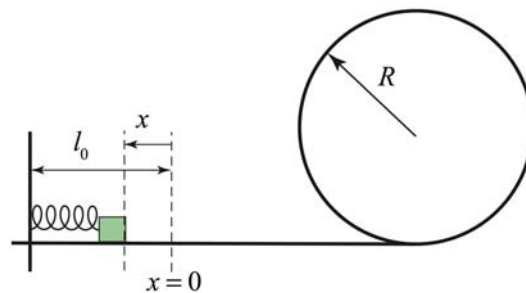# Problem Set 8

1. **Spring-Loop-the-Loop**

   A small block of mass m is pushed against a spring with spring constant k and held in place with a catch. The spring compresses an unknown distance x. When the catch is removed, the block leaves the spring and slides along a frictionless circular loop of radius R.

   When the block reaches the top of the loop, the force of the loop on the block (the normal force) is equal to twice the gravitational force on the mass. How far was the spring initially compressed? Write your answer using some or all of the following: g, k, R, and m.

2. **Slingshot**

A ball of negligible size and mass m hangs from a string of length l. It is hit in such a way that it then travels in a vertical circle. The initial speed of the ball after being struck is v_0. The goal of the first part of this problem is to find the tension in the string when the ball is at the top of the circle. You may assume that there are no external forces other than gravity doing work on the ball and string. Let g denote the magnitude of the gravitational constant.
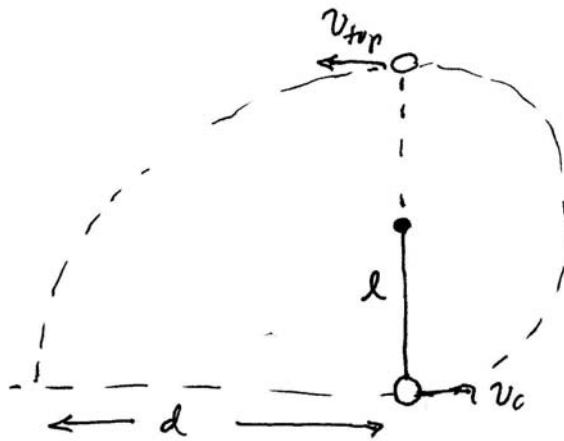
(a) Find the tension in the string when the ball is at the top of the circle. Express your answer in terms of some or all of the following: m, g, l and v_0.

(b) When the ball is exactly at the top of the circle, it detaches from the string and follows the trajectory shown in the figure above. When the ball returns to the level of the bottom of the circle, it is a distance d from the bottom of the circle. Find the distance d. Express your answer in terms of m, g, l, and v_0 as needed.

3. **Objects on a Ring**

   Two objects slide without friction on a circular ring of radius R oriented in a vertical plane. The heavier object (of mass $3m$) is attached to a spring with an unstretched length of zero (admittedly an unphysical assumption) and spring constant k. The fixed end of the spring is attached to a point a horizontal distance $2R$ from the center of the circle. The lighter object (of mass m) is initially at rest at the bottom of the ring. The heavier object is released from rest at the top of the ring, then collides with and sticks to the lighter object. Find the value for m that will allow the combined object (of mass $4m$) to just reach the point A on the ring, but go no higher. Express your answer in terms of some or all of the quantities k, R and the acceleration of gravity g.
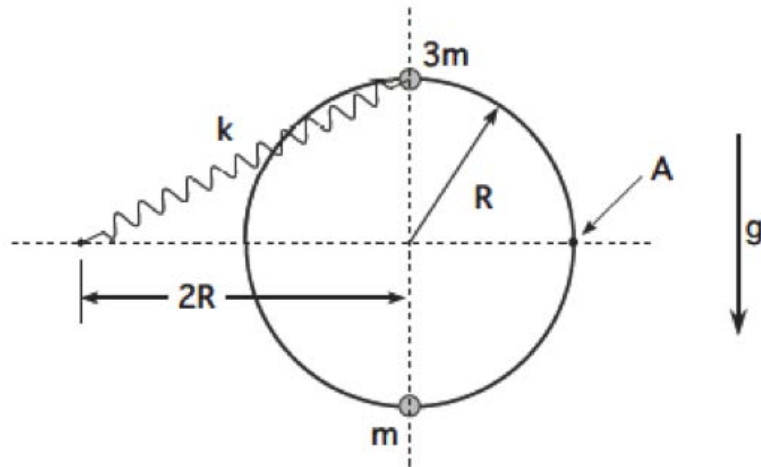

   Express you answer in terms of some or all of the following: k, R and g.

4. **Orbital Collisions**

    A projectile of mass m is fired vertically from the earth's surface with an initial speed that is equal to the escape velocity. The radius of the earth is R_e, the mass of the earth is m_e, and the universal gravitational constant is G.
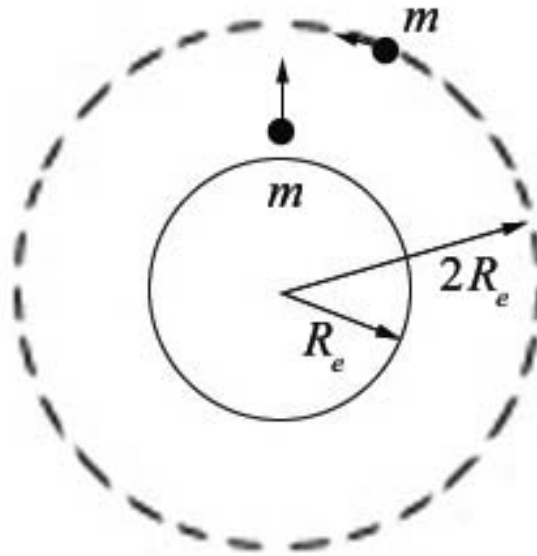
    (a) What is the initial speed of the projectile when it is launched from the surface of the earth? Express you answer in terms of some or all of the following: m, G, m_e and R_e.
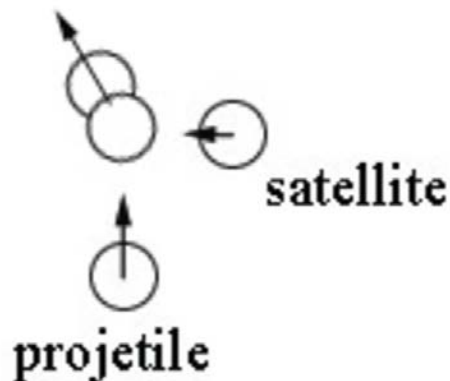
    (b) When the projectile is a distance $2R_e$ from the center of the earth, it collides with a satellite also of mass m that has been orbiting the earth in a circular orbit before

the collision. After the collision the two objects stick together. Assume that the collision is instantaneous, and that the velocities of the projectile and satellite are orthogonal right before the collision. What is the speed of the projectile, just before the collision, when it is a distance $2R_e$ from the center of the earth? Express you answer in terms of some or all of the following: m, G, m_e, and R_e.

(c) What is the speed of the satellite, just before the collision, when it is in a circular orbit of radius $2R_e$? Express you answer in terms of some or all of the following: m, G, m_e and R_e.

(d) What is the speed of projectile and satellite immediately after the collision? Express you answer in terms of some or all of the following: m, G, m_e and R_e.

5. **Exponential Potential Energy Diagram** A particle of mass m moves in one dimension. Its potential energy is given by

$$U(x) = -U_0 e^{-x^2/a^2}$$

where U_0 and a are constants.

(a) Draw an energy diagram showing the potential energy $U(x)$. Choose some value for the total mechanical energy E such that $-U_0 < E < 0$. Mark the kinetic energy, the potential energy and the total energy for the particle at some point of your choosing.

(b) Find the force on the particle as a function of position x. Express you answer in terms of some or all of the following: x, a, and U_0.

(c) Find the speed at the origin $x = 0$ such that when the particle reaches $x = \pm a$, it stops momentarily and reverses the direction of its motion. Express you answer in terms of some or all of the following: x, a, m and U_0.

8.01 Classical Mechanics
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