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8.012 Physics I: Classical Mechanics
Fall 2008

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.012

Fall 2007

Exam 2
Thursday, November 15, 2007

NAME: _____

MIT ID number: _____

Instructions:

1. Do all **FIVE (5)** problems. You have **90 minutes**.
2. **SHOW ALL OF YOUR WORK.** Be sure to **circle your final answer**.
3. Read each question carefully.
4. All work must be done in this booklet. Extra blank pages are provided.
5. **NO** books, notes, calculators or computers are permitted.
6. **A sheet of useful equations is provided on the last page.**

Your Scores

Problem	Maximum	Score	Grader
1	10		
2	25		
3	25		
4	20		
5	20		
Total	100		

Problem 1: Quick Multiple Choice Questions [10 pts]

Answer all five problems. You do not need to show any work. **Do not spend a long time on this section.**

(a) What are the dimensions of the universal gravitational constant G ? Recall that $[M]$ = mass, $[L]$ = length and $[T]$ = time

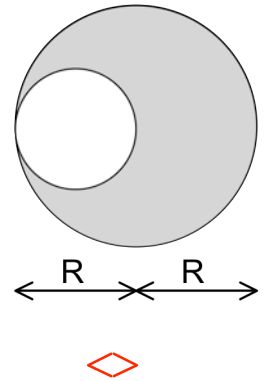
$$[M][L][T]^{-2}$$

$$[M][L]^2[T]^{-1}$$

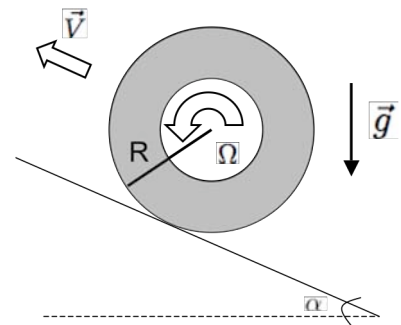
$$[M]^{-1}[L]^2[T]^{-1}$$

$$[M]^{-1}[L]^3[T]^{-2}$$

(b) A uniform disk of radius R has a circular hole of diameter R cut out of it. The hole lies between the outer edge of the disk and the center of the disk, as shown to the right. Mark with an “X” where the center of mass of this disk lies.



(c) A hollow cylinder rolls up an incline without slipping. In which direction is the friction force between the cylinder and incline surfaces acting?

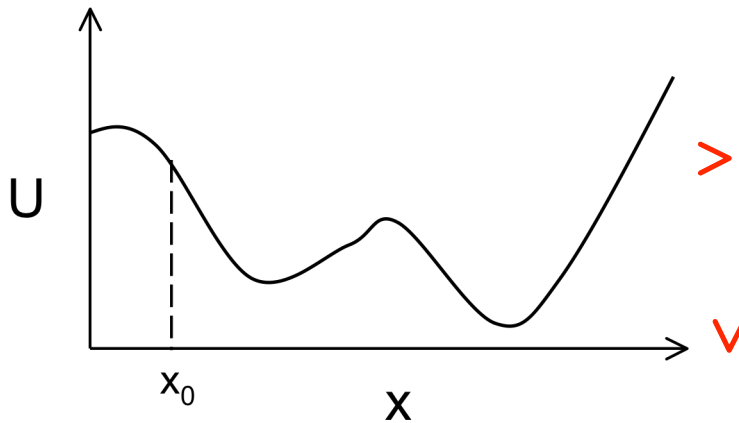


Down the incline

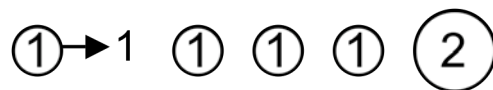
Up the incline

There is no friction force acting on the cylinder

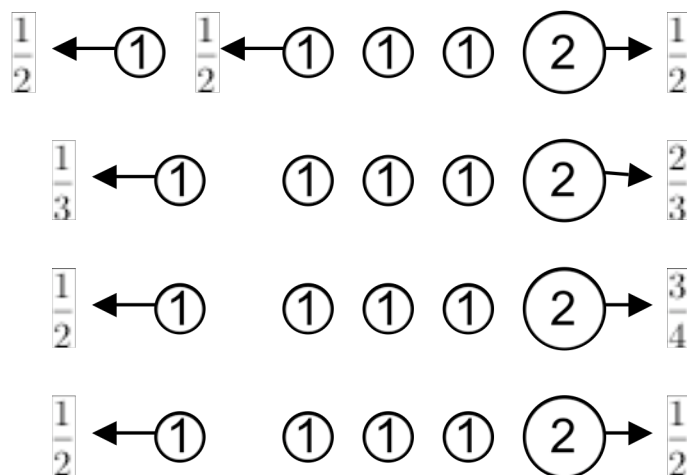
(d) Consider the one-dimensional potential function shown below. Assume that an object in this potential starts at $x = x_0$ at rest. Indicate on the plot the maximum value of x the object can attain and circle the stable equilibrium points in this range.

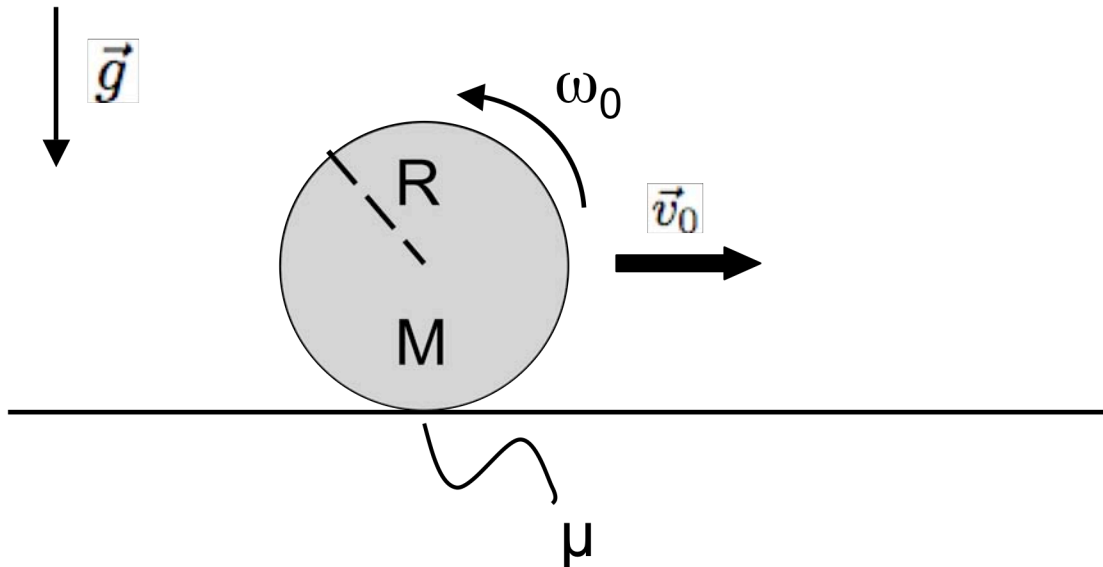


(e) A ball comes in from the left with speed 1 (in arbitrary units) and causes a series of collisions. The initial motion is shown below (the number in the circles indicates the objects' relative masses):



All collisions are elastic. Which of the following indicates the motions of the balls after all of the collisions are completed? (circle the correct diagram)



Problem 2: The Bowling Ball [25 pts]

A bowling ball of radius R , mass M and uniform mass density is thrown down a lane with initial horizontal speed v_0 . The ball is given some backspin – it is spun in the opposite direction of motion – with initial angular rate ω_0 as shown. The maximum coefficient of friction between the ball and lane surfaces is μ . Assume constant gravitational acceleration \vec{g} points downward.

(a) [10 pts] Find the velocity \vec{v} and angular rotation rate ω of the ball as a function of time after it makes contact with the lane, and up to the point when it starts to roll without slipping. Note: directions are important here!

(b) [10 pts] Find the “final” velocity and angular rotation rate of the ball when it starts to roll without slipping.

(c) [5 pts] How must the initial conditions, v_0 and ω_0 , be related for the ball to come to rest?

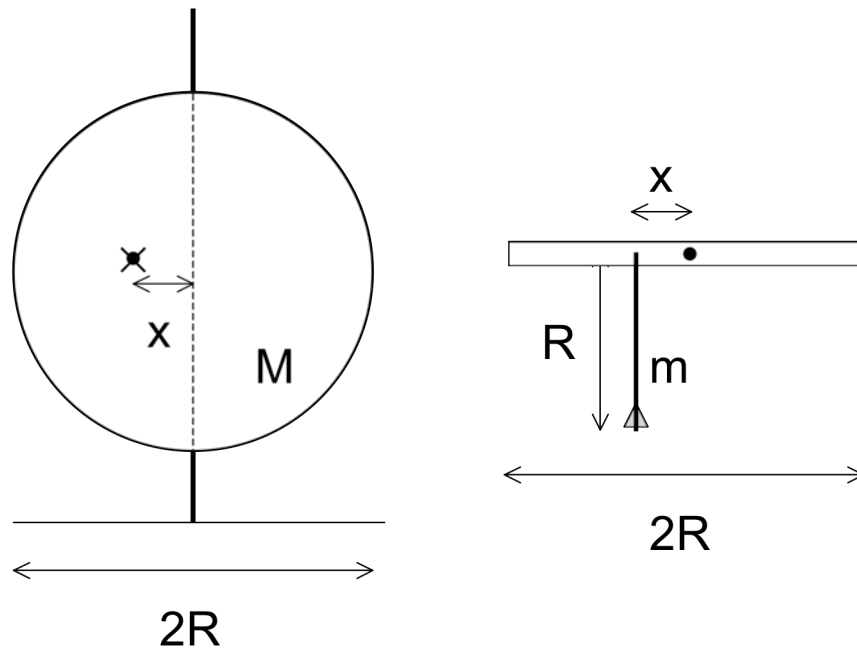
Problem 3: A Generic Potential [25 pts]

Consider the following one-dimensional potential in which an object of mass m moves freely:

$$U(x) = U_0(2s^3 - 3s^2 - 4)$$

where U_0 is a constant with dimensions of energy and $s = x/x_0$ is a normalized dimension of length.

- (a) [5 pts] Sketch this potential as a function of x in the range $-3x_0 < x < 3x_0$.
- (b) [5 pts] Derive values for the equilibrium points and determine which of these are stable.
- (c) [5 pts] Determine the period of small oscillations about the stable equilibrium point.
- (d) [5 pts] If the mass starts from $x = -\infty$ and moves toward positive x , what is the minimum total mechanical energy it needs to reach the stable equilibrium point? What is its velocity at the stable equilibrium point in this case?
- (e) [5 pts] What is the magnitude and direction of the force acting on the mass when it is at $x = 2x_0$?

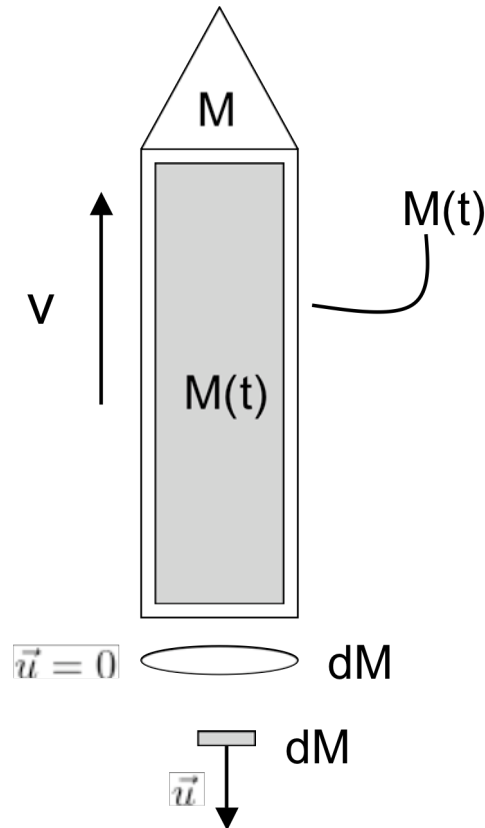
Problem 4: A Ballistic Rotator [20 pts]

Forward view

Top view

A thin arrow with length R , mass m and uniform linear mass density $\lambda = M/R$ is shot with velocity v into a circular target of radius R , mass M , uniform surface mass density $\sigma = M/\pi R^2$, and negligible thickness (i.e., it is essentially a thin disk). The arrow head sticks into the target just off center by a distance $x < R$ as shown above. The target is mounted onto a central frictionless shaft, about which it can spin freely. The target is at rest before it is struck by the arrow.

- (a) [10 pts] Find the angular rotation rate of the target (with arrow embedded) after the arrow strikes it. You can express your answer in terms of I_T , the moment of inertia of the circular target *alone* about the rotation axis shown.
- (b) [5 pts] Show that if $m \ll M$ then to first order the rotation rate depends linearly on both x and v .
- (c) [5 pts] Derive an expression for I_T in terms of M and R . **Warning!** This is a difficult problem, so save it for last!

Problem 5: The Continuous Multi-stage Rocket [20 pts]

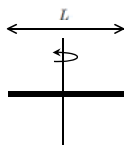
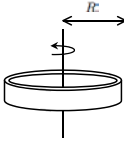
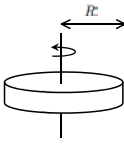
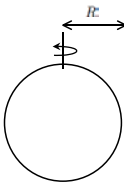
Consider a rocket of total mass $3M$ divided equally into payload (at top), rocket shell, and fuel. The rocket, initially at rest, ejects its fuel at a constant mass loss rate $\gamma = -dM/dt$ and with constant velocity \vec{u} relative to the rocket. However, for each bit of fuel ejected, the rocket also sheds part of its shell, with the same mass loss rate as the fuel but with zero velocity relative to the rocket.

(a) [10 pts] Derive the equation of motion for this rocket, assuming it is moving in free space (i.e., there are no external forces). Note: this is not necessary going to be the same as the standard rocket equation we learned in lecture.

(b) [10 pts] Determine the final speed of the payload after the fuel and shell are ejected in terms of M , u and γ .

USEFUL EQUATIONS

Trajectory for constant acceleration	$\vec{r}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}(0)t + \vec{r}(0)$
Velocity in polar coordinates	$\vec{r} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$
Acceleration in polar coordinates	$\vec{r} = \ddot{r}\hat{r} + r\ddot{\theta}\hat{\theta} + 2\dot{r}\dot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}$
Center of mass of a rigid body	$\vec{R} = \frac{\sum m_i \vec{r}_i}{\sum m_i} = \frac{1}{M} \int \rho \vec{r} dV$
Volume element in cylindrical coordinates	$dV = r dr d\theta dz$
Kinetic energy	$K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$
Work	$W = - \int \vec{F} \cdot d\vec{r} = - \int \vec{\tau} d\theta$
Potential Energy	$U = - \int \vec{F}_c \cdot d\vec{r}$ $\vec{F}_c = -\vec{\nabla}U$ <p>where $\vec{\nabla} \times \vec{F}_c = 0$</p>
Angular momentum	$\vec{L} = \vec{r} \times \vec{p} = I\dot{\theta}$
Torque	$\vec{\tau} = \vec{r} \times \vec{F} = I\ddot{\theta}$

Moment of Inertia	$I_z = \int \rho(x^2 + y^2) dV$ $= \sum_i m_i(x_i^2 + y_i^2)$
Moment of inertia for a uniform bar (about COM)	 $I = \frac{1}{12}ML^2$
Moment of inertia for a uniform hoop (about COM)	 $I = MR^2$
Moment of inertia for a uniform disk (about COM)	 $I = \frac{1}{2}MR^2$
Moment of inertia for a solid uniform sphere (about COM)	 $I = \frac{2}{5}MR^2$
Parallel axis theorem	$I = I_{COM} + MR^2$
Taylor Expansion of f(x)	$f(x) = f(a) + \frac{1}{1!} \frac{df}{dx} \Big _a (x - a)$ $+ \frac{1}{2!} \frac{d^2f}{dx^2} \Big _a (x - a)^2 + \dots$