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Application of the "Reactivity Constraint Approach" to Automatic Reactor Control

John A. Bernard

Massachusetts Institute of Technology, Nuclear Reactor Laboratory

and

Allan F. Henry and David D. Lanning

Massachusetts Institute of Technology, Department of Nuclear Engineering

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Abstract — *The "reactivity constraint approach" is described and demonstrated to be an effective and reliable means for the automatic control of power in nuclear reactors. This approach functions by restricting the effect of the delayed neutron populations to that which can be balanced by an induced change in the prompt population. This is done by limiting the net reactivity to the amount that can be offset by reversing the direction of motion of the automated control mechanism. The necessary reactivity constraints are obtained from the dynamic period equation, which gives the instantaneous reactor period as a function of the reactivity and the rate of change of reactivity. The derivation of this equation is described with emphasis on the recently obtained "alternate" formulation. Following a discussion of the behavior of each term of this alternate equation as a function of reactivity, its use in the design and operation of a nonlinear, closed-loop, digital controller for reactor power is described. Details of the initial experimental trials of the resulting controller are given.*

INTRODUCTION

This paper describes recent theoretical and experimental accomplishments in the on-going development of a general methodology for the closed-loop, nonlinear, digital control of reactor power during transients. The work reported here is one result of a systematic effort being undertaken by the Massachusetts Institute of Technology (MIT) and the Charles Stark Draper Laboratory (CSDL) to apply advanced instrumentation and control techniques to the operation of nuclear reactors. This program was initiated in the expectation that the use of modern digital technology would both improve the performance of current nuclear plants and make the economics of future ones more attractive. Specific arguments supporting the use of closed-loop digital reactor control have been previously enumerated.[1]

Adjustments of the reactor power are normally accomplished by inserting reactivity until a specified stable period has been attained. The power is then allowed to rise (or to fall) exponentially on that period until the desired level is approached. The period is then gradually lengthened by reducing the reactivity, usually in a stepwise fashion, so as to level the power smoothly. The crucial aspect of the control process is that the lengthening of the reactor period must be initiated before attaining the specified power level. Such anticipatory actions are necessary because the rate at which reactivity can be removed is finite, particularly when rods are used at normal speeds. Hence, if changes in the reactor power are to be achieved both efficiently and without challenge to the safety system, some method must be available by which the proper time for initiation of the reactivity removal process can be reliably predicted. The "reactivity constraint approach" is a digital method for accomplishing this task under closed-loop control conditions in the presence of

nonlinear feedback effects, while using a control device of varying differential reactivity worth. The technique functions by restricting the net reactivity so that it is always possible to make the reactor period infinite at the desired termination point of a transient by reversing the direction of motion of whatever control mechanism is associated with the controller. This capability is formally referred to as "feasibility of control." Implementation of the reactivity constraint approach is achieved through use of the dynamic period equation.[2] That equation, as documented below, can be written in either of two forms that are designated as "standard" and "alternate." The design and evaluation of a controller based on the standard formulation has been previously reported, and use of that controller has been demonstrated over a wide range of control capability.[3-6] However, that approach has the disadvantage that the standard equation contains a derivative term that is noisy when evaluated on-line. As a result, it is desirable to delete that term from the relation that is used to determine the proper moment for the initiation of reactivity removal. Arguments justifying the elimination of this derivative-containing term are given in Ref. 7, and while those arguments have been shown experimentally to be valid, they are not rigorous. The research reported in this paper concerns the derivation of an alternate formulation of the dynamic period equation that avoids this problem. Although the focus of this paper is on the alternate dynamic period equation, the discussion of the controller's design and function applies equally well to the standard equation, which is given in the Appendix.

The specific objectives of this paper are

1. to give the derivation of the alternate formulation of the dynamic period equation
2. to present the rationale for the reactivity constraint approach to the automatic control of reactor power
3. to discuss the construction of a digital controller based on the alternate dynamic period equation
4. to present the initial experimental results from the evaluation of that controller.

DYNAMIC PERIOD EQUATION

The instantaneous reactor period $[\tau(t)]$ is defined as $\tau(t) = 1/\omega(t)$, where

$$\dot{T}(t) \equiv \omega(t)T(t) \ , \tag{1}$$

and $T(t)$ denotes the amplitude function, which is a weighted integral of all neutrons present in the reactor. The dynamic period equation gives the instantaneous reactor period as a function of the reactivity and the rate of change of reactivity. Both the standard and alternate formulations are derived from the point kinetics equations by using the same overall approach. The two derivations differ in that (a) each uses a different definition of an effective multigroup decay parameter, and (b) the alternate formulation defers use of the multigroup decay parameter until after completion of the major step in the derivation, which is differentiation of the first point kinetics equation. It is the result of this last difference that the alternate formulation avoids the presence of a derivative-containing term in the resulting expression for the period. Details of the derivation of the standard formulation are given in Refs. 2 and 8. The derivation of the alternate approach is given here.

With the definition of the instantaneous period [Eq. (1)], the first and second point kinetics equations can be written as

$$\omega(t)T(t) = \frac{[\rho(t) - \bar{\beta}]}{l^*} T(t) + \Sigma\lambda_i C_i(t) \tag{2}$$

and

$$\dot{C}_i(t) = \frac{\bar{\beta}_i}{l^*} T(t) - \lambda_i C_i(t) \quad \text{for } i = 1,N \ , \tag{3}$$

where

$\rho(t)$ = net reactivity

$\bar{\beta}$ = effective delayed neutron fraction

$\bar{\beta}_i$ = effective fractional yield of the i'th group of delayed neutrons

l^* = prompt neutron lifetime

λ_i = decay constant of the i'th precursor group

$C_i(t)$ = concentration of the i'th precursor group

N = number of delayed neutron groups, usually six.

The first step in the derivation is to differentiate the first point kinetics equation. Then, using the definition of the instantaneous period to eliminate the derivative of the amplitude function yields

$$\dot{\omega}(t)T(t) + [\omega(t)]^2 T(t) + \frac{\bar{\beta} - \rho(t)}{l^*} \omega(t)T(t) = \frac{\dot{\rho}(t)}{l^*} T(t) + \Sigma\lambda_i \dot{C}_i(t) \ . \tag{4}$$

The next step is to eliminate the rate of change of the precursor concentrations by substitution of the second point kinetics equation. This yields

$$\dot{\omega}(t)T(t) + [\omega(t)]^2 T(t) + \frac{\bar{\beta} - \rho(t)}{l^*} \omega(t)T(t) = \frac{\dot{\rho}(t)}{l^*} T(t) + \frac{\Sigma\lambda_i\bar{\beta}_i}{l^*} T(t) - \Sigma\lambda_i^2 C_i(t) \ . \tag{5}$$

To eliminate the precursor concentrations, an effective multigroup decay parameter is defined as

$$\lambda_e'(t) \equiv \Sigma\lambda_i^2 C_i(t)/\Sigma\lambda_i C_i(t) \ . \qquad (6)$$

It is important to recognize that this effective multigroup decay parameter is a time-dependent quantity. Its value will vary during transients because the relative concentrations of the various delayed neutron precursor groups change depending on the rate at which power is being raised or lowered. Combining this definition with the first point kinetics equation yields

$$\Sigma\lambda_i^2 C_i(t) = \lambda_e'(t)\left[\omega(t)T(t) + \frac{\bar{\beta} - \rho(t)}{l^*}\, T(t)\right] \ . \qquad (7)$$

Substitution of Eq. (7) into Eq. (5) achieves the specified objective, which is elimination of the precursor concentrations. The result is

$$\dot{\omega}(t)T(t) + [\omega(t)]^2 T(t) + \frac{\bar{\beta} - \rho(t)}{l^*}\,\omega(t)T(t) = \frac{\dot{\rho}(t)}{l^*}\,T(t) + \frac{\Sigma\lambda_i\bar{\beta}_i}{l^*}\,T(t) - \lambda_e'(t)\left[\omega(t)T(t) + \frac{\bar{\beta} - \rho(t)}{l^*}\,T(t)\right] \ . \qquad (8)$$

Dividing through by the amplitude function $T(t)$, noting that $\bar{\beta} = \Sigma\bar{\beta}_i$, and rearranging terms yields

$$\omega(t)\left[\frac{\dot{\omega}(t)}{\omega(t)} + \omega(t) + \frac{\bar{\beta} - \rho(t)}{l^*} + \lambda_e'(t)\right] = \frac{\dot{\rho}(t)}{l^*} + \frac{\lambda_e'(t)\rho(t)}{l^*} + \frac{\Sigma\bar{\beta}_i[\lambda_i - \lambda_e'(t)]}{l^*} \ . \qquad (9)$$

Solving for $\omega(t)$ yields

$$\omega(t) = \frac{\dot{\rho}(t) + \lambda_e'(t)\rho(t) + \Sigma\bar{\beta}_i[\lambda_i - \lambda_e'(t)]}{[\bar{\beta} - \rho(t)] + l^*\left[\dfrac{\dot{\omega}(t)}{\omega(t)} + \omega(t) + \lambda_e'(t)\right]} \ . \qquad (10)$$

Noting that the quantity $l^*(\dot{\omega}/\omega + \omega + \lambda_e')$ is small in comparison with the term $(\bar{\beta} - \rho)$ leads to the following expression for the instantaneous reactor period $[\tau(t)]$:

$$\tau(t) \simeq \frac{\bar{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e'(t)\rho(t) + \Sigma\bar{\beta}_i[\lambda_i - \lambda_e'(t)]} \ . \qquad (11)$$

Equation (11) is the alternate formulation of the dynamic period equation, and the quantity $\lambda_e'(t)$ is designated as the alternate effective multigroup decay parameter.

DETERMINATION OF THE EFFECTIVE MULTIGROUP DECAY PARAMETER

A precondition for the implementation of a controller based on the dynamic period equation is that it be possible to evaluate the effective multigroup decay parameter in real time during transients. Two methods for accomplishing this have been established. Both have been used successfully in conjunction with closed-loop control experiments conducted on the MIT research reactor. The first uses validated measurements of the reactor power in conjunction with the second point kinetics equation to obtain an estimate of the concentration of each delayed neutron precursor group at every sampling interval. The effective multigroup decay parameter can then be determined from its definition. This approach has the advantage that the evaluation is made without resorting to approximation. However, given that some of the precursor groups are short-lived, a small sampling interval is required in order to assure numerical stability.

A second approach, which does not restrict the sampling interval, is to develop a correlation between the net reactivity and the effective multigroup decay parameter. Specifically, the decay parameter can be determined approximately by first rewriting Eq. (11) as

$$\lambda_e'(t) = \frac{\tau(t)\,[\Sigma(\bar{\beta}_i\lambda_i) + \dot{\rho}(t)] - [\bar{\beta} - \rho(t)]}{\tau(t)\,[\bar{\beta} - \rho(t)]} \ . \qquad (12)$$

Noting that the term $\Sigma(\bar{\beta}_i\lambda_i)$ is typically more than an order of magnitude greater than that of any specified rate of change of reactivity, the latter is set to zero. Next, the reactivity associated with a given *instantaneous* period $[\tau(t)]$ is estimated by using the inhour formula (which, of course, is valid only for an *asymptotic* period). As a result, a curve of the effective multigroup decay parameter versus the reactivity can be constructed from Eq. (12). Figure 1 depicts the resulting correlation for the 5-MW(thermal) MIT research reactor (MITR-II), which has an effective delayed neutron fraction of 0.00786. (Note: This rather large value of β_{eff} is due to the MITR-II's being heavy water reflected.) As shown in the figure, the magnitude of the decay parameter increases with positive reactivity. This occurs because the concentrations of the short-lived precursor groups remain near their equilibrium values during power increases while those of the longer-lived groups lag behind. Hence, the concentrations of the short-lived groups rise relative to the longer-lived ones during power increases. The opposite occurs during power decreases. The use of this correlation is not limited by the sampling interval. However, its use is restricted to transients for which

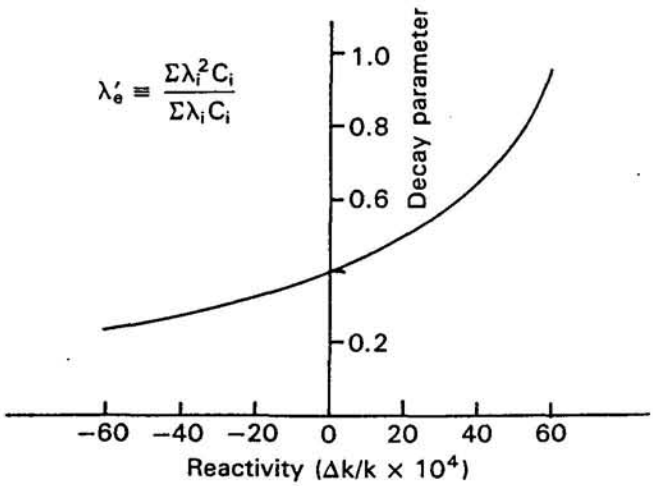

Fig. 1. Effective multigroup decay parameter. This correlation was obtained assuming both zero rates of change of reactivity and asymptotic periods.

the assumptions of small rates of reactivity change and near-asymptotic periods are reasonably valid.

BEHAVIOR OF COMPONENT TERMS

Figure 2 shows the results of a simulation study in which the component terms of the alternate formulation of the dynamic period equation were determined. The reactivity pattern for which this evaluation was performed is representative of that obtained when using the MIT-CSDL nonlinear digital controller[3] (NLDC), which is the implementing algorithm for the previously mentioned reactivity constraint approach. As shown in the lower portion of the figure, such transients are characterized by a continuous insertion, an extended hold, and finally a gradual removal of reactivity.

The simulated reactor was originally at steady state under conditions of delayed neutron equilibrium.

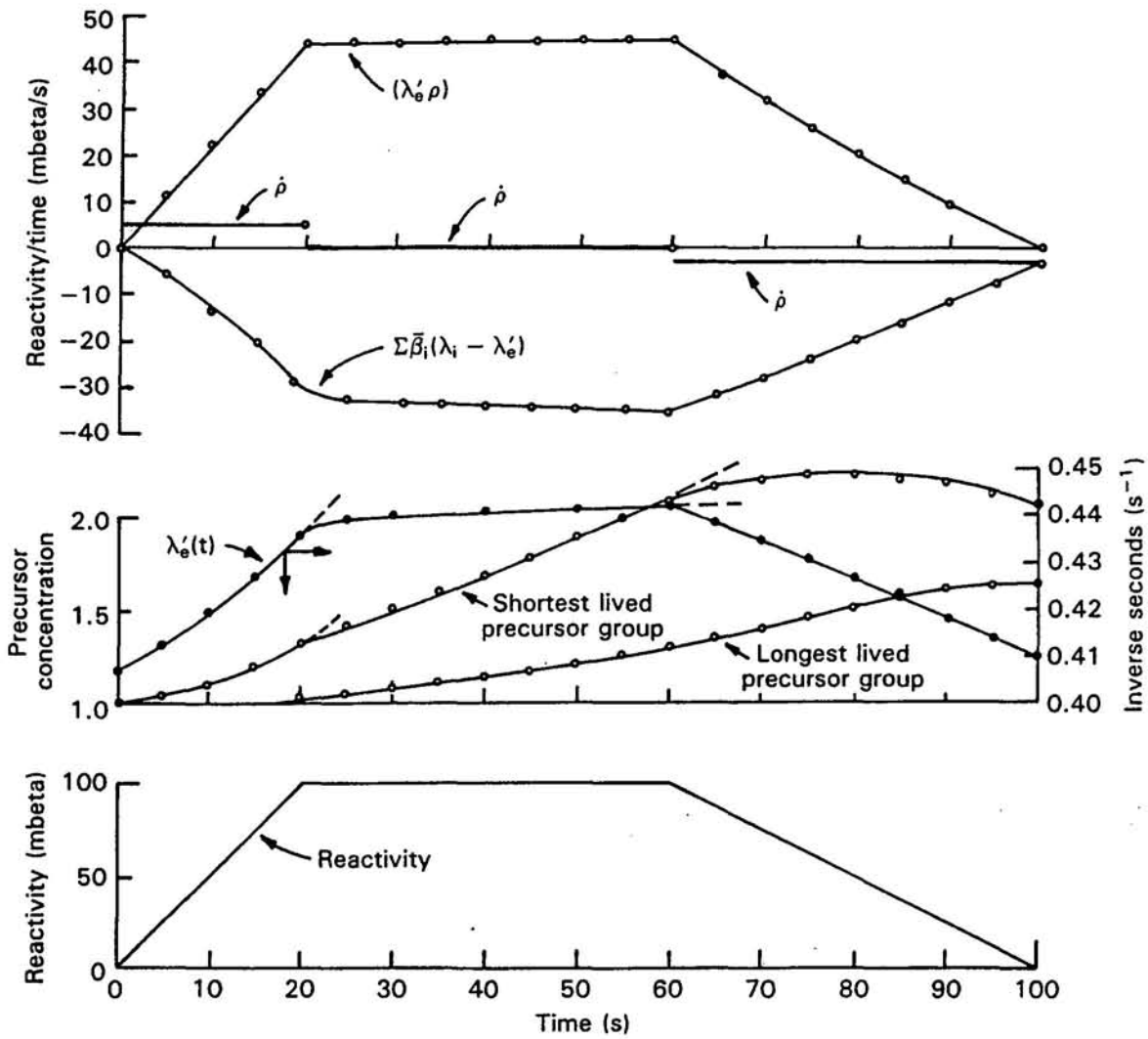

Fig. 2. Behavior of component terms—alternate dynamic period equation.

Hence, the period was infinite and the three component terms in the denominator of the alternate formulation of the dynamic period equation [Eq. (11)] were each equal to zero. The equilibrium value of the effective multigroup decay parameter was 0.407 inverse seconds. Reactivity was then added for 20 s at the rate of 5 mbeta/s (0.0039% $\Delta k/k$/s), held constant at 100 mbeta for 40 s, and then removed for 40 s at the rate of 2.5 mbeta/s. The middle portion of the figure shows the behavior of the effective multigroup decay parameter and the normalized concentrations of the shortest and longest lived delayed neutron precursor groups. As expected, the short-lived group responds rapidly to changes in the reactivity insertion rate while the long-lived group's response is quite sluggish. The behavior of the decay parameter is dominated by that of the short-lived precursor groups and therefore conforms to the reactivity pattern. This is as expected because that parameter is defined in terms of the square of the precursor decay constants and is therefore quite sensitive to changes in the concentrations of the short-lived groups, which have the largest decay constants. The upper portion of the figure depicts the behavior of the three terms that form the denominator of the alternate formulation of the dynamic period equation. The quantity $(\lambda'_e\rho)$ conforms to the pattern of the reactivity with some slight curvature being due to the nonlinearity of the decay parameter. The term $[\Sigma\bar{\beta}_i(\lambda_i - \lambda'_e)]$ is the more interesting. Its behavior follows that of (λ'_e) except that it is of opposite sign. Thus, this term goes rapidly negative, remains almost constant, and then slowly goes to zero.

REACTIVITY CONSTRAINT APPROACH

The objective of the reactivity constraint approach is to provide a means for the closed-loop digital control of reactor power during transients so that there will not be a challenge to the reactor's safety system. Achievement of this goal requires planning because the rate of change of reactor power depends on both the rate of change of reactivity and the amount of reactivity present. This dual dependency, which is characteristic of time-delayed processes, is the result of the interval that elapses between the creation of precursors and the appearance of delayed neutrons. The production of fission products, including precursors, is directly proportional to the transient reactor power. However, such is not the case with the delayed neutron population because of the time dependence of the precursor decay. Upon attaining the desired power level, the rate of change of the delayed neutrons cannot be immediately halted. Rather, when a new constant power level has been reached, the delayed neutron population will continue to change until it attains an equilibrium value corresponding to that power level. Hence, if power is to be leveled smoothly, it is essential to limit the delayed neutron contribution so that, upon attainment of the desired power, the insertion of the control mechanism will make the rate of change of the prompt neutrons sufficiently negative so as to offset the continued rise in the delayed neutron population. This capability is defined here as "feasibility of control." Specifically, a reactor together with a designated control mechanism is defined as being feasible to control if it is possible to transfer the system from a given power level and rate of change of power (i.e., period) to a desired steady-state power level without overshoot (or conversely, undershoot) beyond any specified tolerance bands. Observance of this definition means that not all states (i.e., combinations of reactivity, available rate of change of reactivity, and power) are allowable intermediates through which a reactor may pass while transiting from some initial to some final power. Excluded are both those states that represent actual overshoots and those from which overshoots could not be averted by manipulation of the specified control mechanism. It should be recognized that the concept of feasibility of control is distinct from the more general property of "controllability." This term has a specialized meaning in that a system is said to be controllable if "any initial state can be transferred to any final state in a finite time by some control sequence."[9] This definition does not place any restrictions on intermediate states.

Maintenance of feasibility of control implies that it must be possible to make and maintain the instantaneous period infinite upon attainment of the desired power level. Examination of the dynamic period equation shows that this goal can be realized if the net reactivity is constrained so that the denominator of Eq. (11) can be made less than or equal to zero. [Note: The numerator of Eq. (11) is, for reasons of safety, always positive (i.e., $\rho \ll \bar{\beta}$).] That is, the following condition must be met:

$$\{\lambda'_e(t)\rho(t) + \Sigma\bar{\beta}_i[\lambda_i - \lambda'_e(t)] + \dot{\rho}_f\} \leq |\dot{\rho}_c| \ , \quad (13)$$

where the term $[\rho(t)]$ is the net reactivity, both that added deliberately by the control mechanisms and that present indirectly from feedback effects. The quantity $\dot{\rho}_f$ denotes the rate of change of reactivity due to thermal-hydraulic feedback effects, and the symbol $|\dot{\rho}_c|$ denotes the maximum available rate of change of reactivity that could be obtained were a control mechanism to be moved. As such, $|\dot{\rho}_c|$ is always a nonzero finite number regardless of whether or not the mechanism is actually being moved.

It is worth identifying the origin of each term in Eq. (13) by tracing its appearance through the derivation of the alternate dynamic period equation. Then it is apparent that the $\dot{\rho}$ terms represent the effect of a changing prompt neutron population on the reactor power. Similarly, the terms $\lambda'_e\rho$ and $\Sigma\bar{\beta}_i(\lambda_i - \lambda'_e)$, if taken together, correspond to the net impact of a

changing delayed neutron population and to a changing distribution of precursors within the defined groups. Thus, Eq. (13) is really a statement that delayed neutron effects, which are not subject to direct control, must be restricted to that which can be offset by an induced change in the prompt population.

Experimental evaluation of Eq. (13) has shown that it should not, by itself, be the basis of a general purpose controller. Problems exist relative to the terms $\dot{\rho}_f$ and $\Sigma\bar{\beta}_i(\lambda_i - \lambda_e')$. Specifically, the rate of change of the thermal feedback $\dot{\rho}_f$ is normally negative whenever temperature is increasing. Thus, in the presence of a strong thermal feedback, the condition for halting a power increase can be satisfied with a smaller value of $|\dot{\rho}_c|$ than would otherwise be the case. However, reliance on the $\dot{\rho}_f$ term may cause difficulties when leveling power. For example, assume that the condition specified by Eq. (13) is satisfied during the initial stages of a transient by the presence of a large negative rate of change of reactivity due to thermal feedback. This term will, of course, become zero once the reactor attains steady-state conditions. Should the $\dot{\rho}_f$ term actually go to zero faster than the $\lambda_e'\rho$ term can be reduced, then it will not be possible to satisfy continuously the condition for halting the power increase. That is, even though the inequality represented by Eq. (13) is initially fulfilled, it may not be possible to keep it satisfied once reactivity removal is initiated. Thus, dependence on the $\dot{\rho}_f$ term can create a situation wherein an overshoot is unavoidable. A similar difficulty exists with respect to the quantity $\Sigma\bar{\beta}_i(\lambda_i - \lambda_e')$ because it also opposes power changes and it also may go to zero faster than the $(\lambda_e'\rho)$ term. Therefore, the simplest approach is not to rely on either of these quantities. Such an action is justified because it is conservative. Its net effect is that the controller's response will be slower than would otherwise be the case. [Note: Deletion of the term corresponding to $\Sigma\bar{\beta}_i(\lambda_i - \lambda_e')$ from the standard formulation is not so readily justified as discussed in Ref. 7. This is therefore one of the advantages of the alternate approach.]

If both the $\Sigma\bar{\beta}_i(\lambda_i - \lambda_e')$ and the $\dot{\rho}_f$ terms are deleted, Eq. (13) becomes

$$[\lambda_e'(t)\rho(t)] \leq |\dot{\rho}_c| \ . \tag{14}$$

Equation (14) is an "absolute reactivity constraint." Under this constraint, reversal of the direction of motion of the specified control mechanism will negate the effect of the reactivity present and make the period infinite at any time during a transient. This constraint is overly restrictive because it is generally not necessary to be able to level the power at any time during a transient but only at the desired termination point. A less stringent constraint can be written that specifies that there should be sufficient time available to eliminate whatever reactivity is present beyond the amount that can be immediately negated by reversal of the direction of motion of the designated control mechanism before the desired power level is attained. This requirement, a "sufficient reactivity constraint," can be written for power increases as

$$[\rho(t) - |\dot{\rho}_c|/\lambda_e'(t)]/|\dot{\rho}_c| \leq \tau(t)\ln[P_F/P(t)] \ , \tag{15}$$

where P_F and $P(t)$ are the desired and current power levels, respectively, the quantities $\rho(t)$ and $|\dot{\rho}_c|$ are as previously defined, and $\tau(t)$ is either the observed (instantaneous) reactor period or the asymptotic period that corresponds to the net reactivity, whichever results in a more conservative decision. Note that Eq. (15) is approximate because it assumes that both the reactivity to be annulled and the instantaneous period remain constant during control mechanism movement. Hence, for a sufficient constraint to be of practical value in reactor control, it should be evaluated at a sampling rate of approximately once per second.

CONTROLLER DESIGN AND OPERATION

The MIT-CSDL NLDC is the implementing algorithm for the reactivity constraint approach. It has a multitiered structure consisting of supervisory and predictive routines as well as a suitable man/machine interface. The supervisory algorithm originally used the standard sufficient constraint to provide a method for determining if the control signal should be changed at the *present* time in order to avoid an overshoot at some *future* time. This is the unique feature of the reactivity constraint approach. It permits the on-line determination of the proper moment for initiation of reactivity removal during a transient. Its drawback is that it does not predict the actual power trajectory. Such estimates are therefore the role of the predictive routine, which might embody a model-based linear control technique. As such, the predictive algorithm can project the reactor power forward in time but it cannot determine if it will be possible to halt the transient upon attaining the desired power. Hence, both the supervisory and predictive routines are essential. It should be recognized that the requirement for separate supervisory and predictive control routines stems from the time-delayed nature of reactor dynamics. Specifically, if a system is linear, then establishment of a means to predict the time response of the parameter that is to be controlled is tantamount to determining the sequence of control signals. In contrast, the control of a time-delayed system necessitates the development of special criteria that define the specific conditions under which the rate of change of the controlled parameter can be halted.

License approval has been received from the U.S. Nuclear Regulatory Commission (NRC) for the general use of the NLDC on the MIT research reactor. Also, an experimental protocol has been developed

that permits novel closed-loop control strategies to be tested on-line within the purview of the NLDC. Receipt of this license approval was significant because it implies that the avoidance of challenges to the safety system achieved by incorporation of the concept of feasibility of control in the control algorithm is as acceptable as imposing limitations on the strength of the associated actuators. A diagram of the NLDC is given in Ref. 4. Details of its license approval and the resulting experimental protocol are given in Refs. 6 and 10, respectively.

A controller similar to the NLDC was constructed for use with constraints based on the alternate formulation of the dynamic period equation. This alternate controller retains the original multitiered structure. In particular, the alternate sufficient constraint [Eq. (15)] is used to fulfill a supervisory role. Its function is to evaluate the decisions of the associated control law and to verify that no challenge will be made to the safety system as a result of implementing those decisions. This arrangement permits changes in the demanded power to be readily and safely accomplished. For example, suppose the control law were simply to move the control mechanism at a fixed speed should the deviation between the desired and actual power exceed a specified band. A power increase is desired. Initially, the reactor is at steady state with the control law maintaining the power within the allowed deadband. Once the power setpoint is changed, the control law signals for withdrawal of the control rod. The reactivity constraint is initially satisfied and the withdrawal is permitted. It continues until the constraint becomes limiting. Once this occurs, blade withdrawal is halted even if the control law is signaling for its continuation. The reactor period then lengthens from its dynamic to its asymptotic value. The constraint is again satisfied and further blade withdrawal is possible. This continues until the constraint cannot be satisfied by a cessation of blade withdrawal. Blade insertion then begins. The period lengthens, the constraint is met, and the blade insertion is halted until maintenance of the constraint again requires it. The net effect is that the blade is initially withdrawn continuously, then held more or less constant, and finally inserted in a stepwise fashion.

EXPERIMENTAL EVALUATION

Both the standard and the alternate sufficient reactivity constraints lend themselves to physical interpretation. Relative to the latter, the quantity $(|\dot{\rho}_c|/\lambda_e')$ is the reactivity that can be rapidly negated by reversing the direction of travel of the designated control mechanism. The left side of the constraint is therefore the time necessary to remove whatever reactivity is present beyond the amount that can be negated by reversal of the direction of the control mechanism's motion. This is referred to as the "required time." The right side of the constraint is an estimate of the time remaining to attain full power. This estimate, which is called the "available time," assumes that the current period will be maintained and is therefore conservative during the interval in which power is being leveled.

The initial trial of a controller based on the alternate dynamic period equation was conducted successfully on August 16, 1985. An extensive test program, similar to that described in Refs. 4 and 5, was then conducted. Figure 3 shows some of the results of a closed-loop control test session in which power was raised from 1 to 3 MW using a shim blade with a differential worth of -8.15 mbeta/s (0.0064% $\Delta k/k$/s). Shown in the upper portion of the figure are the reactivity and power trajectories. The lower portion of the figure depicts the required and available times. Note that reactivity removal was initiated when the required and available times became equal, and that this process was begun at 2.25 MW, well in advance of attaining the desired power level. Another interesting feature shown in this figure is that reactor power was leveled and held constant despite the presence of positive reactivity. This was achieved by inserting the control mechanism, thereby causing a negative rate of change in the prompt neutron population that was sufficient to offset the continued rise of the delayed neutrons. Finally, note that the response of the controller was quite rapid during the first 80% of the specified increase and then somewhat sluggish for the remaining 20% of the power change. This was to be expected because the magnitude of the alternate formulation's decay parameter was relatively large. Hence, the amount of reactivity $(|\dot{\rho}_c|/\lambda_e')$ that the alternate controller allowed to be present upon approaching full power was small. This in turn meant that the alternate controller reduced the excess reactivity early in the transient and therefore required considerable time to complete the power change. Thus, while the overall response for this controller was reasonable and certainly comparable to that achieved under manual control, it was somewhat longer than that possible with the standard constraint. The latter had an effective multigroup decay parameter that was approximately five times smaller than that of its alternate counterpart. Thus, the standard constraint would have allowed -100 mbeta (0.0786% $\Delta k/k$) to be present during the final approach to power rather than the 20 mbeta (0.0157% $\Delta k/k$) that was permitted by the alternate constraint. The results of a systematic experimental comparison of controllers based on the standard and alternate sufficient constraints are given in Ref. 11.

SUMMARY AND CONCLUSIONS

A new method for the nonlinear closed-loop digital control of reactor power has been described and

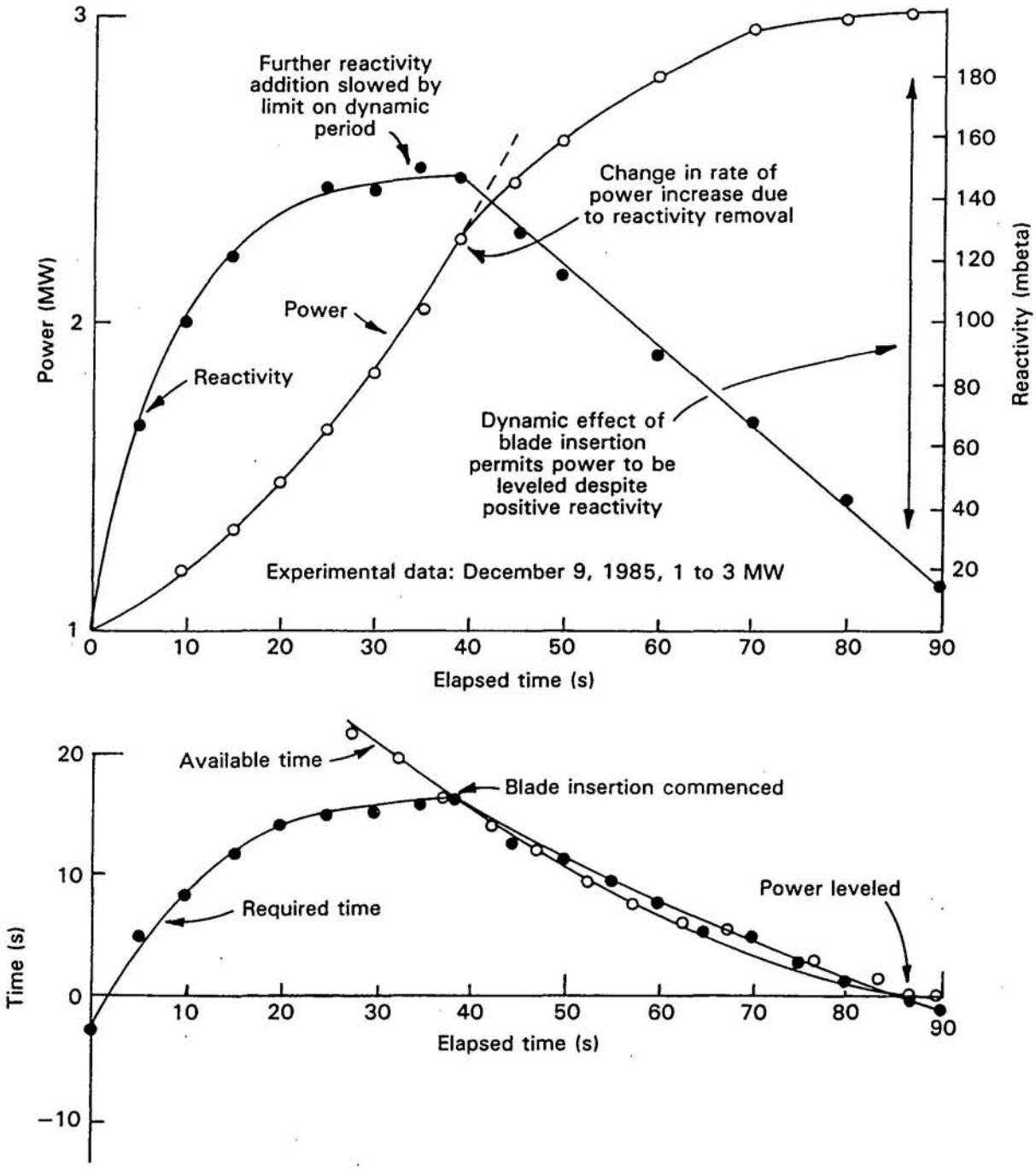

Fig. 3. Power increase via closed-loop digital control using alternate controller.

details of its initial experimental trials reported. This new technique functions within the overall framework of the reactivity constraint approach, a methodology that has received license approval from the NRC for general use on the 5-MW(thermal) MIT research reactor. This new method for reactor control is based on an alternate formulation of the dynamic period equation. As such, it has two major advantages relative to previously reported control schemes based on the standard formulation. First, each of the alternate equation's component terms can be accurately measured on-line in real time. Second, no approximations are required in the development of the associated reactivity constraints. The initial trials of this new controller showed it to be an efficient, reliable means of adjusting the reactor power. Further research is now ongoing in regard to additional uses of the alternate dynamic period equation.

APPENDIX

STANDARD FORMULATION OF THE DYNAMIC PERIOD EQUATION

Details of the standard formulation of the dynamic period equation and its use as the basis of the MIT-CSDL NLDC have been previously reported.[2-8] The following material is provided as a convenience. The standard dynamic period equation is

$$\tau(t) = \frac{\bar{\beta} - \rho(t) + l^*\left[\frac{\dot{\omega}(t)}{\omega(t)} + \omega(t) + \lambda_e(t) - \frac{\dot{\lambda}_e(t)}{\lambda_e(t)}\right]}{\dot{\rho}(t) + \lambda_e(t)\rho(t) + \frac{\dot{\lambda}_e(t)}{\lambda_e(t)}\,[\bar{\beta} - \rho(t)]} , \tag{A.1}$$

where symbols are as previously given except that the standard effective multigroup decay parameter is defined as

$$\lambda_e(t) \equiv \Sigma\lambda_i C_i(t)/\Sigma C_i(t) \ . \tag{A.2}$$

The equilibrium value of (λ_e) is 0.079 inverse seconds for the MIT research reactor. The resulting sufficient reactivity constraint for power increases is

$$\{\rho(t) + [\dot{\lambda}_e(t)/\lambda_e(t)]\,[\bar{\beta} - \rho(t)]/\lambda_e(t) - |\dot{\rho}_c|/\lambda_e(t)\}/|\dot{\rho}_c| \le \tau(t)\ln[P_F/P(t)] \ . \tag{A.3}$$

The term containing the quantity $[\dot{\lambda}_e(t)]$ is difficult to evaluate on-line and may, for the reasons discussed in Ref. 7, be neglected. Hence, the constraint is normally written as

$$[\rho(t) - |\dot{\rho}_c|/\lambda_e(t)]/|\dot{\rho}_c| \le \tau(t)\ln[P_F/P(t)] \ . \tag{A.4}$$

Figures showing closed-loop changes of the reactor power that were accomplished using Eq. (A.4) under conditions similar to those for which the data shown in Fig. 3 were obtained are given in Refs. 5 and 6. [Note: The nomenclature used in this paper occasionally differs from that given in earlier publications. For example, the quantity (λ_e) was originally termed an "effective one-group decay constant" rather than an "effective multigroup decay parameter." The latter terminology is more appropriate because (λ_e) is a time-dependent measure of the relative weighting of the various precursor groups. Also, Eq. (A.1) was previously called the "exact" form of the dynamic period equation. The word exact has now been dropped because it is superfluous. The word standard or alternate has been added to indicate the method of derivation.]

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