8.012 Physics I: Classical Mechanics
Fall 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.012   Fall 2005

FINAL EXAM
Wednesday, December 21, 2005

Name: _____

MIT ID number: _____

INSTRUCTIONS:
- Do all **SEVEN (7)** problems. You have 3 hours.
- **Show all work**, and circle your answer.
- All work must be done in this booklet.
- No books, notes, or calculators permitted.


USEFUL RELATIONS:
- Velocity in polar coordinates: $\dot{\mathbf{r}} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\boldsymbol{\theta}}$
- Acceleration in polar coordinates: $\ddot{\mathbf{r}} = (\ddot{r} - r\dot{\theta}^2)\,\hat{\mathbf{r}} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\boldsymbol{\theta}}$
- Fictitious force for rotating coordinates: $\mathbf{F}_{\text{fict}} = -2m\,(\boldsymbol{\Omega} \times \mathbf{v}_{\text{rot}}) - m\,\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$
- Effective potential for central force: $U_{\text{eff}}(r) = U(r) + \frac{L^2}{2\mu r^2}$

Problem	Maximum	Score	Grader
1	15		
2	15		
3	15		
4	15		
5	15		
6	10		
7	15		
TOTAL	100		

1. **Problem 1 of 7**
   **Sliding a crate.** (15 points)

   A mover is trying to slide a uniformly filled crate of length L and height H across the floor. There is friction with coefficient μ between the crate and the floor. The mover exerts a horizontal force F at the upper back edge of the crate. If $\mu > \mu_0$, the crate will tip over before it slides. Calculate the critical friction coefficient μ_0. Gravity is directed downward.
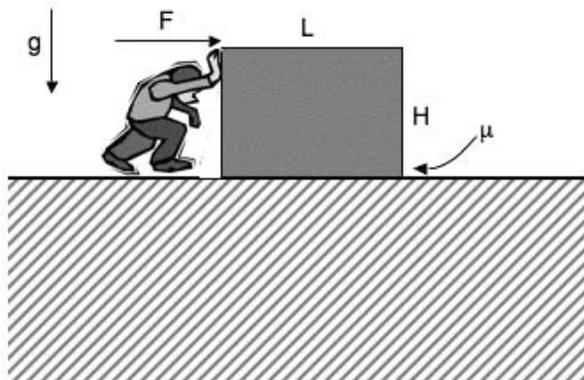
2. **Problem 2 of 7**
   **Reel of tape on a frictionless surface.** (15 points)

   A long length of tape is wrapped around a reel which is at rest on a frictionless surface. The tape itself has negligible mass, but the reel has mass M, radius R, and moment of inertia I_0 about its center. The end of the tape is pulled horizontally with a constant force F, as shown in the figure. Calculate x, the horizontal displacement of the reel from its initial position when a length s of tape has unwound from the reel.
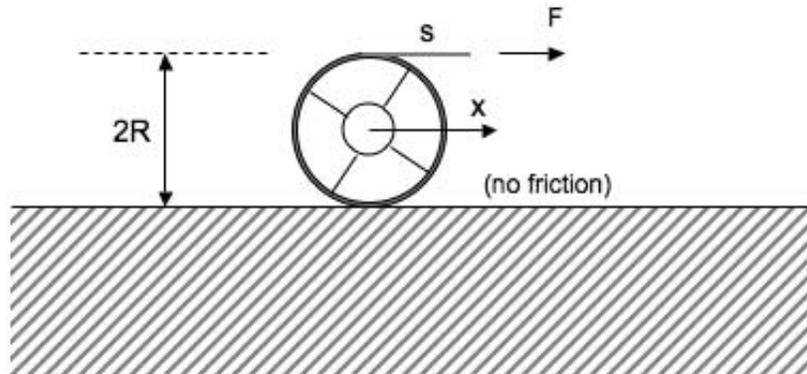
3. **Problem 3 of 7**
   Falling rod. (15 points)

   A uniform rod of length L and mass M is free to rotate about a pivot at its lower end. An attached bead of negligible mass is free to move without friction along the rod, but a massless collar fixed on the rod at a distance h from the pivot requires the bead's distance from the pivot to be $\geq h$. Initially, the rod is at rest and nearly vertical, and the bead is resting on the collar. The rod is then released and falls over. Compute the angle θ from the vertical at which the bead loses contact with the collar. (You should give the full expression for any moment of inertia in your answer.) Gravity is directed downward.
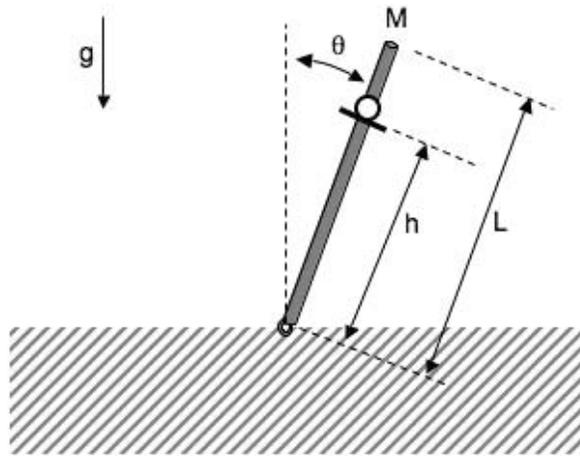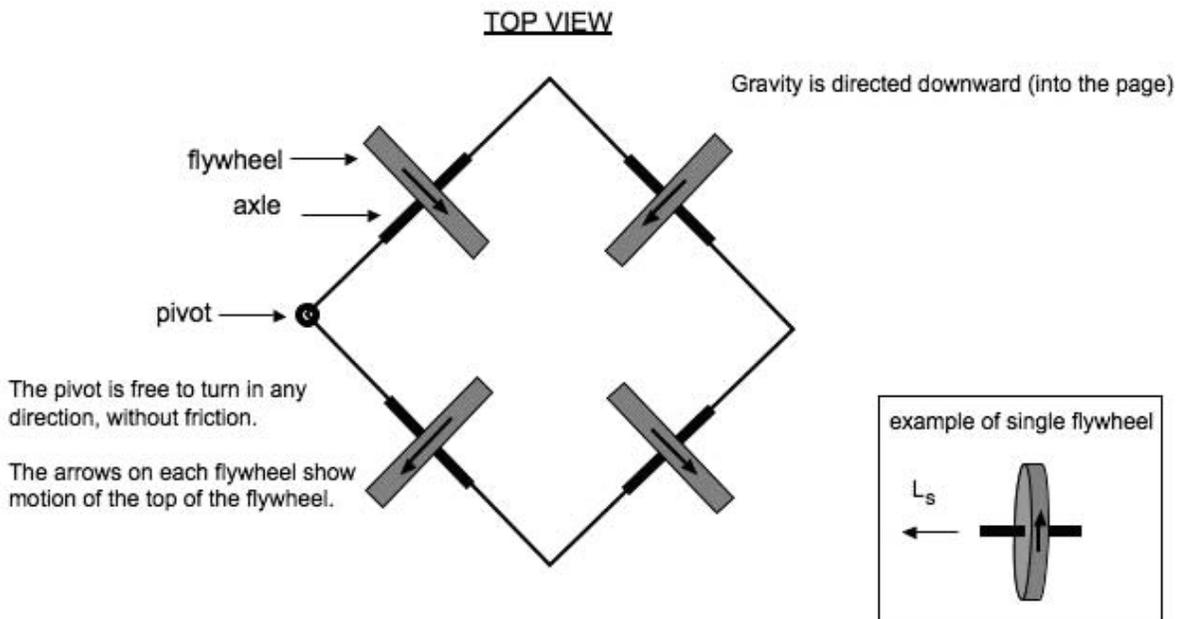
4. **Problem 4 of 7**
   Compound gyroscope. (15 points)

   A compound gyroscope consists of four identical flywheels mounted at the midpoints of the edges of a rigid square frame of length L. Each flywheel has moment of inertia I_0 and is spinning rapidly at the same angular velocity ω, with the sense of rotation shown in the figure. The entire assembly (frame plus flywheels) has total mass M, and the center of mass is at the geometric center of the square frame. One corner of the frame is attached to a frictionless pivot, so that the entire frame is free to rotate about the pivot in any direction. In the top-view figure, gravity is directed into the page.

   (a) (10 points) Find the frequency of uniform precession Ω and indicate its direction.

   (b) (5 points) Now suppose the entire compound gyroscope is placed inside an elevator. The elevator accelerates upward (opposite to gravity, so out of the page in the figure) with uniform acceleration A. Find the new frequency of uniform precession.

TOP VIEW

Gravity is directed downward (into the page)

flywheel

axle

pivot

The pivot is free to turn in any direction, without friction.

The arrows on each flywheel show motion of the top of the flywheel.

example of single flywheel

L_s

5. **Problem 5 of 7**

   Particle in a central force field. (15 points)

   A mass m moves under the influence of an attractive central force Ar^4 with angular momentum L, where A and L are positive constants. Define the potential energy to be zero at the origin. For what total energy will the motion be circular, and what is the radius of this circular orbit?

6. **Problem 6 of 7**

   Colliding blocks. (10 points)

   A mass M_1 is sliding with velocity v_0 along a frictionless table toward three masses M_2, M_3, and M_4 which are sitting at rest in the path of M_1, as shown. The masses get successively much smaller, so that

   $$M_4 \ll M_3 \ll M_2 \ll M_1,$$

   where the symbol "$\ll$" means "much smaller than". What is the velocity of mass M_4 after all the collisions have happened? Assume that all the collisions are perfectly elastic.

   Hint: This problem is easiest if you consider each collision individually and make a judicious choice for the coordinate frame each time.
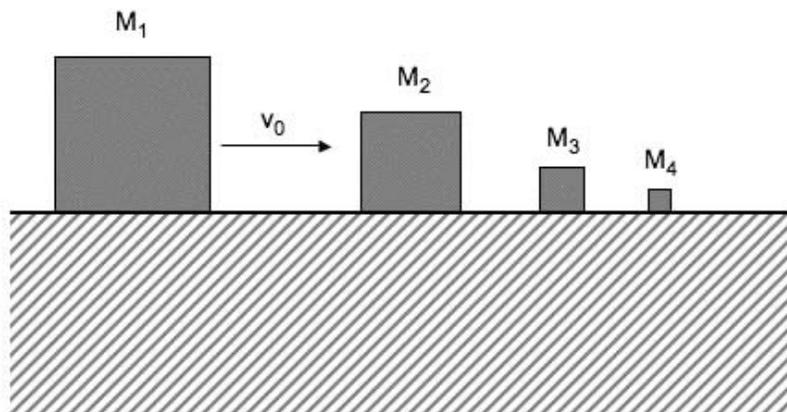
7. **Problem 7 of 7**

   Projectile near two gravitating spheres. (15 points)

   Two identical spheres of mass M and radius R are held fixed at a separation distance D, as shown. A small projectile of mass m is fired with initial velocity v_0 from the surface of one sphere directly toward the other. What is the minimum value of v_0 such that the projectile just reaches the second sphere? You should assume that the projectile is attracted gravitationally by each sphere, and that *no other forces are acting.*