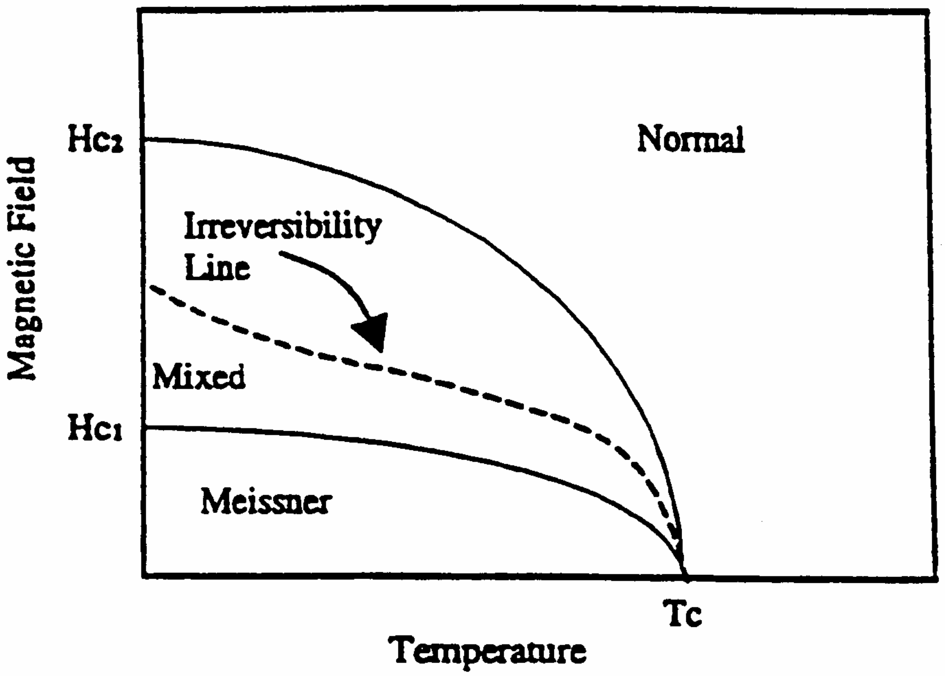


Superconducting electron density distribution: $\mathbf{n_s}$
Coherence length: ξ
Penetration depth: λ

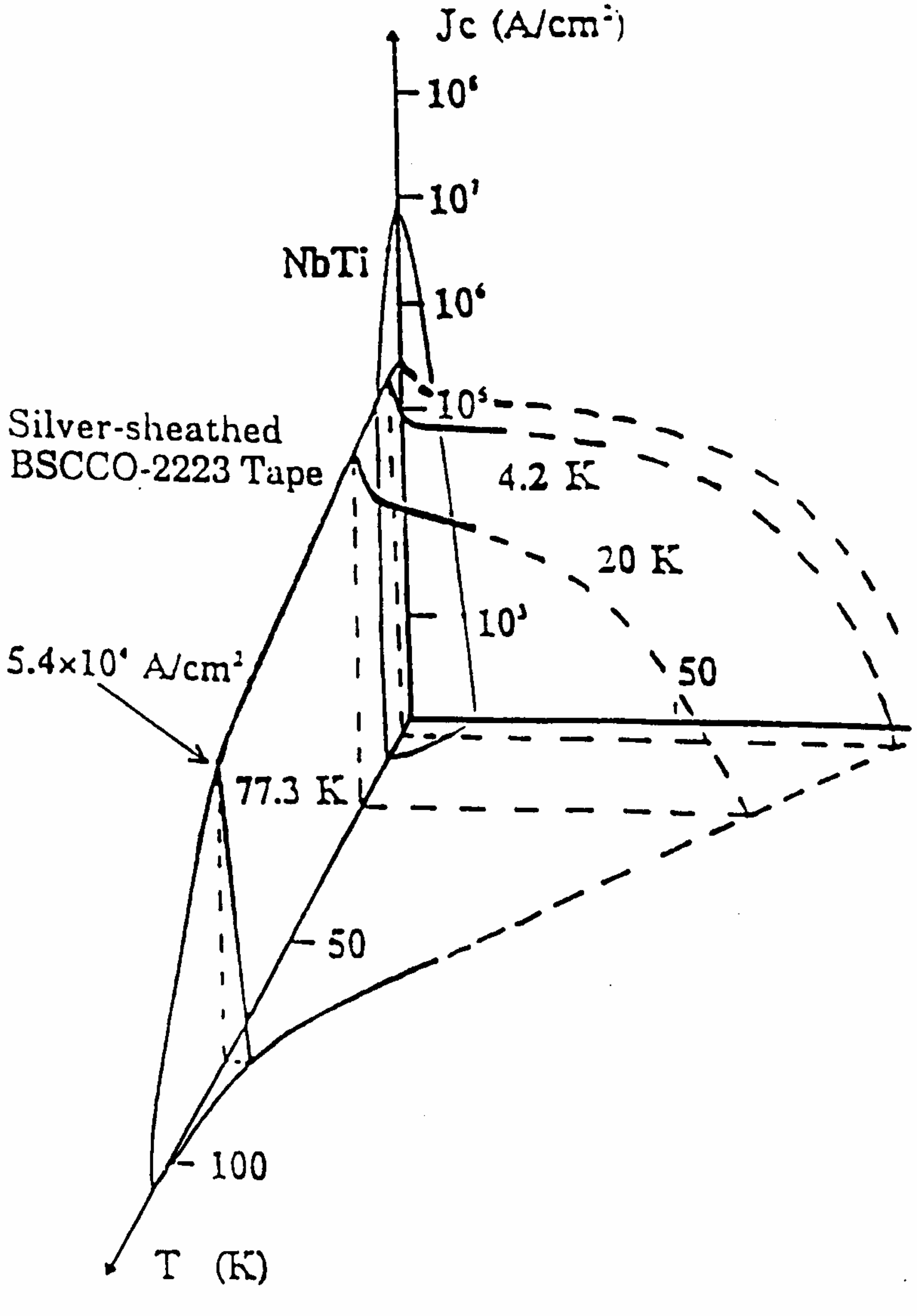
Magnetization Plot with the Mixed State



Magnetic Field vs Temperature Plots



Critical Surfaces



A Brief History
(•: science; ✱: technology)

1900s

✱ Liquefaction of helium ($T_s = 4.22$ K), by KO (1908).

1910s

✱ First liquid helium "cryostat" by KO (1911).

● Discovery by KO of Type I superconductors (1911).

✱ First SCM by KO failed (1913).

1930s

● First Type II superconductor, W. de Haas and J. Voogd.

● Meissner effect, by W. Meissner and R. Oschenfeld (1933).

● Electromagnetic theory ("penetration depth" λ), by F. and H. London (1935).

1940s

✱ First "large-scale" helium liquifier, by S. Collins (1946).

1950s

● "Coherence length" (ξ), introduced by A.B. Pippard.

● GLAG (Ginzburg, Landau, Abriskov, Gorkov) theory – magnetics of Type II superconductors $(\kappa \equiv \delta/\xi > 1/\sqrt{2})$.

● Many A-15 (metallic superconductors, chiefly in the U.S.

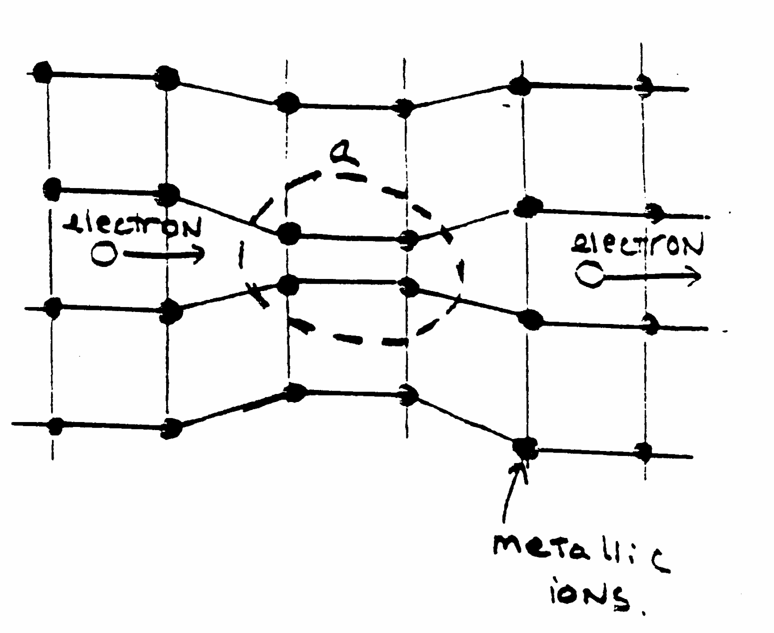
● Cooper pair, by L.N. Cooper (1956).

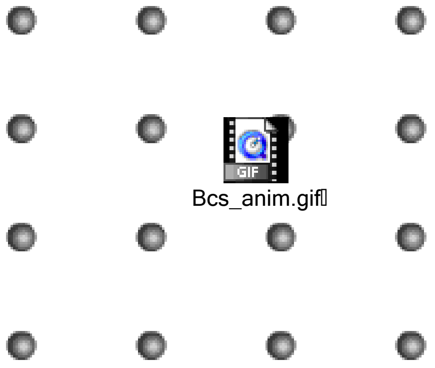
A Brief History (continued)

1950s

- BCS (Bardeen, Cooper, Schreiffer) theory – microscopic theory of superconductivity (1957).
- Flux quantization.

✳ “Toy” superconducting magnets (SCM) (1956).





A Brief History (continued)

1960s

✳ High-field, high-current superconductors (1961) - “pinning” of the “islands” (fluxoids).
● Josephson tunneling, B.D. Josephson (1962).
✳ Birth of superconducting magnet technology (mid-1960s).
✳ Start of large SCM for research – MHD, HEP (R&D).

1970s

✳ Maglev (“linear motor”)
✳ Superconducting generators; transmission lines (R&D).
✳ Commercial NMR magnets.
✳ Fusion and particle accelerator magnets.

1980s

✳ Commercial MRI magnets.
● Discovery of HTS.

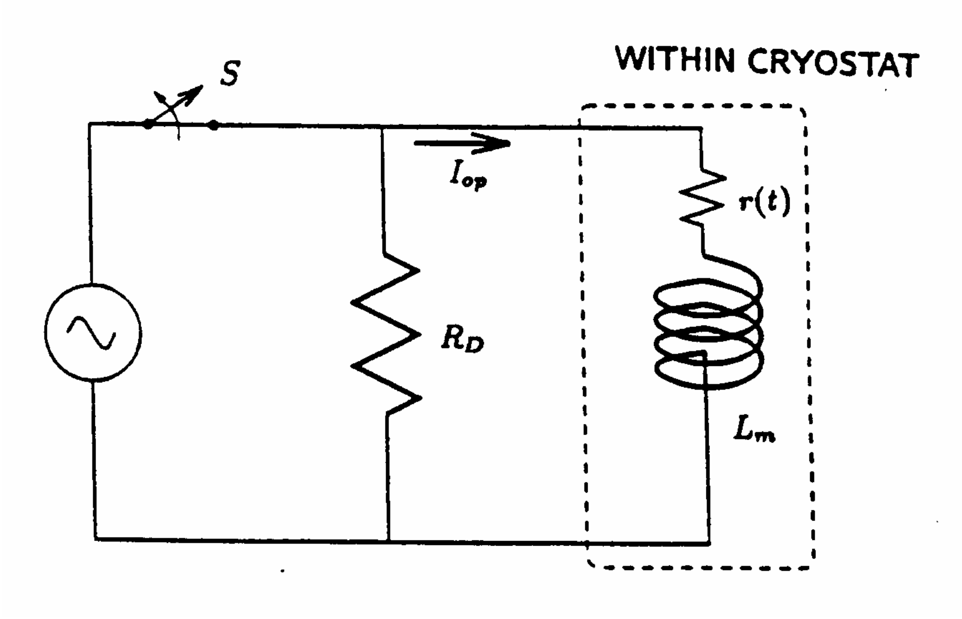
Pros and Cons of Superconductors

Positive Aspects

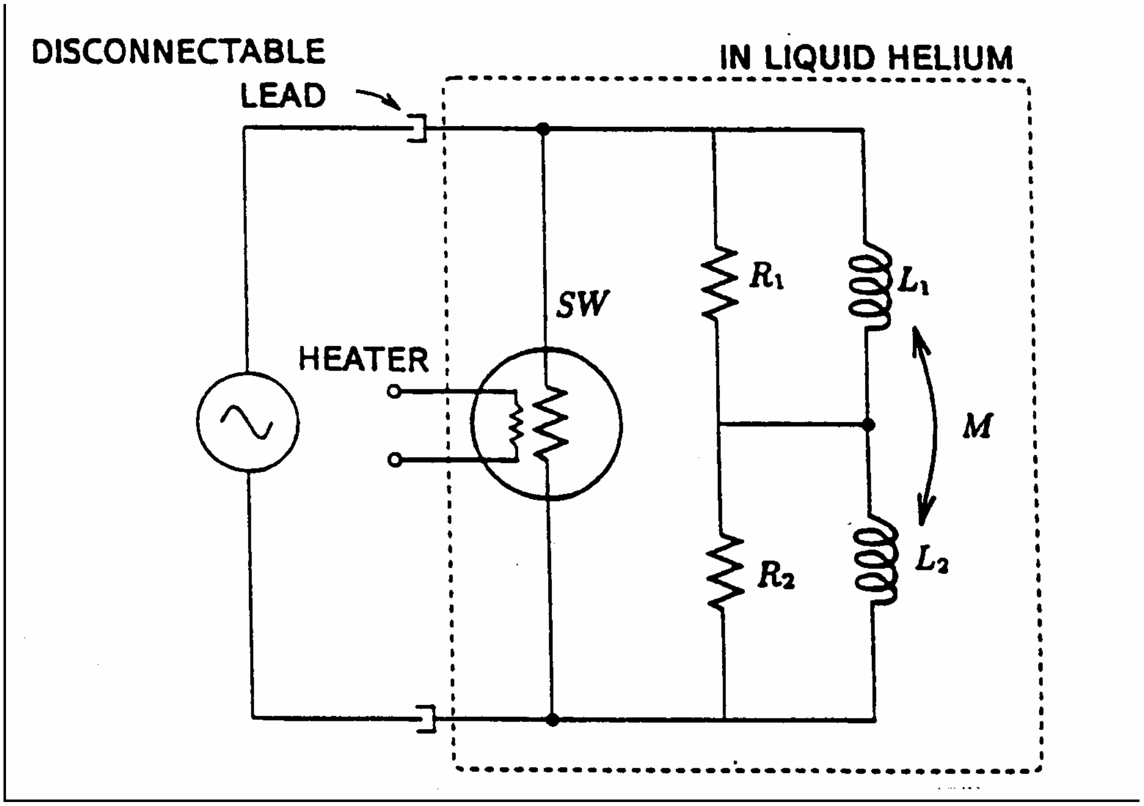
- **R = 0 under DC conditions**
  - **Can generate a large magnetic field.**
    Dissipation = $I^2R = 0$.
  - **Can generate a large magnetic field over large volumes.**
  - **Can generate a "persistent" magnetic field.**

$$\frac{dB}{dt} = 0$$

Driven System ($I^2R=0$)



Persistent-Mode System ($dB/dt = 0$)



Pros and Cons of Superconductors

Negative Aspects

- **T_c << room temperature.**
  - **Require refrigeration and good thermal insulation.**

- **Mixed state (Type II).**
  - **$R \neq 0$, under time-varying conditions.**

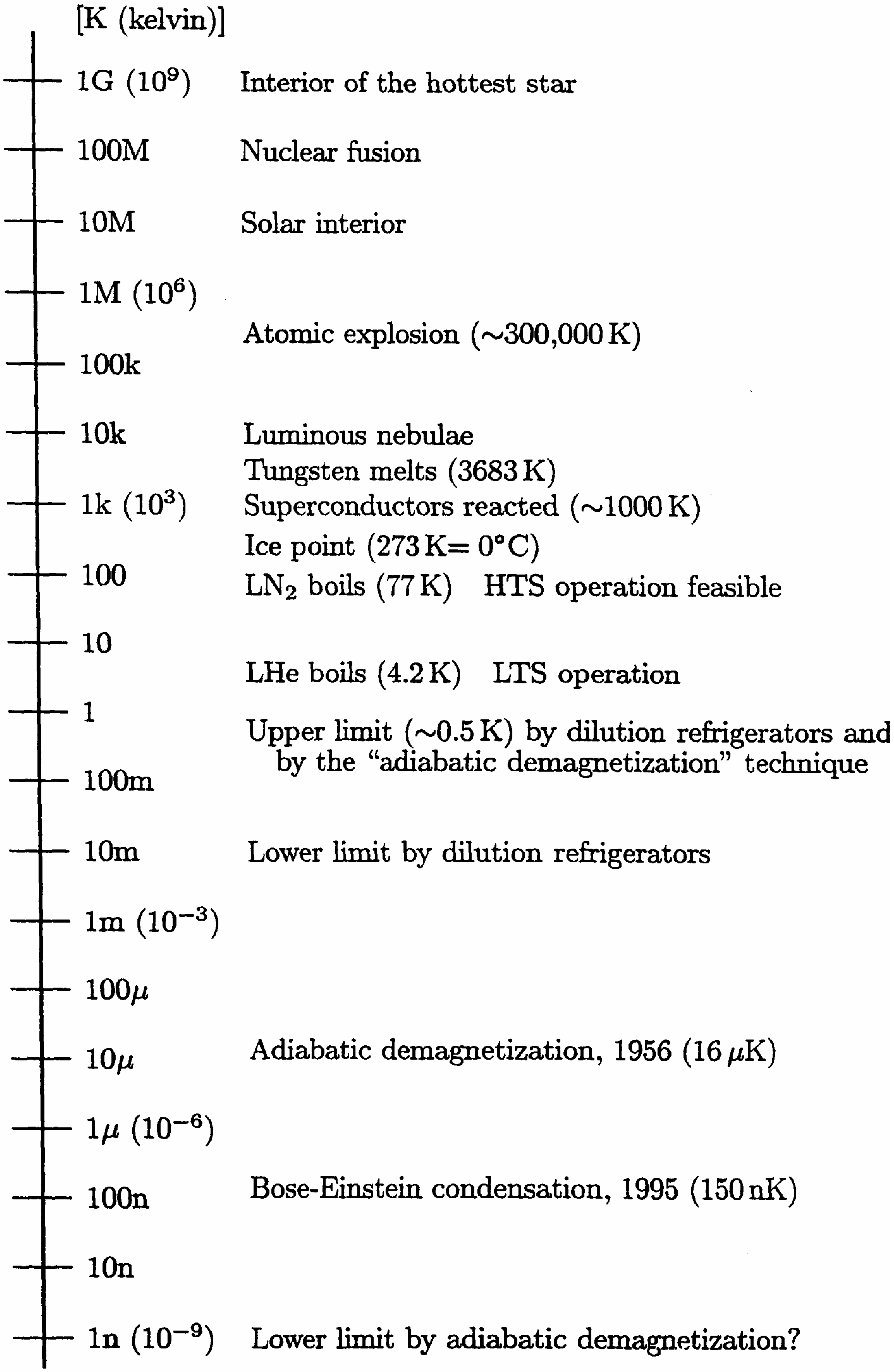
- **Expensive vs copper, aluminum, steel, organic materials.**
  - **100-1000 times more than copper.**

Magnetic Field Spectrum

[T (tesla)]

1G (10^9) Neutron stars
1k (10^3) Upper limit (?) of man-made fields
"Explosion" pulse fields ($\sim$500 T) lasting ≤ 0.000001 s
100 Pulse fields (80 T) lasting $\sim$0.00001–0.001 s
Highest "DC" fields (40 T); SCM ($\sim$25 T) for NMR
10 SCM for thermonuclear reactors ($\sim$10 T)
SCM for maglev, generators, motors ($\sim$5 T)
1 SCM for MRI ($\sim$1 T)
Upper limit by permanent magnets ($\sim$0.3 T)
100m
10m (100 gauss) Toy magnets
Safe limits ($\sim$50 gauss) for floppy disks, pace-makers
1m (10 gauss)
FDA limits (5 gauss) for "environmental" fields
100μ (1 gauss)
Earth field ($\sim$0.7 gauss)
10μ
1μ (10^{-6})
100n
10n
1n (10^{-9}) NDT for microcracks
100p "Heart" field
10p
1p (10^{-12})
"Brain" field ($\sim$500 fT)
100f
10f
1f (10^{-15}) Lower limit of SQUID sensitivity

Temperature Spectrum



[K (kelvin)]	
1G (10^9)	Interior of the hottest star
100M	Nuclear fusion
10M	Solar interior
1M (10^6)	
	Atomic explosion (~300,000 K)
100k	
10k	Luminous nebulae
	Tungsten melts (3683 K)
1k (10^3)	Superconductors reacted (~1000 K)
	Ice point (273 K= 0°C)
100	LN_2 boils (77 K) HTS operation feasible
10	
	LHe boils (4.2 K) LTS operation
1	
	Upper limit (~0.5 K) by dilution refrigerators and by the "adiabatic demagnetization" technique
100m	
10m	Lower limit by dilution refrigerators
1m (10^{-3})	
100μ	
10μ	Adiabatic demagnetization, 1956 (16 μK)
1μ (10^{-6})	
100n	Bose-Einstein condensation, 1995 (150 nK)
10n	
1n (10^{-9})	Lower limit by adiabatic demagnetization?

Applications of Superconductivity

Energy

- Generation & Storage
  - Fusion; Generators; SMES; Flywheel
- Transmission & Distribution
  - Power Cable; Transformer; FCL
- End Use
  - Motor

Transportation

- Maglev

Medicine

- MRI; NMR; SQUID (biomagnetism); Magnetic Steering; Biological Separations

Space & Ocean

- Sensors; SQUID; Undersea Cables; Maglifter

High Tech

- Magnetic Bearings; SOR; Magnetic Separation

Information/Communication

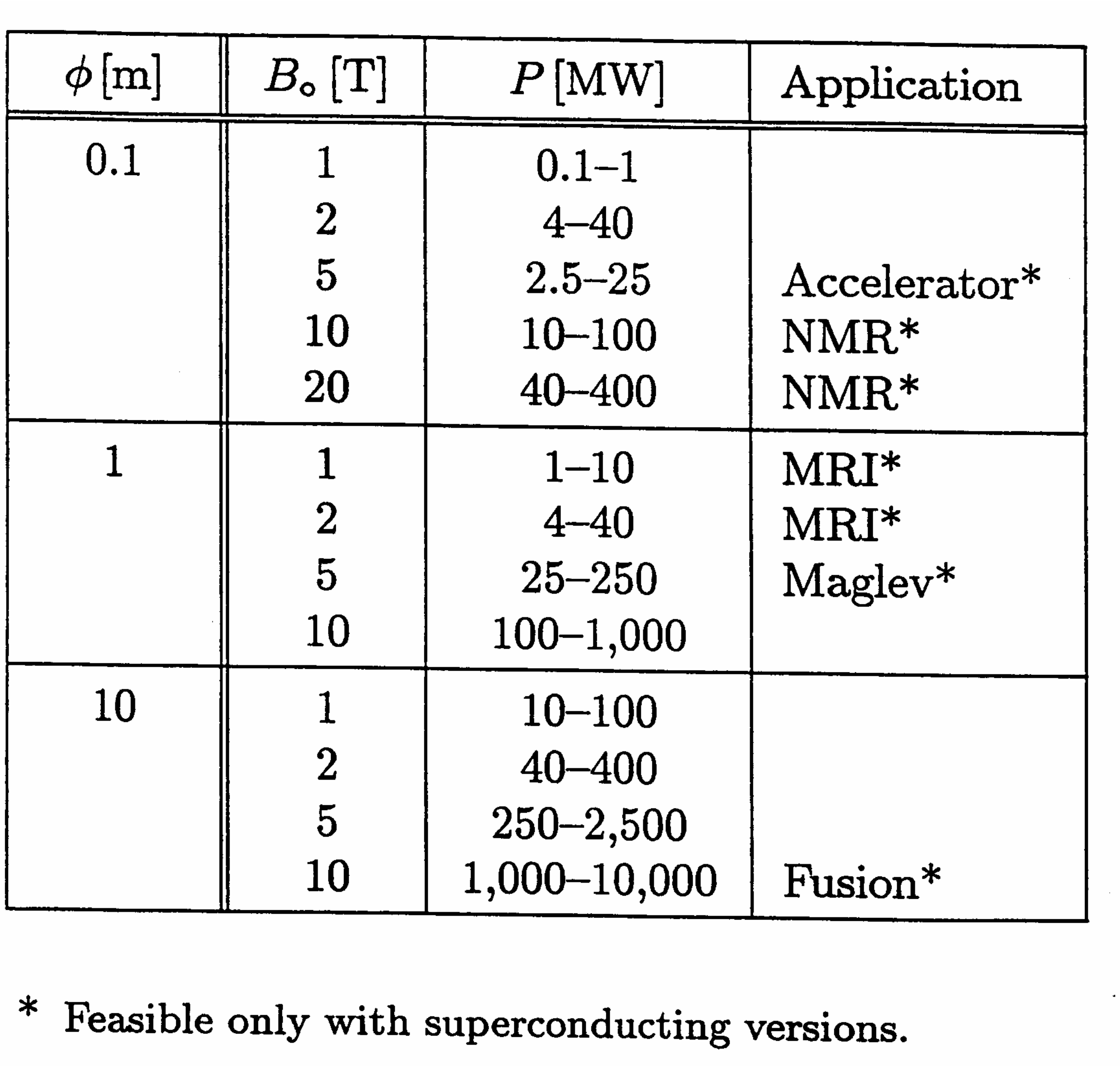
- Electronics; Filters

Research

- NMR; HEP Accelerators; High-Field Magnets, Proton Radiography

Power Requirements for Copper Solenoids (at Room Temperature)

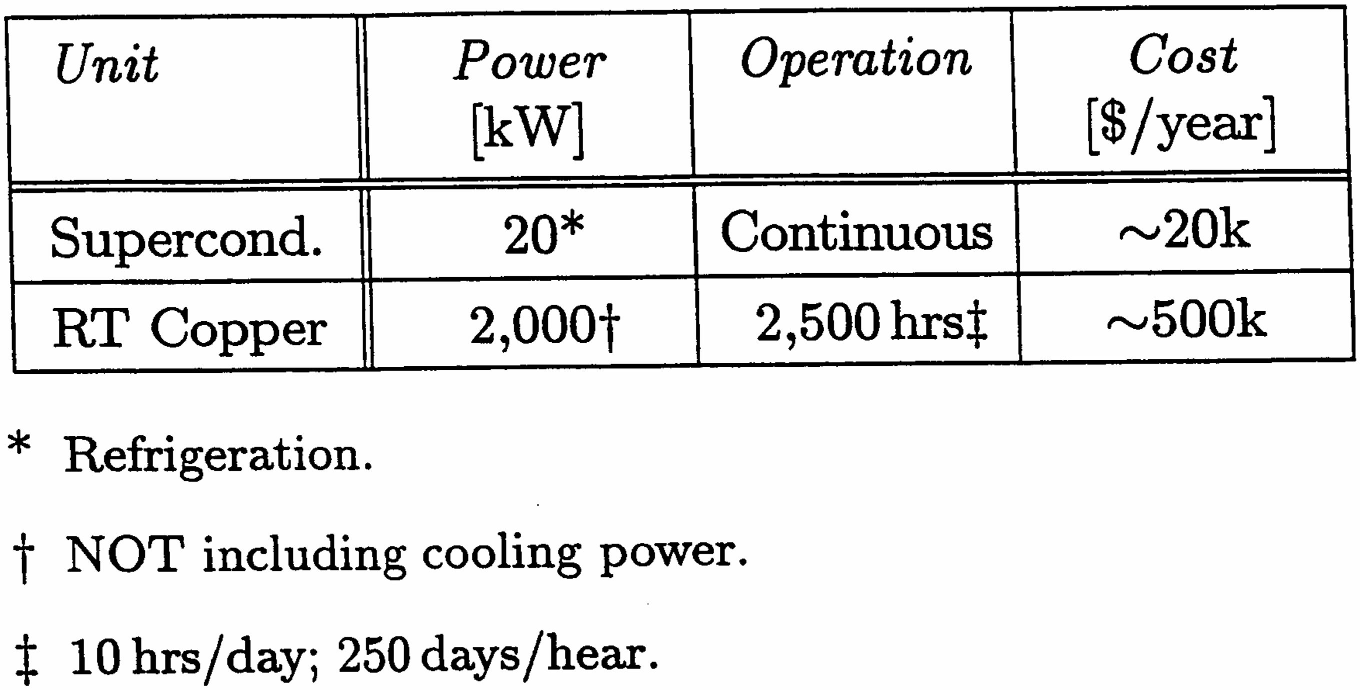
- **Power Required $\propto$ Resistivity x Diameter x Field^2**



ϕ [m]	$B_\circ$ [T]	P [MW]	Application
0.1	1	0.1–1	
	2	4–40	
	5	2.5–25	Accelerator*
	10	10–100	NMR*
	20	40–400	NMR*
1	1	1–10	MRI*
	2	4–40	MRI*
	5	25–250	Maglev*
	10	100–1,000	
10	1	10–100	
	2	40–400	
	5	250–2,500	
	10	1,000–10,000	Fusion*

* Feasible only with superconducting versions.

For 1-T Whole-Body MRI Units: Superconducting vs Room-Temp Copper



Unit	*Power* [kW]	*Operation*	*Cost* [$/year]
Supercond.	20*	Continuous	~20k
RT Copper	2,000†	2,500 hrs‡	~500k

* Refrigeration.

† NOT including cooling power.

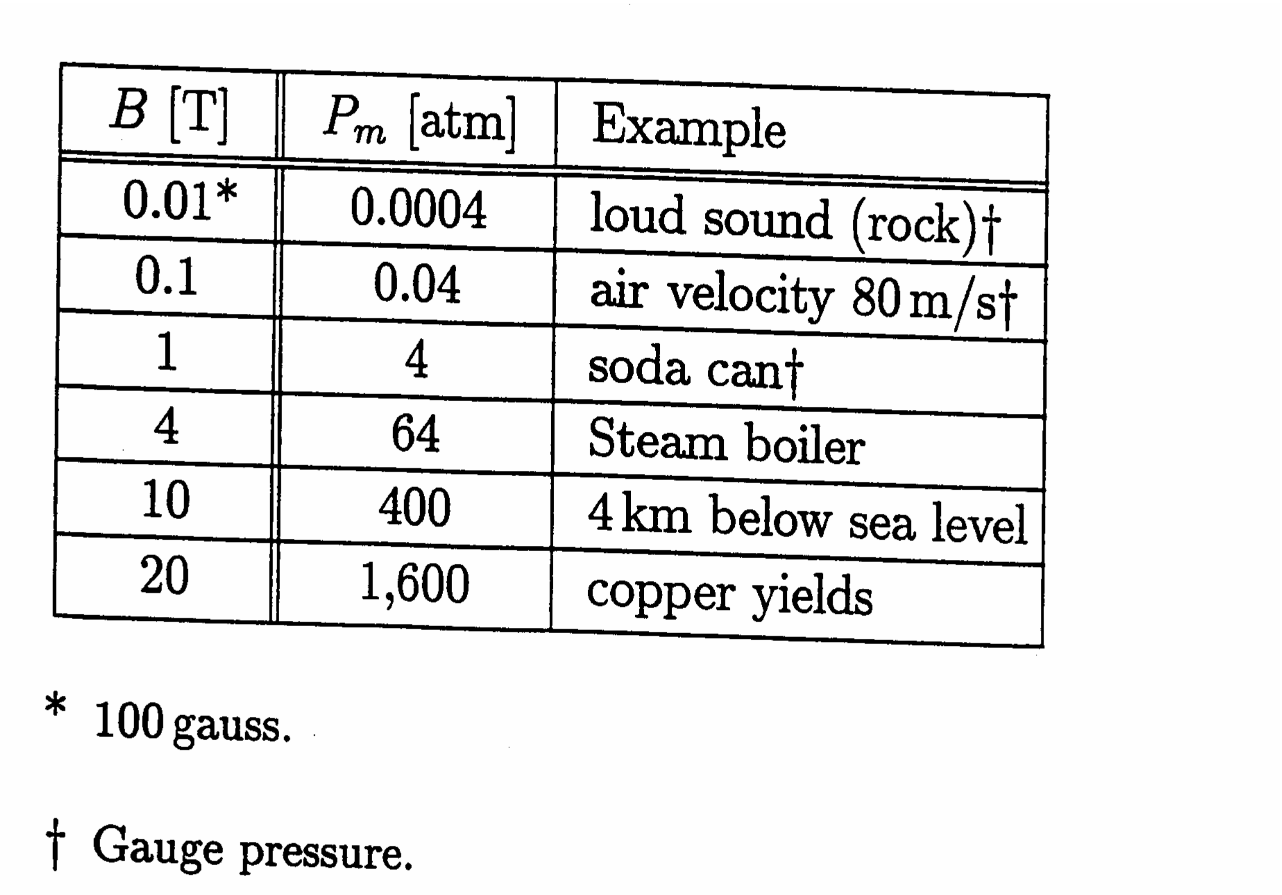
‡ 10 hrs/day; 250 days/hear.

Magnetic Pressure, Density, Force

Magnetic Pressure

$$P_m = \frac{B^2}{2\mu_o} \quad \text{[N/m}^2\text{]}$$

Where $\mu_o = 4\pi \times 10^{-7}$ H/m and B in tesla.



B [T]	P_m [atm]	Example
0.01*	0.0004	loud sound (rock)†
0.1	0.04	air velocity 80 m/s†
1	4	soda can†
4	64	Steam boiler
10	400	4 km below sea level
20	1,600	copper yields

* 100 gauss.

† Gauge pressure.

Fusion

The thermal or kinetic pressure, p_k, of the plasma must be confined with the magnetic pressure, p_m, exerted on the plasma by a magnetic field. The magnetic confinement of hot plasma requires that

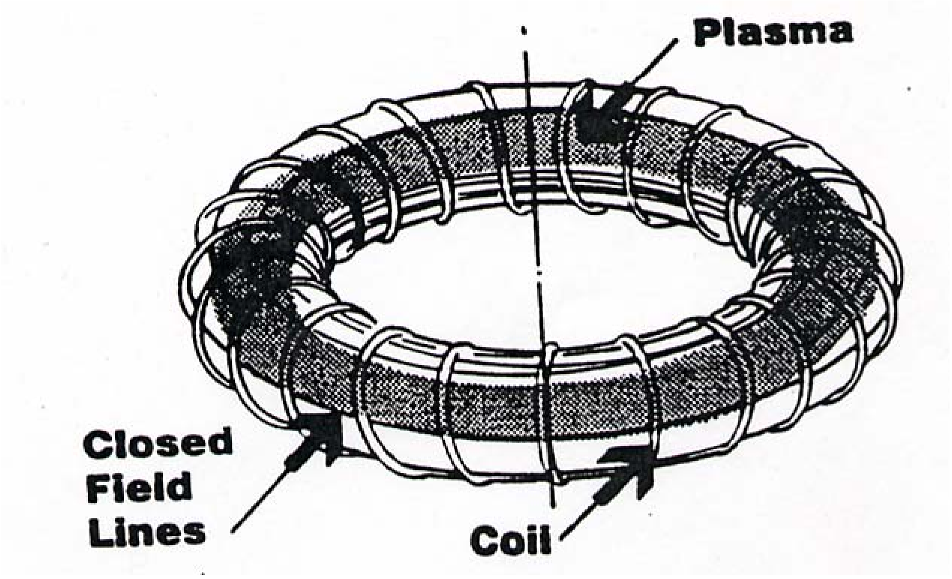
$$p_k = n_p k_b T_p << p_m = \frac{B_f^2}{2\mu_o}$$

$$n_p k_b T = \beta \frac{B^2}{2\mu_o}$$

$$B = \sqrt{\frac{2\mu_o n_p k_b T_p}{\beta}}$$

With $n_p = 10^{21}$ m^{-3}, $T = 10^8$K, $k_B = 1.38 \times 10^{-23}$ J/K and $\beta = 0.05$,

$B \simeq$ 8.3 T (~ 10 T)
P_m = 400 atm (10 T)
P_{sun} ~ 3 × 10^{11} atm ~ 3 × 10^5 T)



Maglev

$$\mathbf{P_m = 4\ atm\ (\sim 1\ T)}$$

Other support pressures [atm]:

Shoes:	0.1 - 0.3
Bicycle Tires:	4 – 6
High-speed train steel wheels:	5,000

Magnetic Energy Density

$$E_m = \frac{B^2}{2\mu_o}[J/m^3]$$

SMES (Superconducting Magnetic Energy Storage)
B limited to ~ 5 T for practical considerations.

Illustration:
$B = 5\ \mathrm{T} \rightarrow e_m = 2 \times 10^7\ \mathrm{J/m^3}$.

Boston Edison has a peak-power demand, $\Delta P_{pk\text{-}av}$, on a hot summer day of typically ~ 2,500 MW lasting (Δt) of ~10 h. Namely,

$$\Delta E_{pk} \sim 2.5 \times 10^9\mathrm{W} \times 10\ \mathrm{h} \times 3{,}600\ \mathrm{s/h}$$
$$\sim 10^{14}\ \mathrm{J}$$

Because $V_{arena} \sim 1 \times 10^6\ \mathrm{m^3}$, the number of SMES units each the size of a large arena storing a magnetic energy of $E_{arena} \sim 2 \times 10^{13}$ J:

$$N_{SMES} \sim 4 \text{ Arenas}$$

Magnetic (Lorentz) Force

$$F_{Lorentz} = (length) \times I \times B \text{ [Newtons]}$$

Electrical Devices: Generators and motors.

B limited to ~ 5 T for practical considerations.

Magnetic Force on Elementary Particles

High-Energy Physics Accelerators

$$\text{Centrifugal force} \equiv \vec{F}_{cf} = \frac{M_p v^2}{R_a}\vec{i}_r \cong \frac{M_p c^2}{R_a}\vec{i}_r = \frac{E_p}{R_a}\vec{i}_r$$

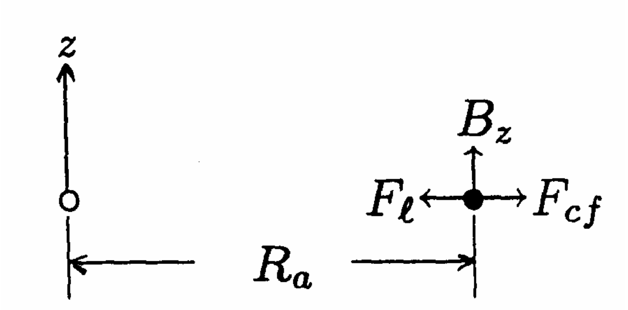
$$\text{Centripetal force} = \text{Lorentz force} \equiv \vec{F}_l = -qcB_z\vec{i}_r$$

$$\frac{E_p}{R_a} = qcB_z$$

$$R_a = \frac{E_p}{qcB_z}$$

With $E_p = 20$ TeV (3.2 μJ), $q = 1.6 \times 10^{-19}$ C, $c = 3 \times 10^{8}$ m/s, and $B_z = 5$ T,

Ra ~ 13 km or a diameter exceeding 25 km.



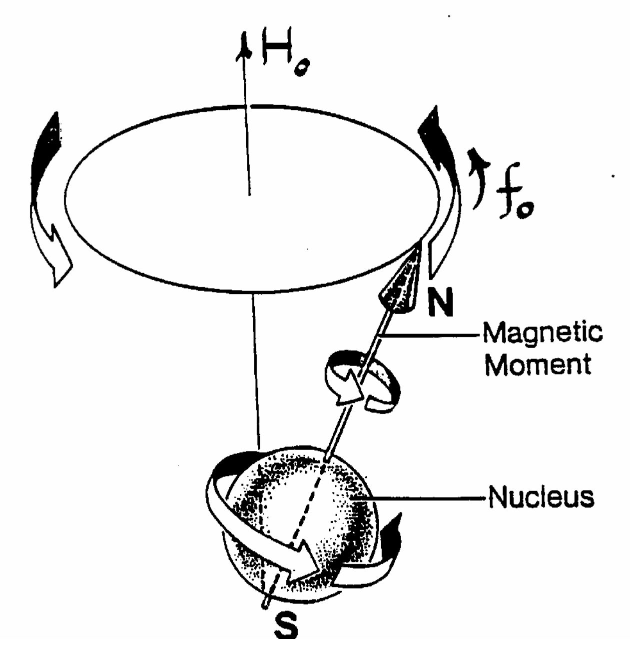
Magnetic Torque on Nuclei

Nuclear Magnetic Resonance (NMR)
and
Magnetic Resonance Imaging (MRI)

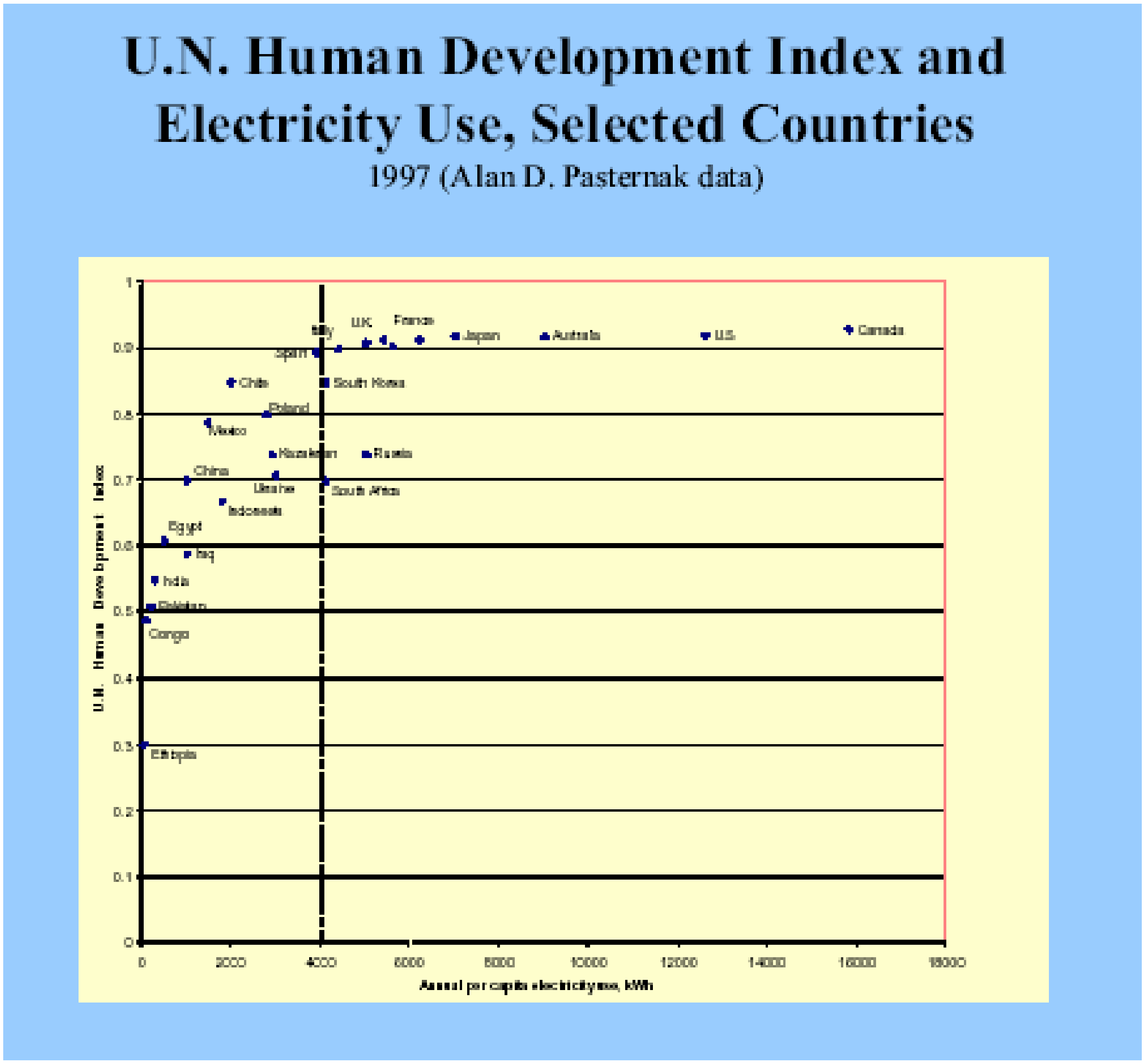
$$\gamma\vec{\mu}\times\vec{H}_o = \frac{d\vec{\mu}}{dt}$$

Larmor frequency ≡ $f_o = \gamma H_o$

Where γ is a gyromagnetic ratio.
For hydrogen (proton): f_o = 42.58 MHz/tesla.

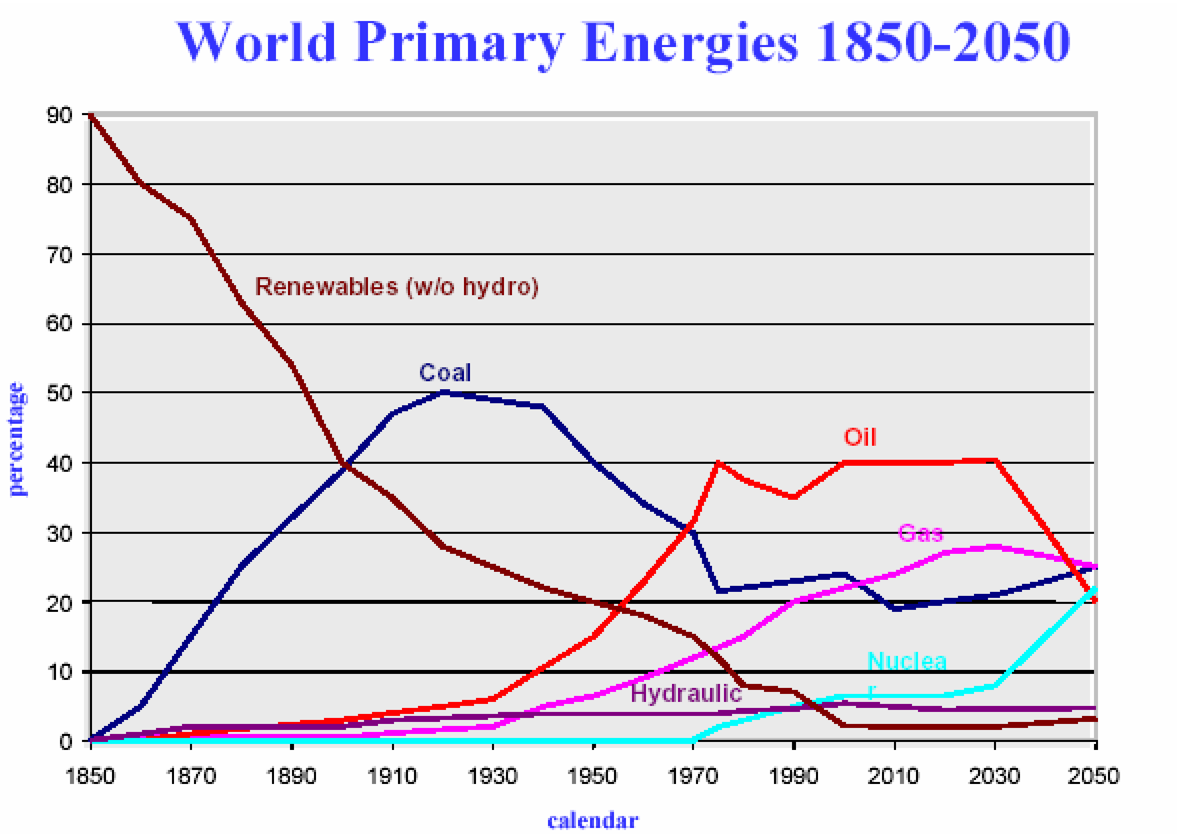


Applications to Electricity

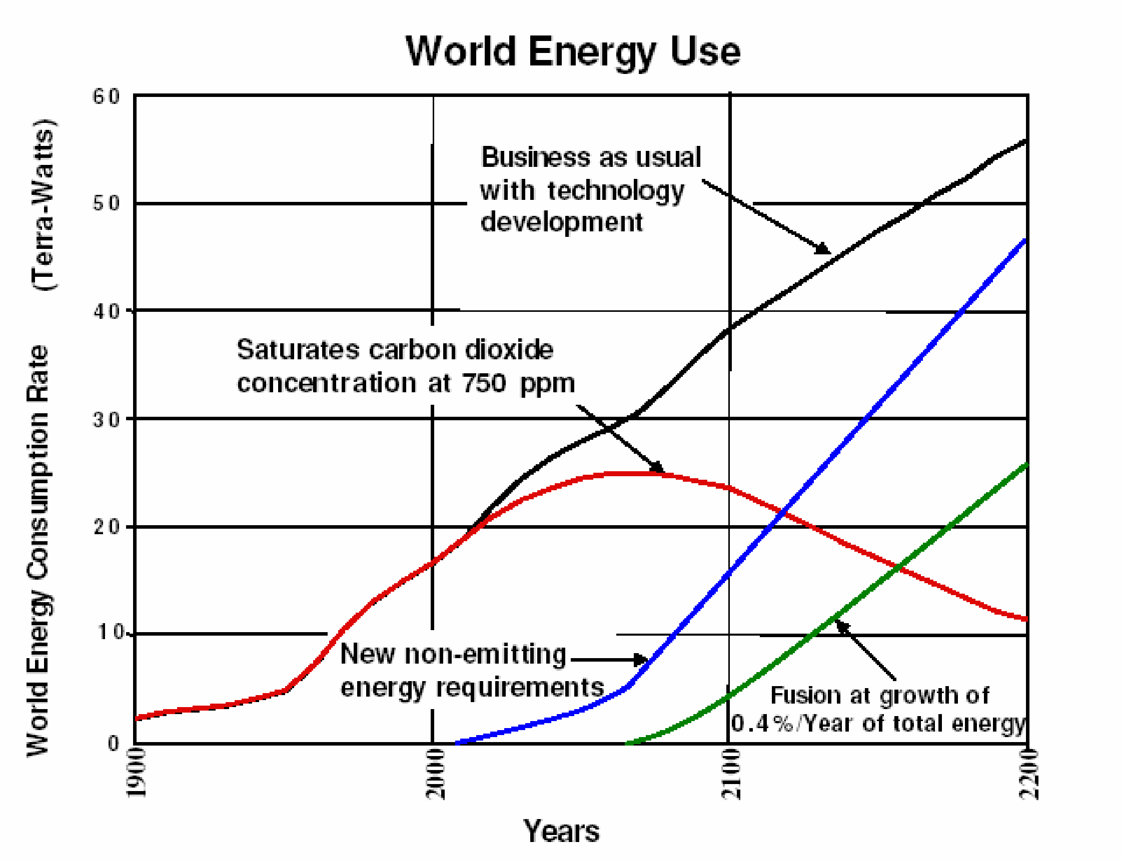


U.N. Human Development Index and Electricity Use, Selected Countries

1997 (Alan D. Pasternak data)

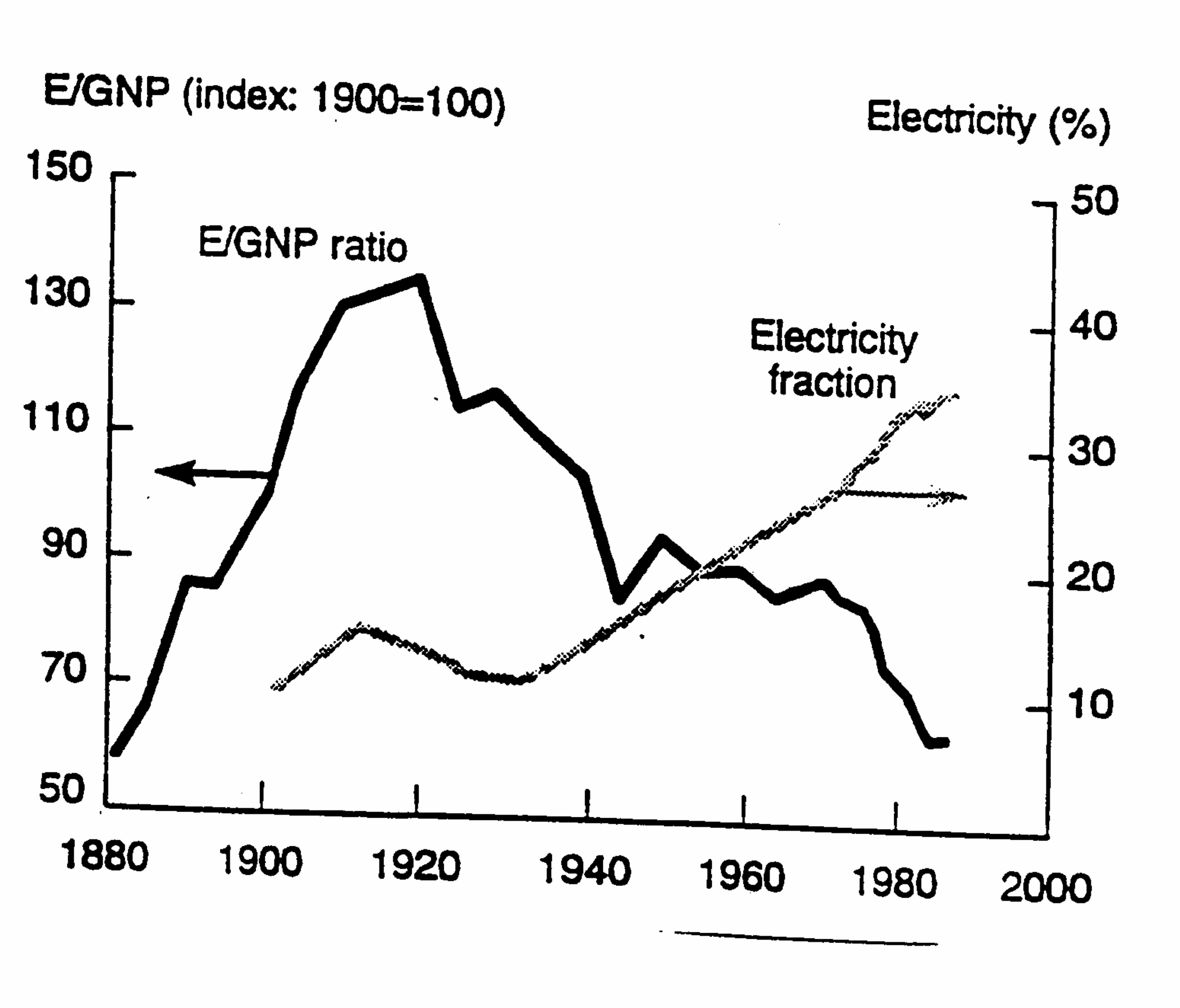


World Primary Energies 1850-2050



World Energy Use

Applications to Electricity



E/GNP ratio, energy consumption/GNP vs year
Electricity fraction, electric energy/total energy vs year
Both for the U.S.

Prospects – February 2003

Application	’79–’88	’89–’98	’99–’09	’10⟶
Fusion	↗	↗	⤨*	↗
Generator	↗	↗	⤨	↗
SMES	↗	↗	⤨	?
Flywheel	—	↗	⤨	?
Cable	→	↗	⤨	?
Transformer	—	↗	⤨	?
FCL	→	↗	⤨	?
Motor	—	↗	⤨	↗
Transportation	↗	↗	↗	?
NMR	⇗	⇗	⇗	↗↗
MRI	⇗	⇗	⇗	↗↗

↗ Upbeat; in the R&D phase.

↘ Downbeat; R&D will probably continue.

⤨ Upbeat but struggling.

⟶ Dormant.

⇗ Commercially successful.

↗↗

Enabling vs Replacing

Technology	***Feature***	***Competitor***	***Criterion***
Enabling	**Yes**	**No**	**Feature**
Replacing	**No**	**Yes**	**Cost**

Does Superconductivity Make A Technology Enabling?

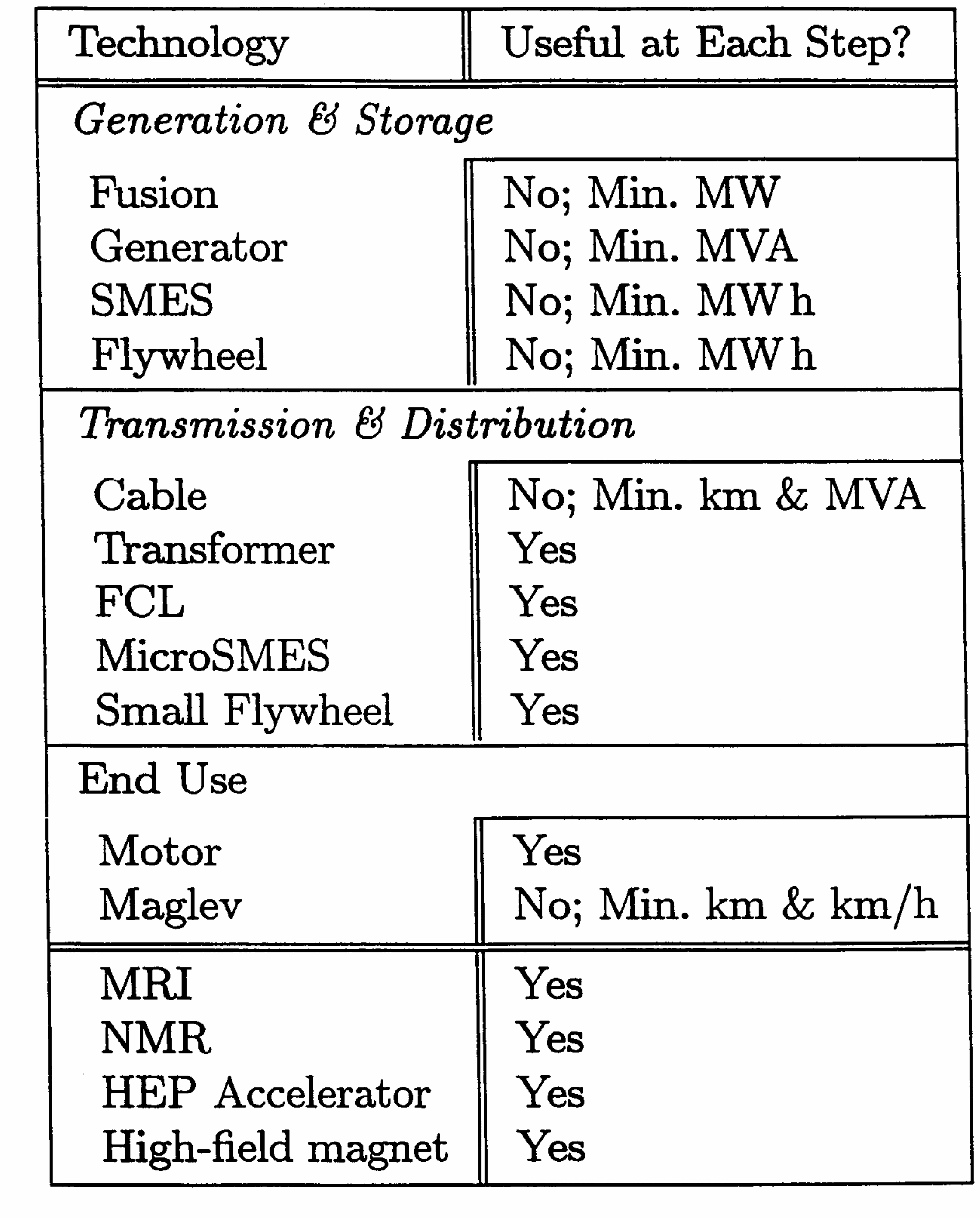
Technology	In General	Yes, but Under...
Generation & Storage		
Fusion	Yes	>2050 AD
Generator	No	Large power
SMES	(Yes)*	Large energy
Flywheel	No	Large energy
Transmission & Distribution		
Cable	No	Large power†
Transformer	No	Compact unit
FCL	No	Compact unit
MicroSMES	(Yes)*	Compact unit
Small Flywheel	No	Compact unit
End Use		
Motor	No	Large power
Maglev	No	High speed
MRI	Yes	>0.5 T
NMR	Yes	>2 T
HEP Accelerator	Yes	>2 T
High-field magnet	Yes	>2 T

* Not enabling as an energy storage device.

† And limited space, *e.g.*, underground facilities.

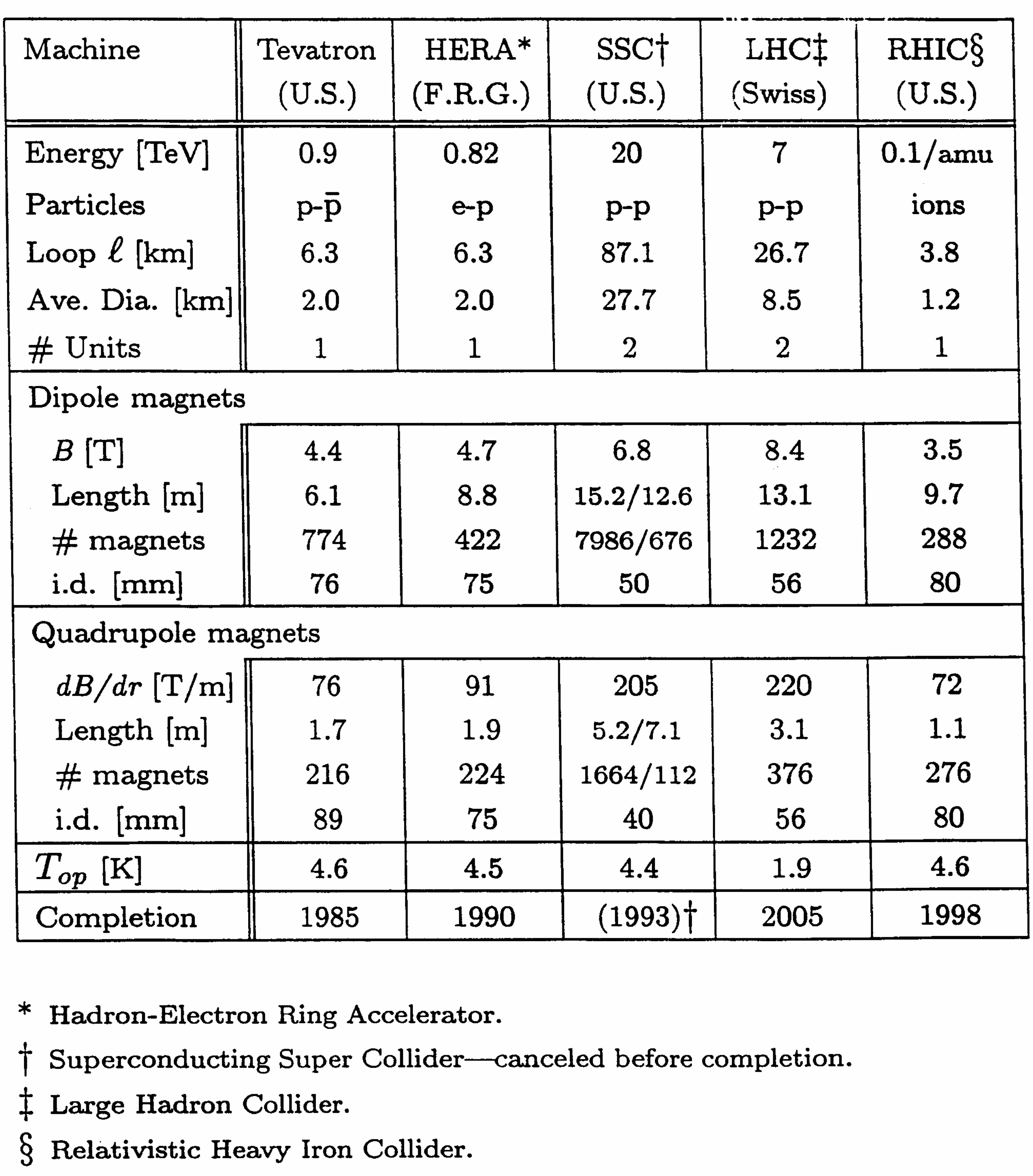
Stages of Development Towards a Commercial Product

- Step-by-step progression from low to high grade.
- Highly beneficial if the device is useful *at each step*.



Technology	Useful at Each Step?
Generation & Storage	
Fusion	No; Min. MW
Generator	No; Min. MVA
SMES	No; Min. MW h
Flywheel	No; Min. MW h
Transmission & Distribution	
Cable	No; Min. km & MVA
Transformer	Yes
FCL	Yes
MicroSMES	Yes
Small Flywheel	Yes
End Use	
Motor	Yes
Maglev	No; Min. km & km/h
MRI	Yes
NMR	Yes
HEP Accelerator	Yes
High-field magnet	Yes

Superconducting (NbTi) Particle Accelerators



Machine	Tevatron (U.S.)	HERA* (F.R.G.)	SSC† (U.S.)	LHC‡ (Swiss)	RHIC§ (U.S.)
Energy [TeV]	0.9	0.82	20	7	0.1/amu
Particles	p-$\bar{p}$	e-p	p-p	p-p	ions
Loop ℓ [km]	6.3	6.3	87.1	26.7	3.8
Ave. Dia. [km]	2.0	2.0	27.7	8.5	1.2
# Units	1	1	2	2	1
Dipole magnets					
B [T]	4.4	4.7	6.8	8.4	3.5
Length [m]	6.1	8.8	15.2/12.6	13.1	9.7
# magnets	774	422	7986/676	1232	288
i.d. [mm]	76	75	50	56	80
Quadrupole magnets					
dB/dr [T/m]	76	91	205	220	72
Length [m]	1.7	1.9	5.2/7.1	3.1	1.1
# magnets	216	224	1664/112	376	276
i.d. [mm]	89	75	40	56	80
T_{op} [K]	4.6	4.5	4.4	1.9	4.6
Completion	1985	1990	(1993)†	2005	1998

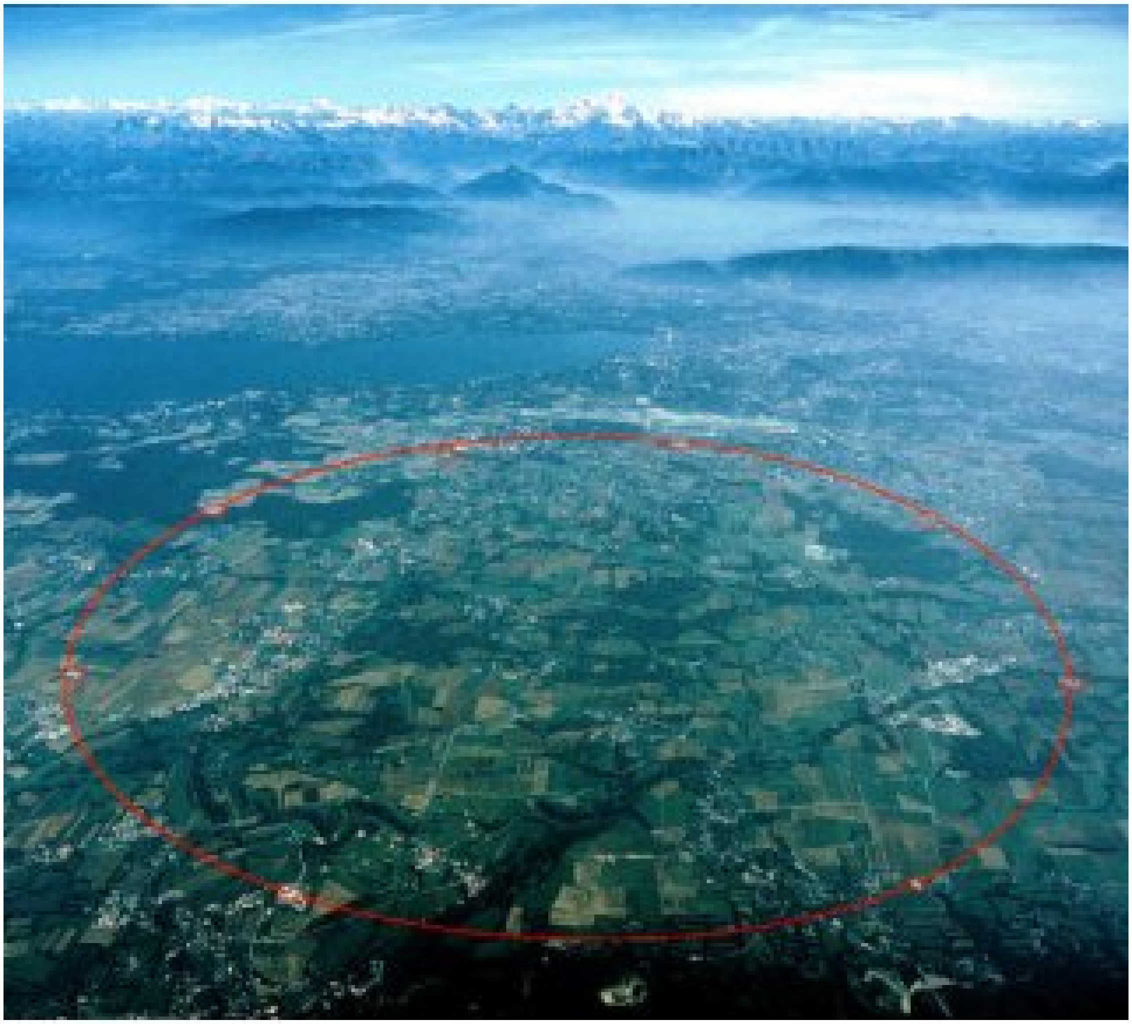
* Hadron-Electron Ring Accelerator.

† Superconducting Super Collider—canceled before completion.

‡ Large Hadron Collider.

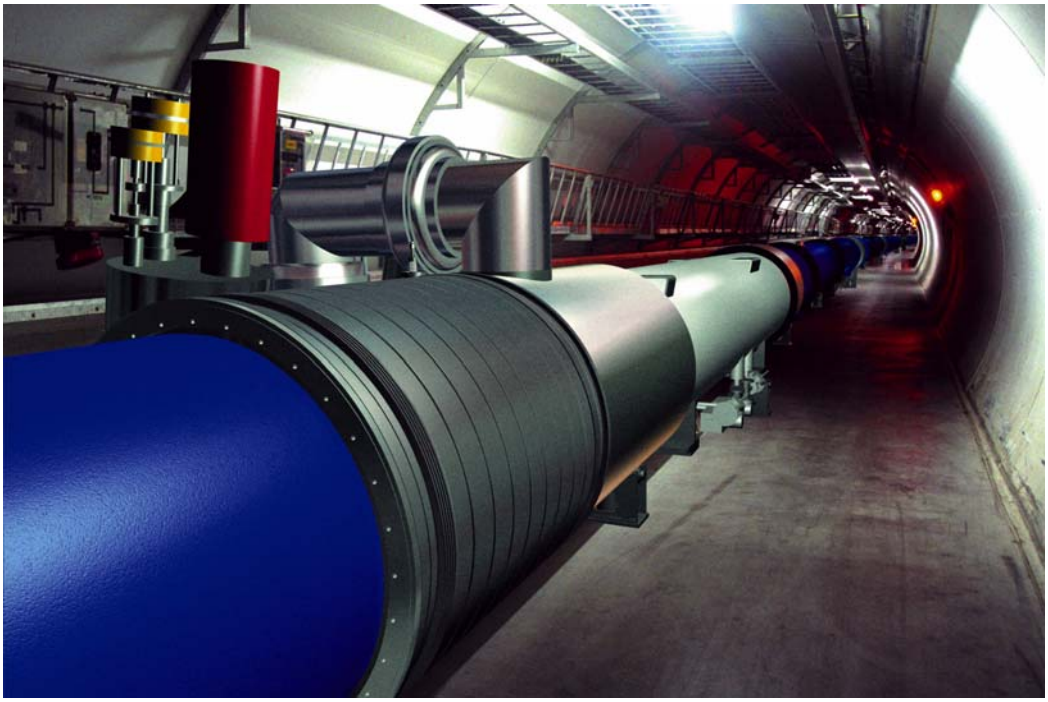
§ Relativistic Heavy Iron Collider.

Superconducting (NbTi) Particle Accelerators



Large Hadron Collider
at CERN, Geneva

Superconducting (NbTi) Particle Accelerators



Superconducting Magnets for Fusion

See "Fusion Magnets" presentation

Challenges for Superconductivity

Cons: Revisited

- **T_c << room temperature.**
  - **Require refrigeration and good thermal insulation.**

- **Mixed state (Type II).**
  - **$R \neq 0$, under time-varying conditions.**

- **Expensive vs copper, aluminum, steel, organic materials.**
  - **100-1000 times more than copper.**

1. Operating Temperature

- All devices operate at room temperature
  Examples: motors,; camera; refrigerators; wrist watches; CD players, automobiles; pianos; airplanes; etc.

- *Exceptions*: Superconducting devices.

Challenge

- Develop “magnet-grade” HTS that enable: $T_{op} = T_{RT}$ (Room Temperature)

Operating Temperatures: HTS vs LTS

Two Views

- Reference temperature at 0 K:

$$\frac{[T_{op}]HTS}{[T_{op}]LTS} \approx \frac{80K}{4K} = 20$$

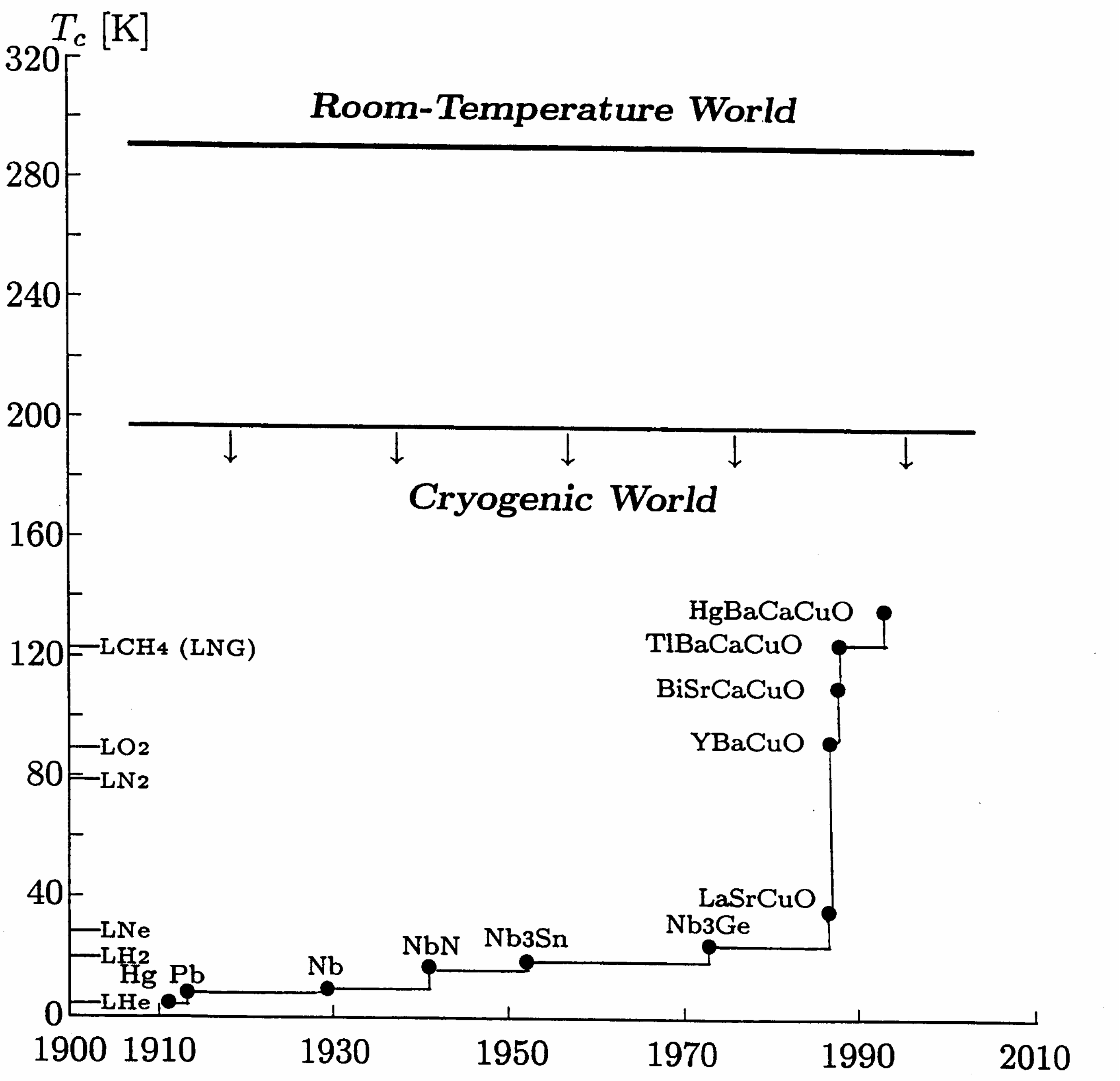
A 20-fold improvement.

- Reference temperature at 22 C:

$$\frac{[T_{op}]HTS}{[T_{op}]LTS} \approx \frac{-291C}{-218C} = 1.33$$

Only a modest 33% improvement.

Progress of Tc:
Impressive but still a lot to go.



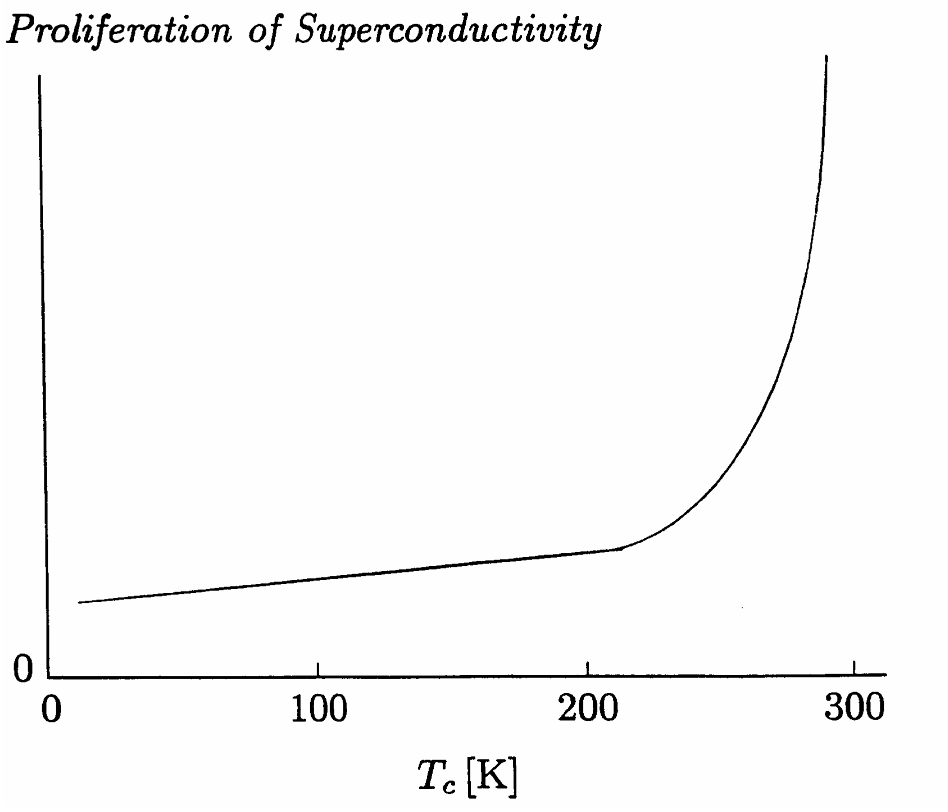
1. Operation at Cryogenic Temperature

Refrigeration and thermal insulation required.

- Refrigeration power – manageable.
- Thermal insulation – fundamental hindrance.
  - o Needed: quantum improvements in insulation techniques.

Corollary

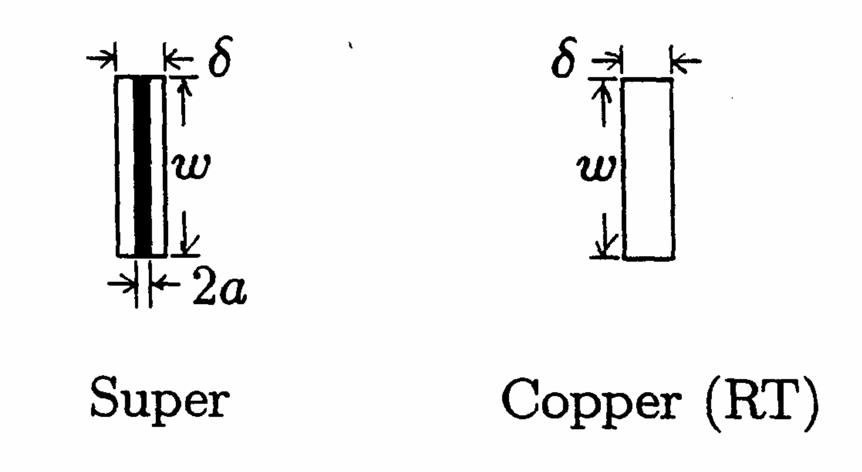
- Proliferation of superconductivity nonlinear with T_c.



2. Mixed State (Type II)

Illustration: Losses in Tapes

Superconductor and copper under $I_t = I_o \sin(2\pi f_o t)$.



Superconductor at Top < T_{RT}: Hysteresis Dissipation Density.

$$p_{hy}\Big|T_{op} = \frac{f_o \mu_o I_t^3}{6w^2 I_c}[W/m^3]$$

Copper at T_{RT}: Ohmic Dissipation Density

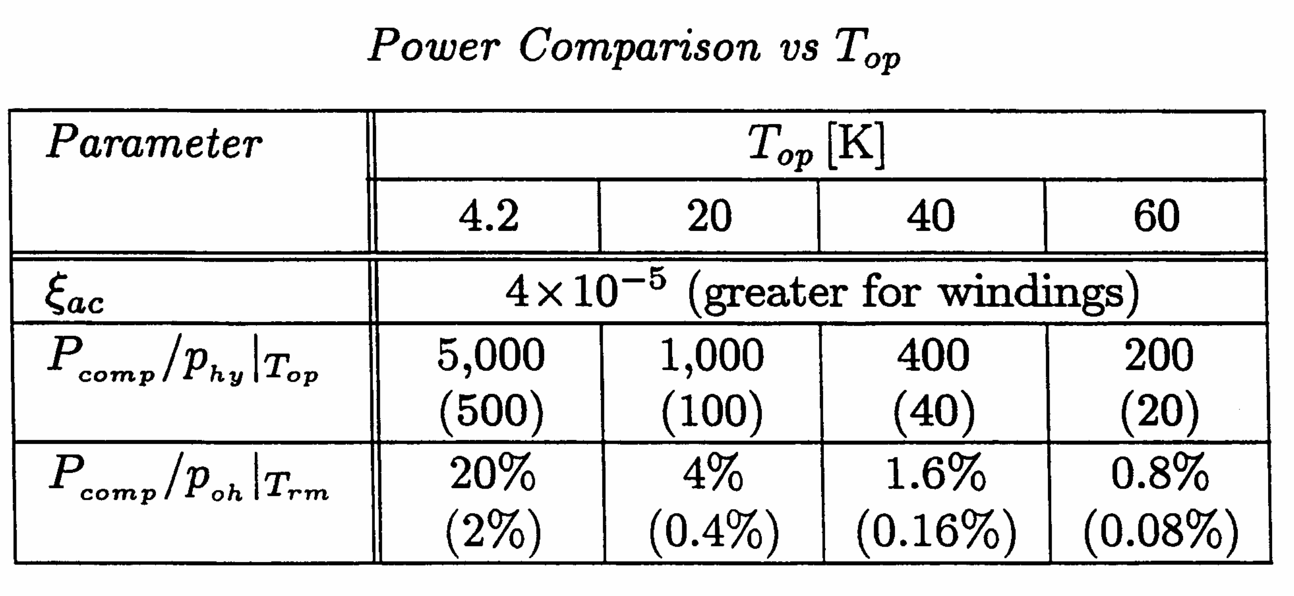
$$p_{oh}\Big|T_{RT} = \frac{\rho_{cu} I_t^2}{2w^2 \delta^2}[W/m^3]$$

Power Density Ratio

$$\frac{p_{hy}\big|T_{op}}{p_{oh}\big|T_{RT}} \equiv \xi_{ac} = \frac{f_o \delta^2}{3}\left(\frac{\mu_o}{\rho_{cu}}\right)\left(\frac{I_t}{I_c}\right)$$

With $f_o = 60$ hz; $\delta = 0.25$ mm; $\rho_{cu} = 2 \times 10^{-8}$ Ωm; $I_t/I_c = 0.5$:

$$\xi_{ac} = 4 \times 10^{-5}$$



Power Comparison vs T_{op}

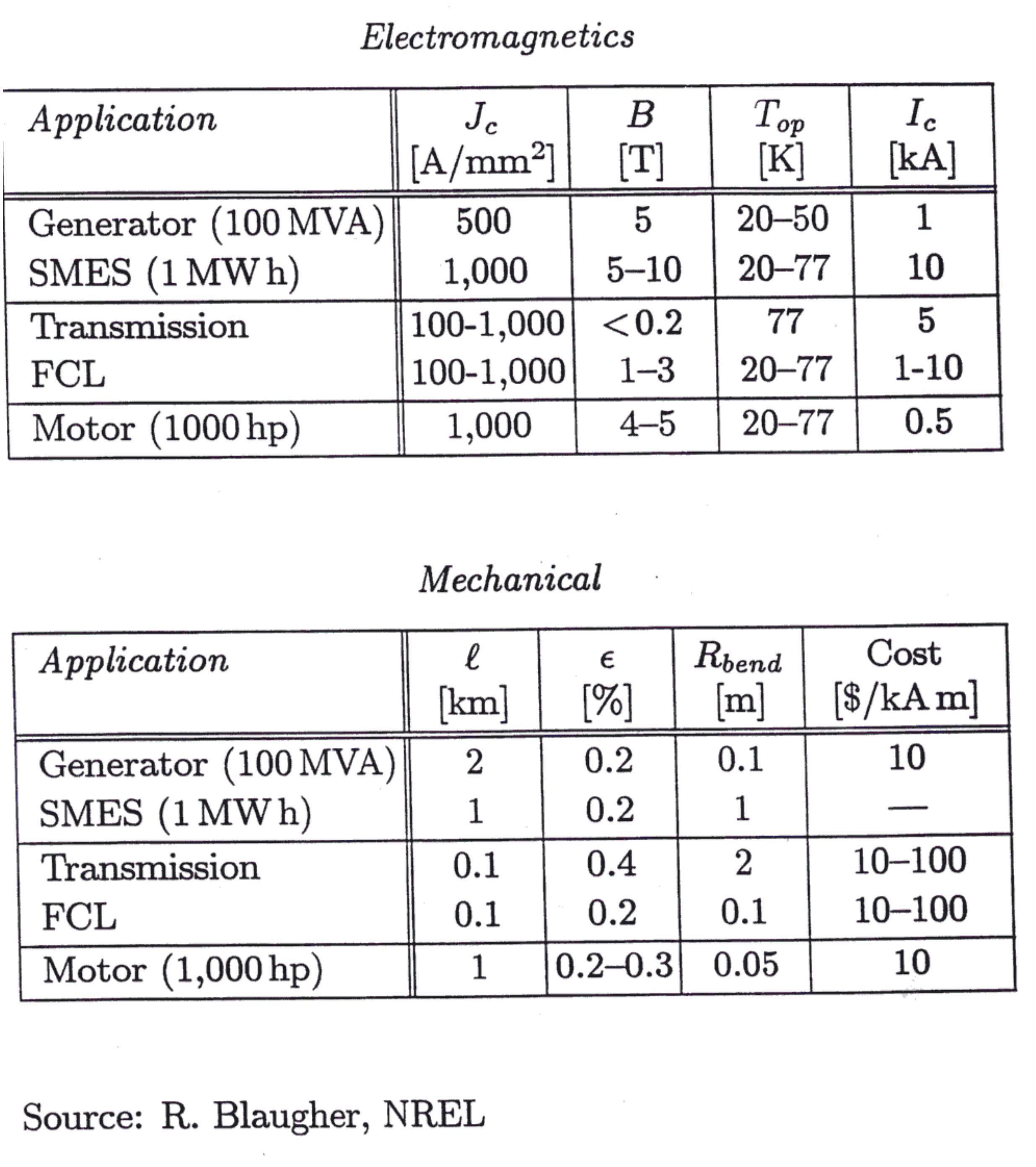
Parameter	T_{op} [K]			
	4.2	20	40	60
ξ_{ac}	4×10^{-5} (greater for windings)			
$P_{comp}/p_{hy}\|T_{op}$	5,000 (500)	1,000 (100)	400 (40)	200 (20)
$P_{comp}/p_{oh}\|T_{rm}$	20% (2%)	4% (0.4%)	1.6% (0.16%)	0.8% (0.08%)

Observation: Hysteresis losses, though nonzero, are manageable with good refrigerators.

3. Conductor

- **Performance improvement.**
- **Cost reduction.**
- **T_{op} optimization.**
- **Operating mode.**

Performance Requirements



Electromagnetics

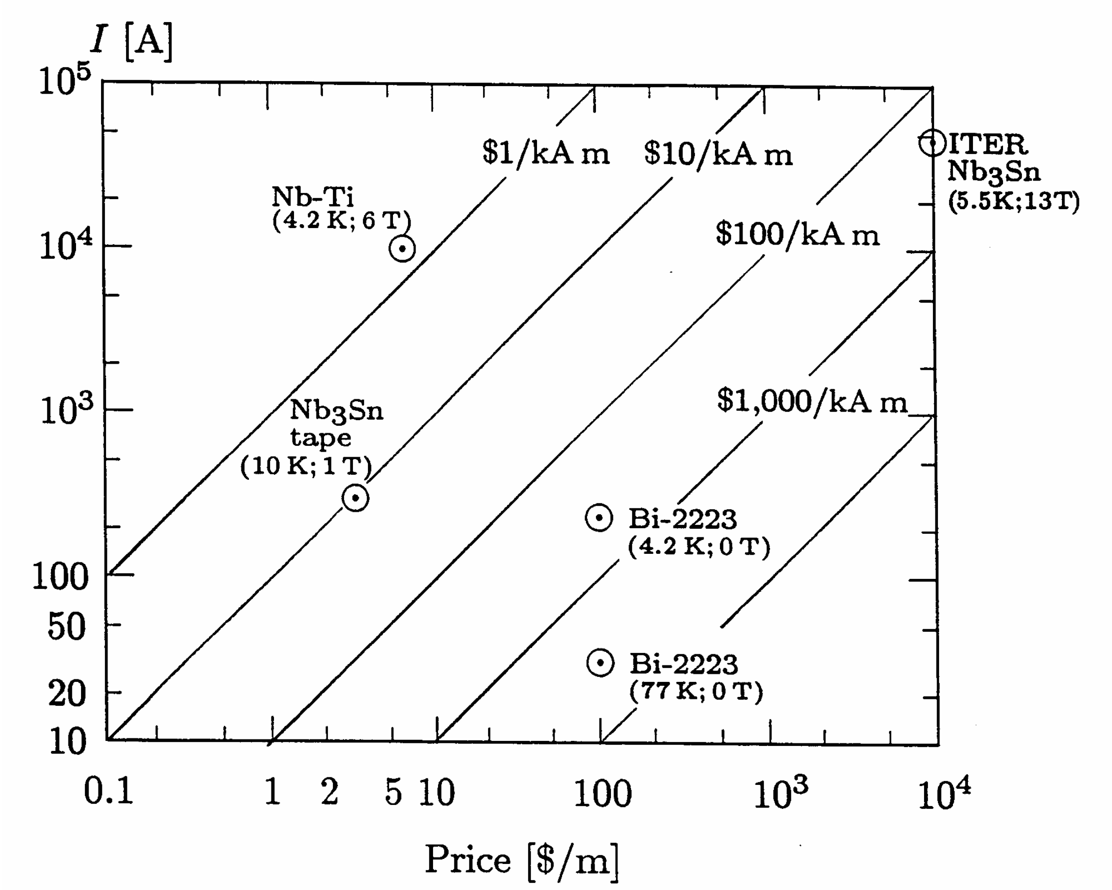
Application	J_c [A/mm^2]	B [T]	T_{op} [K]	I_c [kA]
Generator (100 MVA)	500	5	20–50	1
SMES (1 MW h)	1,000	5–10	20–77	10
Transmission	100-1,000	<0.2	77	5
FCL	100-1,000	1–3	20–77	1-10
Motor (1000 hp)	1,000	4–5	20–77	0.5

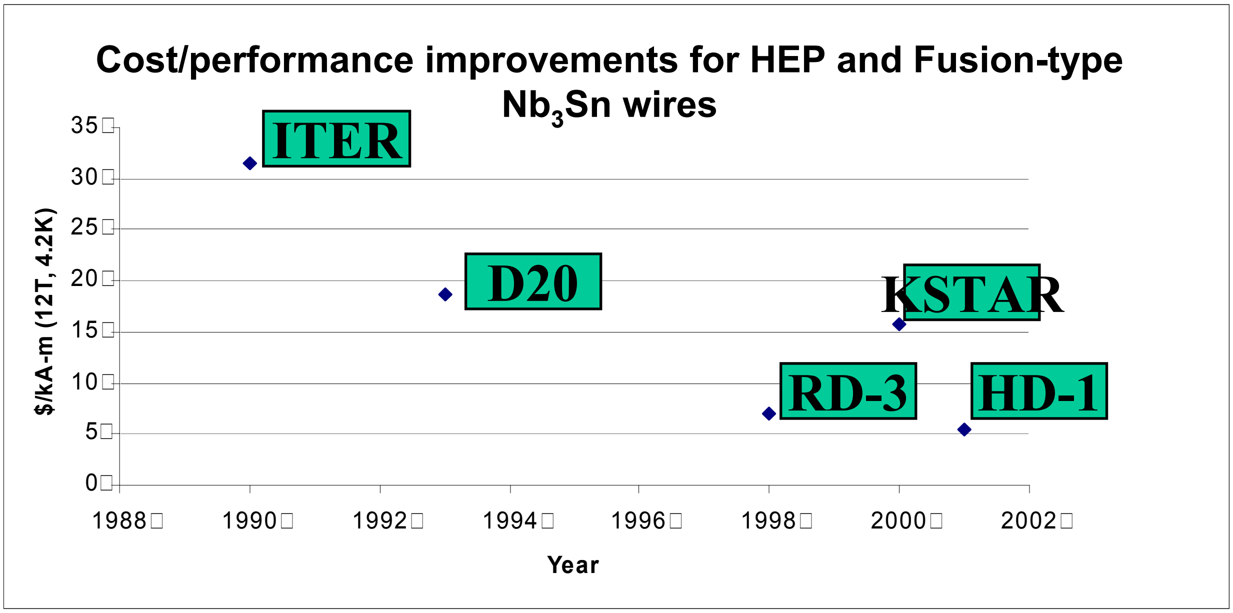
Mechanical

Application	ℓ [km]	ϵ [%]	R_{bend} [m]	Cost [\$/kA m]
Generator (100 MVA)	2	0.2	0.1	10
SMES (1 MW h)	1	0.2	1	—
Transmission	0.1	0.4	2	10–100
FCL	0.1	0.2	0.1	10–100
Motor (1,000 hp)	1	0.2–0.3	0.05	10

Source: R. Blaugher, NREL

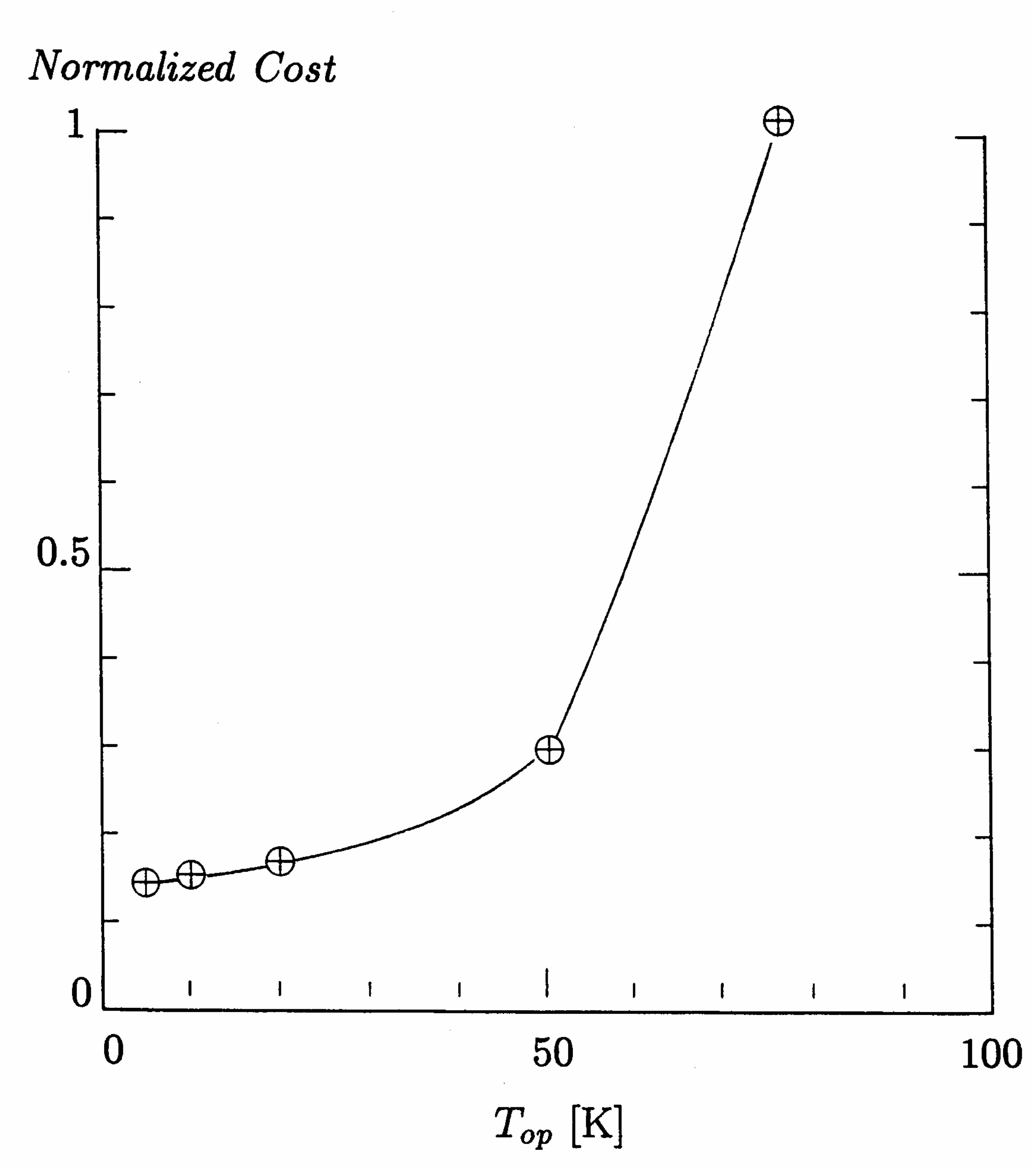
Conductor Costs



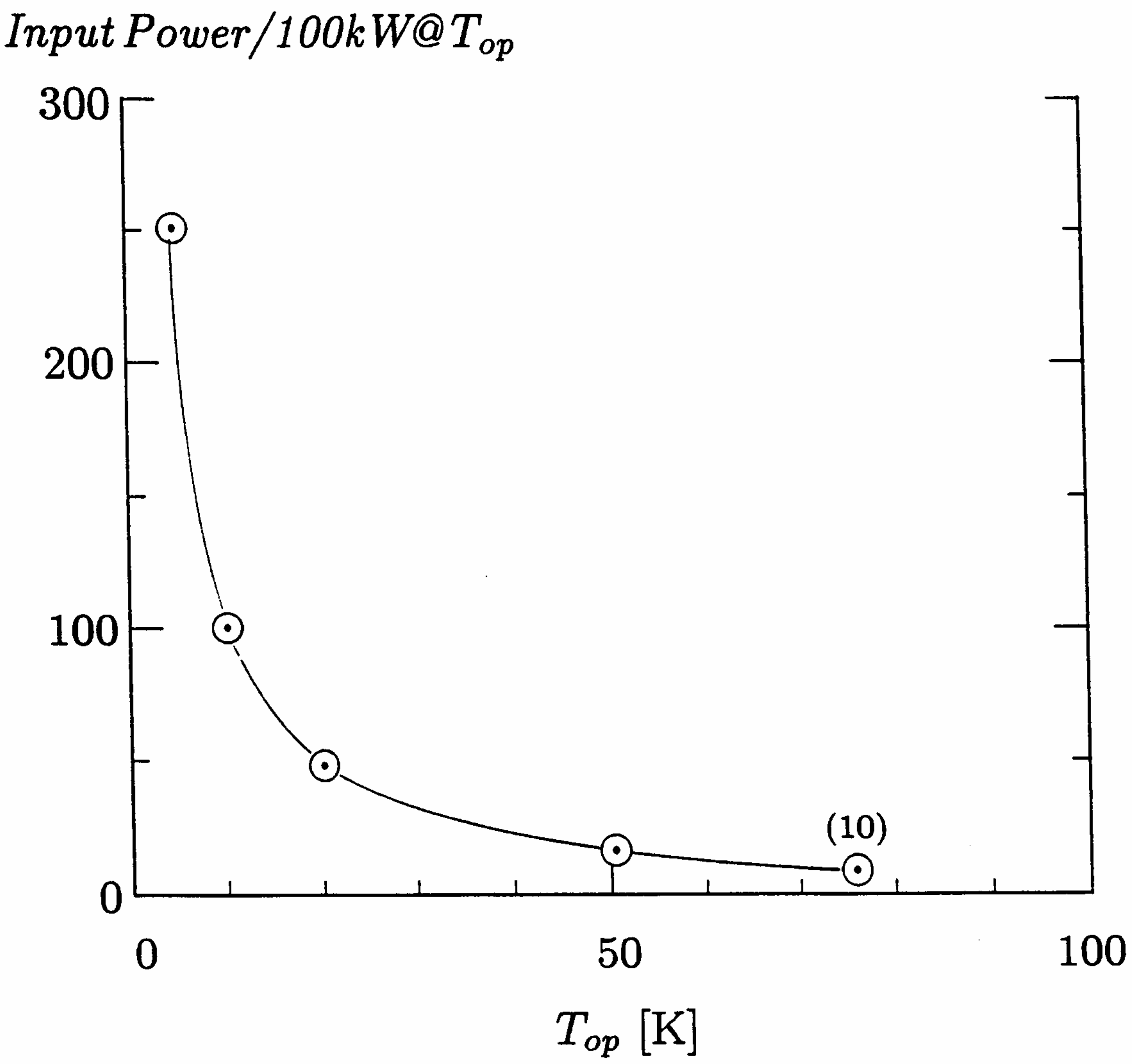


Cost of Silver-Sheathed Bi-2223 Tape

(Normalized to 77 K Cost: $1/J_c|_{77K}$)



100-kW Refrigerator Performance



Optimum Operating Temperature

- $J_c(T)$ important.

- Refrigerator performance vs T important.

- Optimum T_{op}:
  - o ~15 K for systems with large refrigerators
  - o Up to ~40 K for systems with cryocoolers
  - o 77 K – probably *NOT* and optimum Top.
  - o LN_2 useful for high-voltage applications.