Reactor Operation at Power

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Outline

A. Subcritical Reactor Behavior

- Neutron Sources
- Source - Detector Geometry
- Neutron Life Cycle
- Subcritical Multiplication

B. Critical Operation w/o Feedback

- Prompt and Delayed Neutrons
- Reactivity
- Power-Period Relation

First Lecture

- Dynamic Period Equation
- Step and Ramp Reactivity Transients

C. Critical Operation w/Feedback

- Coolant Temperature (Moderator Coefficient)
- Fuel Temperature (Doppler Effect)
- Xenon
- Estimated Critical Position
- Examples of Reactor Transients
- Laboratory Exercise

Second Lecture

Reactor Kinetics

In order to understand the time-dependent behavior of a reactor we need equations that describe the response of the prompt and delayed neutron populations to changes in reactivity. This problem is mathematically complex because the neutron population in a reactor is a function of both space (i.e., position in the core) and time. For many practical situations, we can assume that the spatial and temporal behavior are separable. This allows us to write equations of reactor kinetics as a function of time alone. This approach is acceptable for purposes of personnel training and for routine reactor operation including transients. It is often NOT acceptable for reactor design analysis and for some safety studies.

Reactor Kinetics (Continued)

The space-independent equations of reactor kinetics, which are often called the "point kinetics" equations are:

$$\frac{dn(t)}{dt} = \frac{(\rho(t) - \overline{\beta})}{\ell^*} n(t) + \sum_i^N \lambda_i C_i(t)$$

$$\frac{dC_i(t)}{dt} = \frac{\overline{\beta}_i n(t)}{\ell^*} - \lambda_i C_i(t)$$

where $n(t)$ = is the reactor power,

$\rho(t)$ = is the net reactivity,

$\overline{\beta}$ = is the effective delayed neutron fraction,

ℓ^* = is the prompt neutron life time,

λ_i = is the decay constant of the i<u>th</u> precursor group,

C_i = is the concentration of the i<u>th</u> precursor group, and

N = is the number of delayed neutron precursor groups.

Reactor Kinetics (Continued)

Quantity	Physical Meaning
$\rho(t)$	Fractional change in the total neutron population per generation.
$\overline{\beta}$	Fraction of neutrons that are delayed.
$\left(\rho(t)-\overline{\beta}\right)$	Fractional change in the prompt neutron population per generation.
$1/\ell^*$	Number generations per unit time.
$n(t)$	Total neutron population.
$\frac{\left(\rho(t)-\overline{\beta}\right)}{\ell^*}n(t)$	Change in prompt neutron population per unit time.
$\lambda_i C_i$	Rate of decay of delayed neutron precursors. This equals the rate of appearance of the delayed neutrons.
$\frac{\overline{\beta}_i n(t)}{\ell^*}$	Rate of production of delayed neutron precursors per unit time.

Reactor Kinetics (Continued)

— The first kinetics equation describes the behavior of the neutrons. It states that the rate of change of the total neutron population equals the sum of the rates of change of the prompt neutrons and the delayed ones. The second kinetics equation describes the behavior of the precursors. It says that the rate of change of the precursors is the difference between their production and loss.

— These equations are not too useful to a reactor operator. One can not measure precursors and it is difficult to visualize the consequences of two simultaneous differential equations. A more useful approach is to combine these equations through a process of differentiation and substitution to obtain the dynamic period equation. This equation relates the reactor period, which is measurable, to the reactivity.

Dynamic Period Equation

1. It is useful to relate reactivity to period. Most text books do this by use of the Inhour Equation which is valid only a long time after reactivity changes. A more general relation, one that is valid under all conditions, is the dynamic period equation. (This relation was developed at MIT in the mid-1980s and is the basis of MIT's very successful program on digital control of reactors.) A simplified version is:

$$\tau(t) = \frac{\overline{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)}$$

where $\tau(t)$ = is the reactor period,

$\overline{\beta}$ = is the effective delayed neutron fraction,

$\rho(t)$ = is the net reactivity,

$\dot{\rho}(t)$ = is the rate of change of the net reactivity, and

$\lambda_e(t)$ = is the standard, effective multi-group decay parameter.

Dynamic Period Equation (cont.)

Quantity	Meaning
$\dot{\rho}(t)$	— Rate of change of reactivity. This is proportional to the prompt neutron population. Changes in the velocity of a control device therefore have an immediate effect on the period.
$\lambda_e(t)\rho(t)$	— This term is proportional to the delayed neutron population. Reactivity can not be changed on demand. Rather, a control device's position has to be altered or the Burnable poison concentration has to be adjusted. This takes time.

Dynamic Period Equation (cont.)

2. It is important to note that the reactor period depends on both the rate of change of reactivity ($\dot{\rho}$) and the total reactivity (ρ). The former corresponds to prompt neutron effects; the latter to delayed ones. Hence:

    (i) The speed at which one changes reactivity alters the period. This is the basis of power cutbacks that involve high speed rod insertions.

    (ii) The reactor period is a function of the power history because the decay term reflects the power level that existed when the delayed neutron precursors were created. This is one reason why it is important to approach a final power level slowly.

    (iii) The relation between period and reactivity is not readily solved. Hence, operators may find predictive displays to be of use.

Dynamic Period Equation (cont.)

$$\tau(t) = \frac{\left(\overline{\beta} - \rho(t)\right) + \ell^* \left[\frac{\dot{\omega}(t)}{\omega(t)} + \omega(t) + \lambda_e(t) - \frac{\dot{\lambda}_e(t)}{\lambda_e(t)}\right]}{\dot{\rho}(t) + \lambda_e(t)\rho(t) + \frac{\dot{\lambda}_e(t)}{\lambda_e(t)}\left(\overline{\beta} - \rho(t)\right)}$$

where the standard, effective, multi-group decay parameter is defined as:

$$\lambda_e(t) \equiv \Sigma\lambda_i C_i / \Sigma C_i(t) \quad \text{for } i = 1, N$$

and where symbols not previously defined are:

$\dot{\omega}(t)$	is the rate of change of the inverse of the dynamic reactor period,
$\omega(t)$	is the inverse of the dynamic reactor period,
$\dot{\lambda}_e(t)$	is the rate of change of the standard, effective, multi-group decay parameter,
$C_i(t)$	is the concentration of the i<u>th</u> precursor group normalized to the initial power, and
N	is the number of groups of delayed neutrons, including photo-neutrons.

Step Change in Reactivity

— Step changes of reactivity can be analyzed analytically (i.e., without the need for a computer). However, such changes do not usually occur on operating reactor. Exceptions might be the sudden injection of a cold slug of water into a steam generator as might occur if the hot well level control valve on a condenser failed open.

— A step insertion of reactivity such that $\rho < \overline{\beta}$ will cause:

(a) A rapid increase in the prompt neutron population. This is called the "prompt-jump" and it represents the start of a nuclear runaway. But the runaway can not continue because the reactivity is less than the delayed neutron fraction.

(b) A rise on a period corresponding to the growth of the delayed neutrons.

Step Change in Reactivity (Continued)

— The magnitude of the step change is given by:

$$P_f = P_i\left(\frac{\overline{\beta}}{\overline{\beta}-\rho}\right)$$

where P_f and P_i are the power levels before and after the step insertion. Note that the effect of a step insertion depends on the initial power level. Suppose a reactivity of 0.2 $\overline{\beta}$ is inserted as a step and also assume the initial power to be 10% of allowed. The final power is 12.5% of allowed – a minor change. But what if the initial power had been 90% of allowed. The final power would now be 112.5% of allowed – a serious problem.

— The power behavior following the prompt jump is given by:

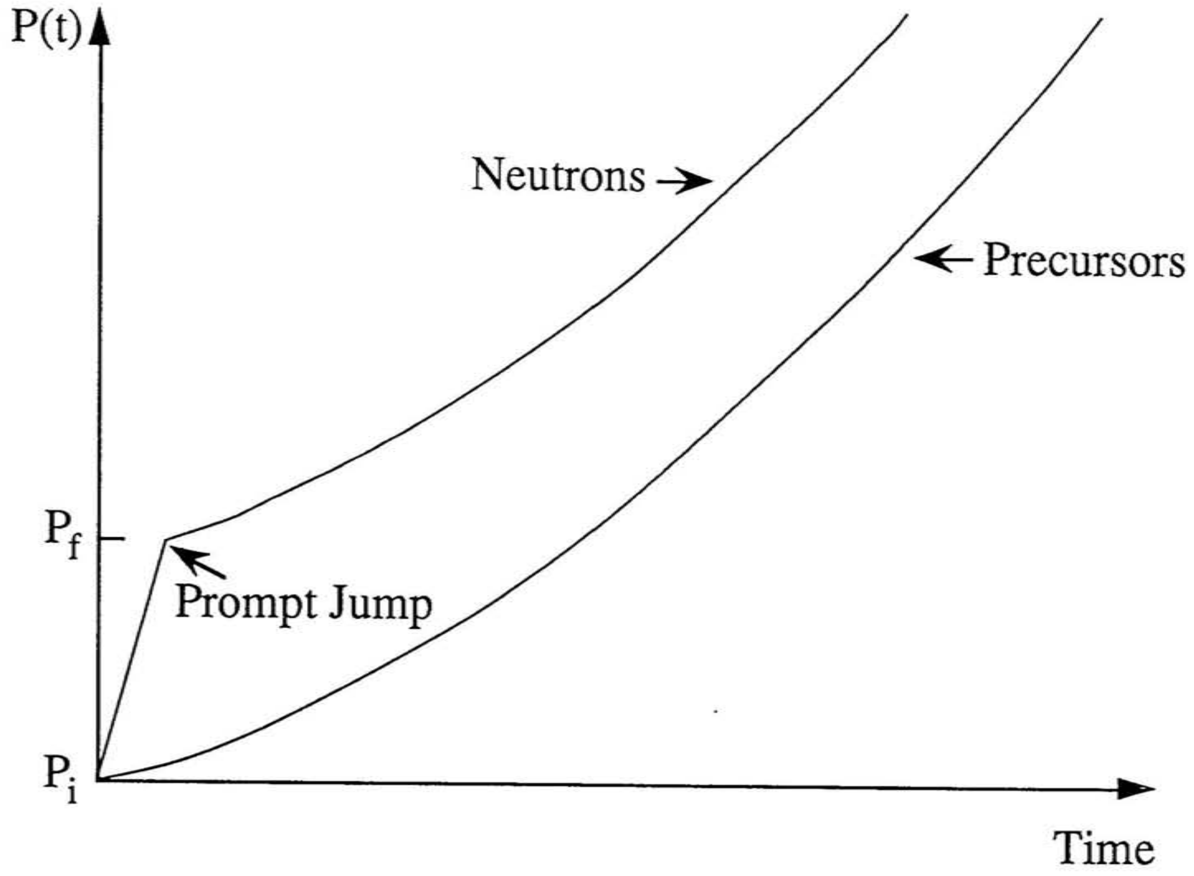
$$P(t) = P_f e^{t/\tau}$$

$$\tau(t) = \frac{\overline{\beta}-\rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} \cong \frac{\overline{\beta}-\rho}{\lambda_e \rho}$$

where $\dot{\rho}(t)$ is zero, $\lambda_e(t)$ can be assume to be 0.1 inverse seconds, and $\rho(t)$ is the step reactivity insertion.

Step Change in Reactivity (Continued)

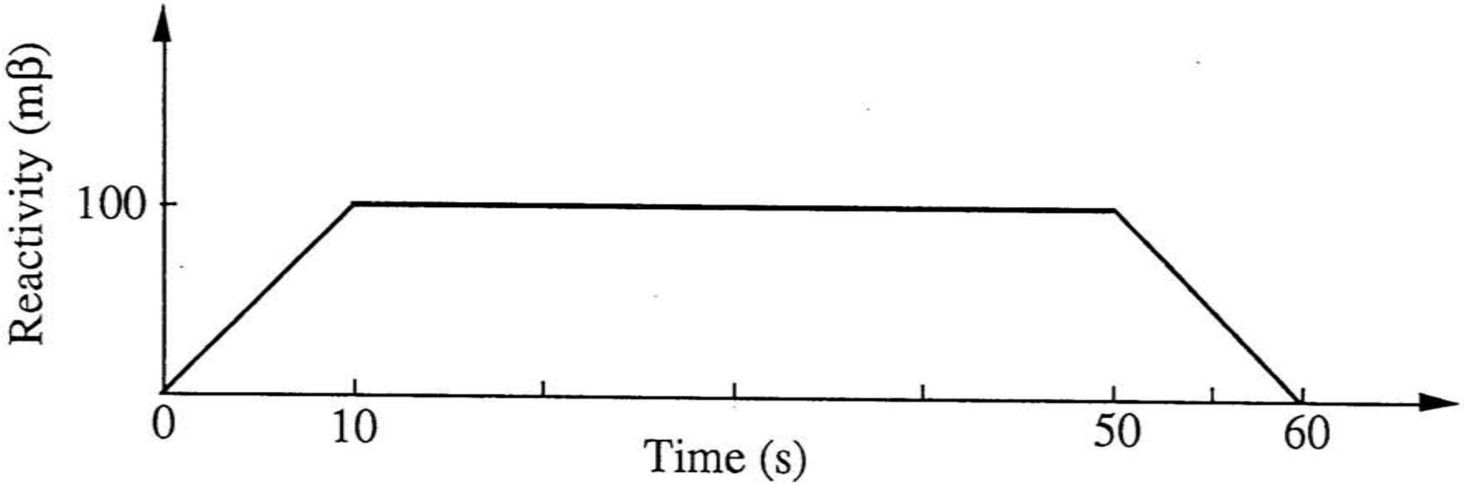
The time-dependent behavior of the reactor power and precursors following a step insertion of reactivity has the following appearance:



Ramp Insertion of Reactivity

Ramp reactivity insertions are common in nuclear reactors. These occur when control devices are moved or when the concentration of the soluble boron is adjusted. Even an approximate solution of the power following a ramp insertion requires a computer. The following is a qualitative analysis.

a) Reactivity – We assume that the reactivity insertion has the following shape:



Reactivity is inserted at the rate of 10 mbeta/s for ten seconds, held constant at 100 mbeta for 50 seconds, and then removed at the rate of -10 mbeta/s for 10 seconds.

Ramp Insertion of Reactivity (Continued)

b) Period – The period is defined as the power divided by the rate of change of power. Hence, it is infinite at steady-state and therefore difficult to plot. So, the inverse of the period is plotted.

It is useful to calculate the period immediately before and immediately after each change in the reactivity insertion. Thus, calculations are done at $t = 0^-, 0^+, 10^-, 10^+, 50^-, 50^+, 60^-$, and 60^+ where (-) and (+) refer to immediately before and after the indicated time. The relation:

$$\tau(t) = \frac{\overline{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)}$$

will be used. Also, assume $\lambda_e(t)$ to equal $0.1\ s^{-1}$.

Ramp Insertion of Reactivity (Continued)

(i) At $t = 0^-$, both the reactivity and its rate of change are zero. So, the period is:

$$\tau(t) = \frac{\overline{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} = \frac{1.0 - 0.0}{0.0 + (0.1)(0.0)} = \infty$$

(ii) At $t = 0^+$, the reactivity is still zero. But the rate of change of reactivity is now +10 mbeta/s or 0.01 Beta/s. Hence,

$$\tau(t) = \frac{\overline{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} = \frac{1.0 - 0.0}{0.01 + (0.1)(0.0)} = 100 \text{ s}$$

So, the mere act of initiating a reactivity insertion has immediately placed the reactor on a positive period of 100 s.

Ramp Insertion of Reactivity (Continued)

(iii) At $t = 10^-$, there is 100 mbeta of reactivity present and reactivity is still being added at the rate of 10 mbeta/s. Thus,

$$\tau(t) = \frac{\overline{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} = \frac{1.0 - 0.1}{0.01 + (0.1)(0.1)} = 45 \text{ s}$$

So, the period has gone from 100 s to 45 s. Power is rising faster. What happens when the reactivity insertion stops?

(iv) At $t = 10^+$, we have:

$$\tau(t) = \frac{\overline{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} = \frac{1.0 - 0.1}{0.0 + (0.1)(0.1)} = 90 \text{ s}$$

So, the effect of stopping the reactivity insertion is to lengthen the period in a stepwise manner.

Ramp Insertion of Reactivity (Continued)

(v) At $t = 50^-$, $\tau(t) = 90$ s because conditions are the same as at $t = 10^+$. This means that the period was a constant for $10 < t < 50$ seconds. So, during that segment of the transient, the power rose on a pure exponential.

(vi) At $t = 50^+$, the reactivity is still 100 mbeta, but the rate of change of reactivity is now -10 mbeta/s. Thus,

$$\tau(t) = \frac{\overline{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} = \frac{1.0 - 0.1}{-.01 + (0.1)(0.1)} = \infty$$

The effect of initiating the reactivity removal has then to halt the power increase.

Suppose instead of -10 mbeta/s the rate of removal had been only half that. In that case, $\tau(t)$ would be 180 seconds. The power would still be rising. This is an extremely important observation. The fact that the control devices are being inserted does NOT necessarily mean that the reactor power is decreasing. It may still be rising but at an even decreasing rate. This behavior is the result of the delayed neutron precursors. This is one reason why the operation of a nuclear reactor requires skill and experience. Operators must preplan their actions.

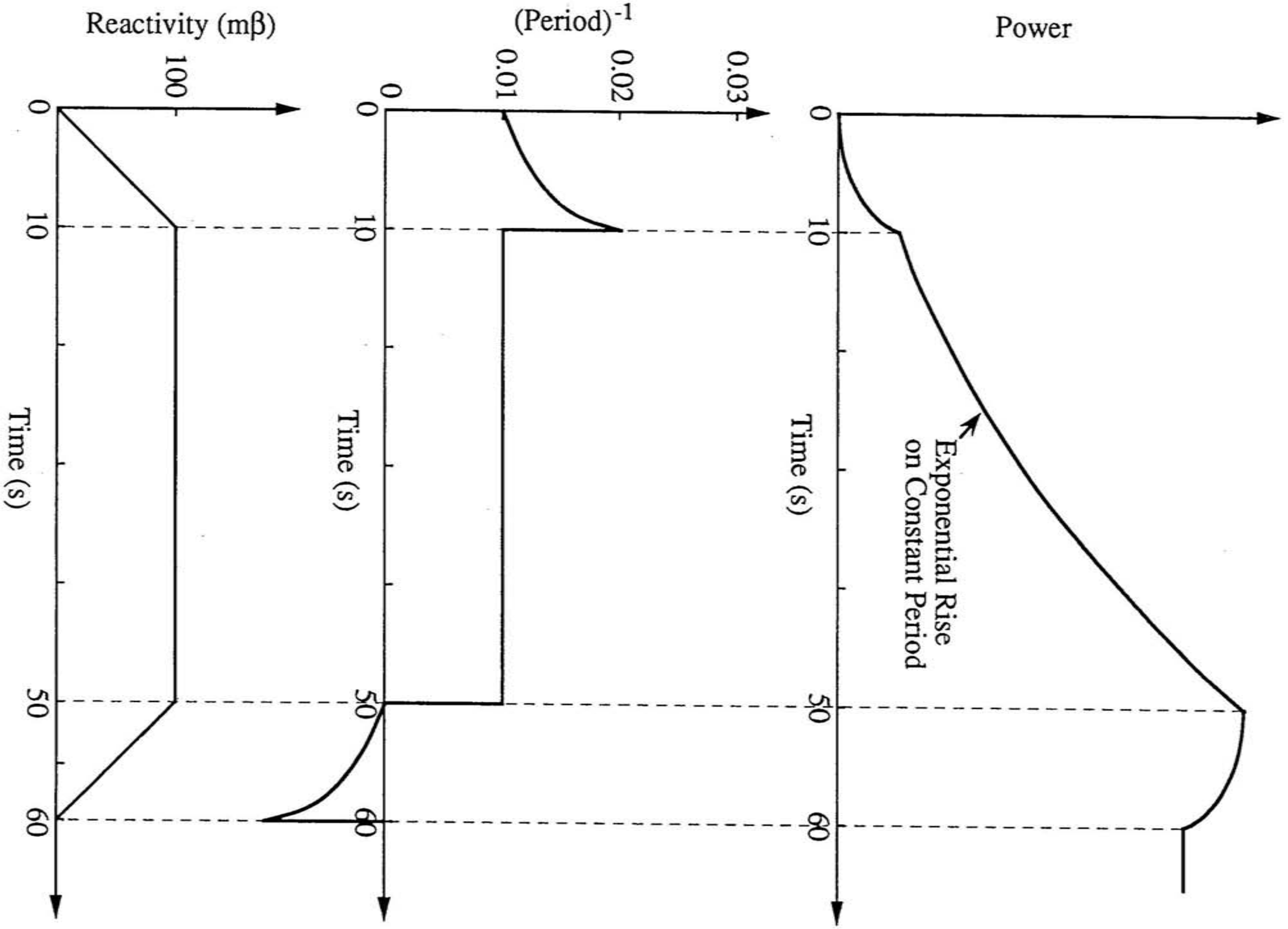
Ramp Insertion of Reactivity (Continued)

(vii) At $t = 60^-$, the reactivity is zero, but the rate of change of reactivity is still -10 mbeta/s. So,

$$\tau(t) = \frac{\overline{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} = \frac{1.0 - 0.0}{-.01 + (0.1)(0.0)} = -100 \text{ s}$$

(viii) At $t = 60^+$, $\tau(t)$ is again infinite.

c) Power – The shape of the power profile can be estimated from the periods using the power-period relation $\left(\rho(t) = \rho_0 \exp(t/\tau)\right)$. However, this equation is only valid for a constant period and that is only true for $10 < t < 50$ seconds. The figure on the next page gives the result.



Reactivity-Period-Power Profile

Point of Adding Heat – Hot Operating

Crucial concepts are:

- Types of power-dependent feedback
  - Coolant Temperature
  - Fuel Temperature (Doppler Effect)
  - Void Coefficient
  - Xenon
- Time scales of each feedback mechanism
- Estimated Critical Position

Coolant Temperature

1. As the temperature of the primary coolant increases, it becomes less dense. This causes:

   a) Less neutron moderation

   b) Increased neutron leakage

   Hence, as coolant temperature rises, negative reactivity is generated. This makes reactors self-regulating. Time scale is primary loop circuit time ~ 30 s.

2. Negative temperature coefficients are required for all U.S., European, and Japanese reactors. They provide a safety feature. But, under certain accidents they make the situation worse:

   a) Steam line break

   b) Control rod drop

   In both of these cases, the reactor cools off and the negative coefficient causes a positive reactivity insertion.

Void Coefficient

The primary coolant in both PWRs and BWRs performs several functions. These are:

(1) Removal of heat.

(2) Moderation of neutrons so as to continue the neutron chain reaction.

(3) Shielding during maintenance.

If the coolant becomes less dense, it becomes less efficient as a moderator. One way to decrease coolant density is to increase its temperature. Another is to create voids in it. This occurs in BWRs when steam bubbles form. BWRs can be controlled by adjusting the recirculation flow which in turn controls the rate at which voids are swept out of the core.

Reactor Regulation

Negative coefficients of reactivity promote self-regulation of a reactor. Consider a PWR and suppose that the demand on the turbine increases. The following sequence then occurs:

(i) Turbine first stage steam pressure decreases.

(ii) Steam flow from the steam generator increases. This causes steam generator pressure to drop.

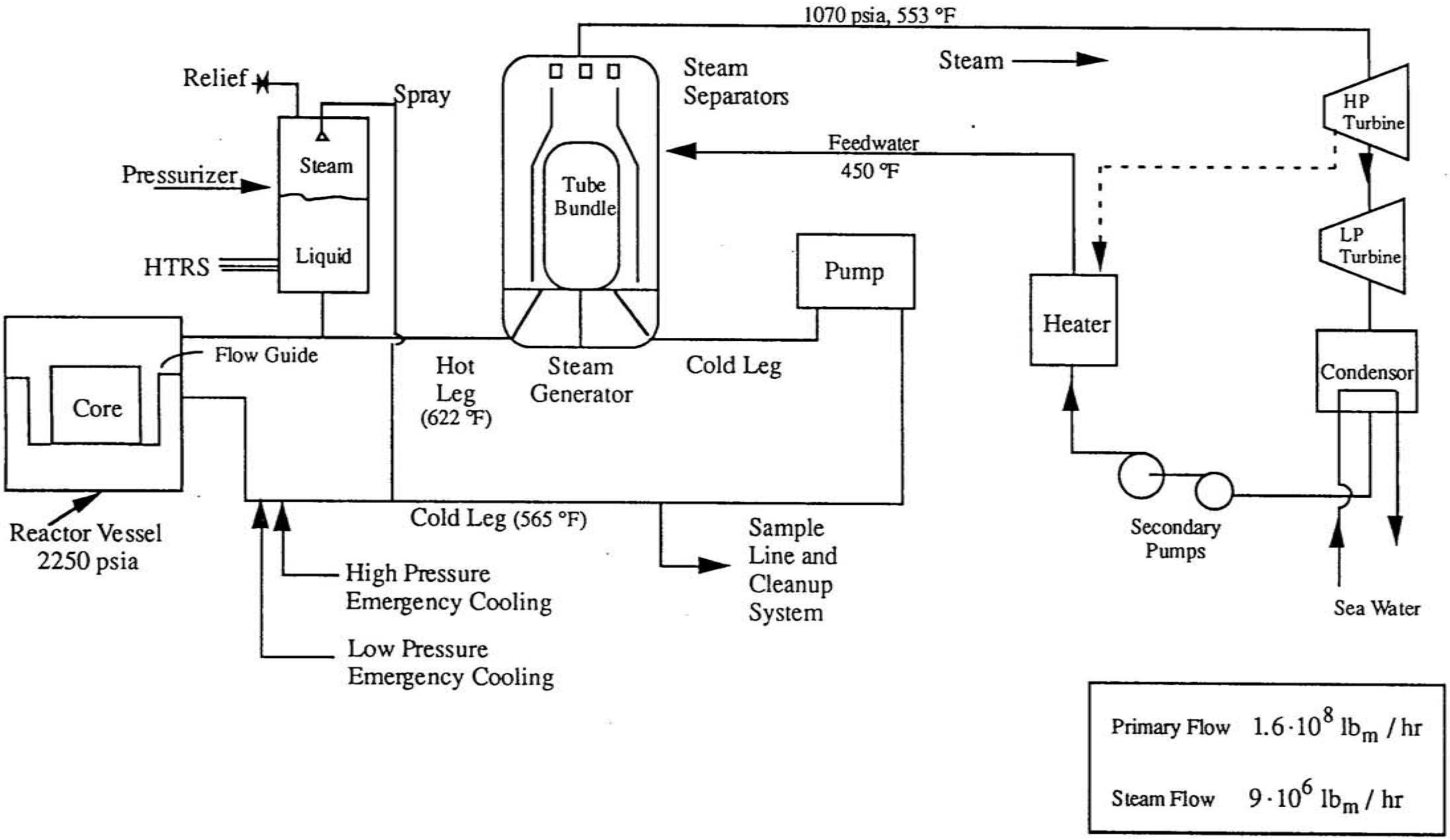
(iii) The steam generator is a saturated system. So, its temperature also drops.

(iv) The decrease in steam generator temperature causes a decrease in the cold leg temperature of the primary coolant.

(v) Cooler primary coolant enters the reactor core. This denser coolant increases neutron moderation.

(vi) Reactor power increases and so does the temperature of the hot leg.

(vii) Hotter primary coolant reaches the steam generator. Steam generator temperature and pressure rise and the steam supply equals the demand.



Pressurized Water Reactor (PWR)

Reactor Regulation (Continued)

1. The sequence on the previous viewgraph is often abbreviated as:

$$\text{Demand} \uparrow P_{sg} \downarrow T_{sg} \downarrow T_{CL} \downarrow \rho \uparrow \text{Power} \uparrow T_{HL} \uparrow T_{sg} \uparrow P_{sg} \uparrow$$

2. The final result is that the reactor power has increased to equal the demand. Also, the difference between the hot and cold leg temperatures has increased but the average of these two temperatures is unchanged.

Doppler Effect

1. As the temperature of the fuel increases, the U-238 resonances broaden and capture more neutrons in reactions that do not lead to fission. Hence, negative reactivity is generated. Time scale is seconds or less.

2. The Doppler effect is an inherent safety feature that may prevent fuel damage during an accident. Some research reactors (TRIGAs) are designated to eject control rods thereby causing very rapid power increases to hundreds of MWs. The Doppler effect shuts these reactors down. (Note: TRIGAs use a special fuel that accentuates the U-238 resonance absorption.)

Xenon

1. Xenon is a fission product that absorbs neutrons. It is produced whenever a reactor is at power and, after about 40 hours, reaches an equilibrium value. Xenon peaks 11 hours after shutdown and then decays away over several days.

2. Reactors must be designed with enough fuel to offset the effect of Xenon. This increases the cost of the reactor and the complexity of the control system.

3. At some points in a refueling cycle, there may not be enough excess fuel to restart during peak Xenon. Such reactors are referred to as 'Xenon-precluded.'

Xenon Peaking

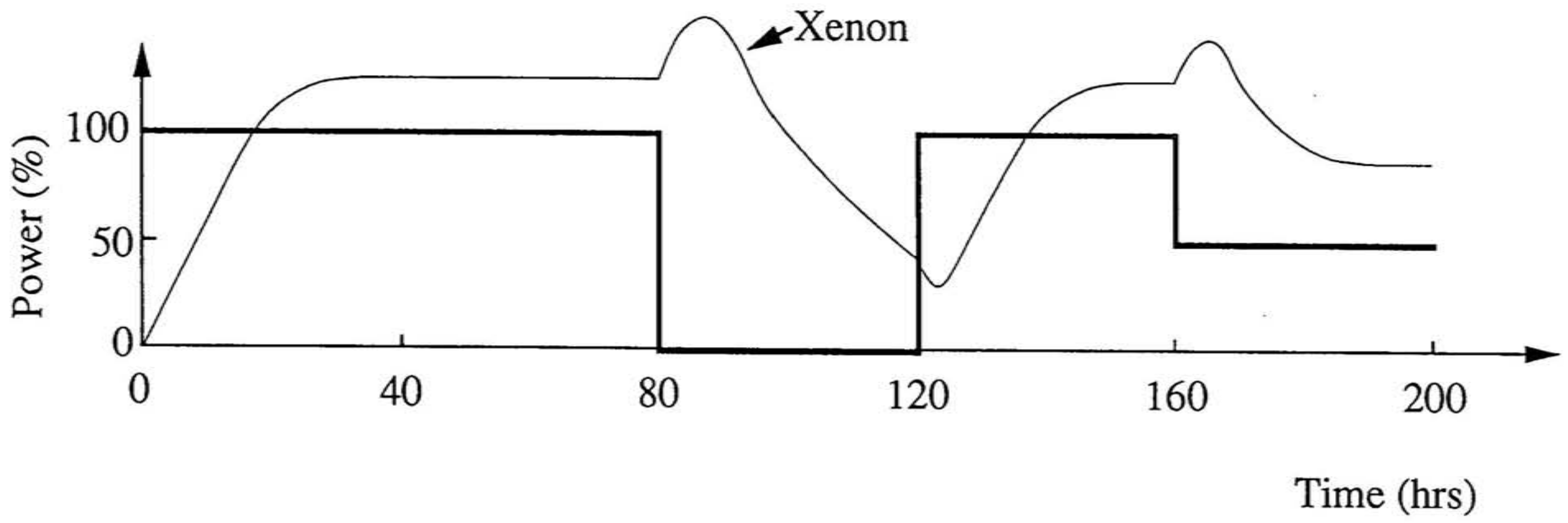
1. There are two sources of Xenon: direct from fission and indirect from the decay of iodine. The latter is the major source.

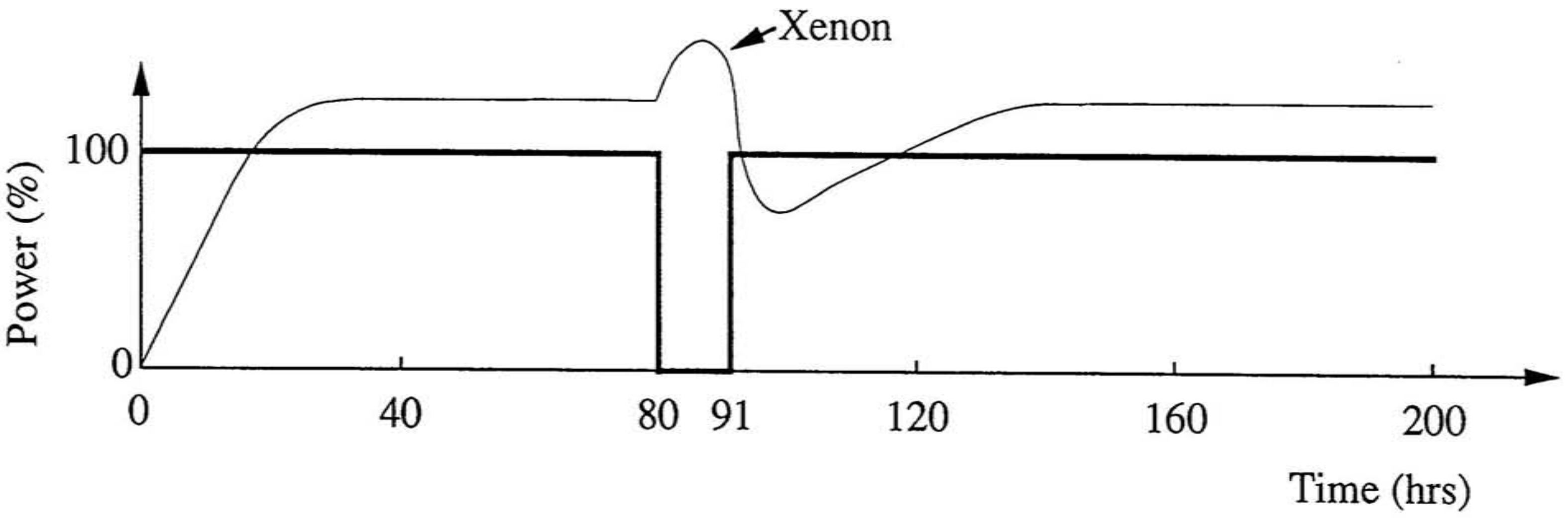
2. There are two sinks for Xenon: burnup and decay to Cesium which does not act as a neutron poison. Burnup is the major sink.

3. What happens on a reactor shutdown? The major sink for Xenon is removed while the major source remains active because of the inventory of the fission product iodine. So, Xenon rises. The rise continues until the supply of iodine is exhausted.

4. What happens when the reactor is restarted? The major sink (burnup) is resumed. But the major supply (iodine decay to Xenon) is below its equilibrium value because the reactor has been off-line. So, Xenon decreases.

Effect of Power History on Xenon





Effect of Xenon on Reactor Operation

1. At end of core life, there may not be enough reactivity to override peak Xenon. Such reactors are referred to as being 'Xenon-precluded.' If a scram occurs, a restart may not be possible for 30-40 hours.

2. For reactors at 50% power, the Xenon concentration is 70% of its equilibrium value.

3. Xenon may undershoot its equilibrium value. This requires the operator to manipulate the control devices in unusual patterns. Power peaking problems can develop.

Estimated Critical Position

- One means of estimating criticality is a '1/M' plot. Another is to compute an estimated critical position or ECP.

- The reactor's last known critical position is recorded including data on rod positions, poison concentration, power history (Xenon), and coolant temperature.

- Prior to startup, the current values of the poison concentration, Xenon, and coolant temperature are recorded. The reactivity associated with the change in each of these parameters is computed and summed. This allows determination of the rod position at which criticality can be expected.

- ECPs are much faster than '1/M' plots are and used for most startups.

Examples of Transients

— The figures that follow are from the MIT program on the closed-loop digital control of reactors. These illustrate many of the physical features associated with rapid ramp insertions/removals of reactivity including:

- Effect of rapid rates of change of reactivity on the rate of rise of power (1st figure/2nd figure).
- Doppler effect (2nd figure for t > 15 s. Power is constant and rods are being pulled out. Reactivity is decreasing because of the rise in fuel temperature).
- Effect of delayed neutrons in power decreases (3rd figure. In absence of rod movement, the period is limited to -80 s. Here, a shorter period (-10 s) is sustained briefly because of rod insertion).
- Need for validated signals (4th figure).
- Need for the preplanning of reactor maneuvers (5th figure).

[Five slides containing the above-referenced figures have been removed for copyright reasons.]

Laboratory Exercise

1. Each person will have an opportunity to sign in at the MIT Research Reactor and raise the power level.

2. The reactor will then be shutdown. Observe the ~80 s period that corresponds to the longest lived precursor group.

3. The reactor will then be started up via a '1/M' plot.