14.3 Changes in Potential Energies of a System

Consider an object near the surface of the earth as a system that is initially given a velocity directed upwards. Once the object is released, the gravitation force, acting as an external force, does a negative amount of work on the object, and the kinetic energy decreases until the object reaches its highest point, at which its kinetic energy is zero. The

gravitational force then does positive work until the object returns to its initial starting point with a velocity directed downward. If we ignore any effects of air resistance, the descending object will then have the identical kinetic energy as when it was thrown. All the kinetic energy was completely recovered.

Now consider both the earth and the object as a system and assume that there are no other external forces acting on the system. Then the gravitational force is an internal conservative force, and does work on both the object and the earth during the motion. As the object moves upward, the kinetic energy of the system decreases, primarily because the object slows down, but there is also an imperceptible increase in the kinetic energy of the earth. The change in kinetic energy of the earth must also be included because the earth is part of the system. When the object returns to its original height (vertical distance from the surface of the earth), all the kinetic energy in the system is recovered, even though a very small amount has been transferred to the Earth.

If we included the air as part of the system, and the air resistance as a non-conservative internal force, then the kinetic energy lost due to the work done by the air resistance is not recoverable. This lost kinetic energy, which we have called thermal energy, is distributed as random kinetic energy in both the air molecules and the molecules that compose the object (and, to a smaller extent, the earth).

We shall define a new quantity, the change in the internal *potential energy* of the system, which measures the amount of lost kinetic energy that can be recovered during an interaction.

When only internal conservative forces act in a closed system, the sum of the changes of the kinetic and potential energies of the system is zero.

Consider a closed system, $\Delta E_{sys} = 0$, that consists of two objects with masses m_1 and m_2 respectively. Assume that there is only one conservative force (internal force) that is the source of the interaction between two objects. We denote the force on object 1 due to the interaction with object 2 by $\vec{\mathbf{F}}_{2,1}$ and the force on object 2 due to the interaction with object 1 by $\vec{\mathbf{F}}_{1,2}$. From Newton's Third Law,

$$\vec{\mathbf{F}}_{2,1} = -\vec{\mathbf{F}}_{1,2} . \tag{14.3.1}$$

The forces acting on the objects are shown in Figure 14.5.

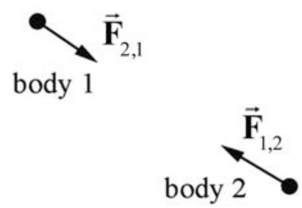

Figure 14.5 Internal forces acting on two objects

Choose a coordinate system (Figure 14.6) in which the position vector of object 1 is given by $\vec{\mathbf{r}}_1$ and the position vector of object 2 is given by $\vec{\mathbf{r}}_2$. The relative position of object 1 with respect to object 2 is given by $\vec{\mathbf{r}}_{2,1} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2$. During the course of the interaction, object 1 is displaced by $d\vec{\mathbf{r}}_1$ and object 2 is displaced by $d\vec{\mathbf{r}}_2$, so the relative displacement of the two objects during the interaction is given by $d\vec{\mathbf{r}}_{2,1} = d\vec{\mathbf{r}}_1 - d\vec{\mathbf{r}}_2$.

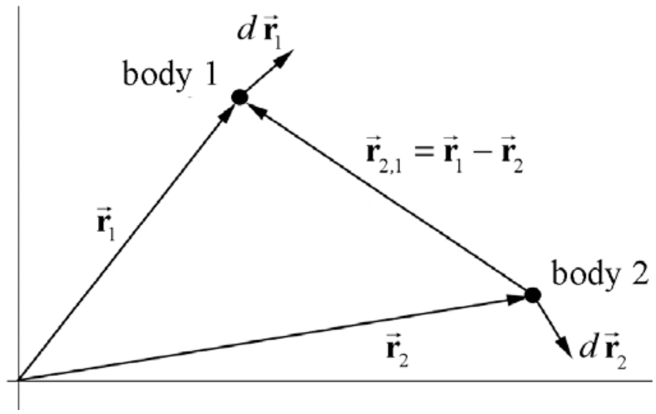

Figure 14.6 Coordinate system for two objects with relative position vector $\vec{\mathbf{r}}_{2,1} = \vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2$

Recall that the change in the kinetic energy of an object is equal to the work done by the forces in displacing the object. For two objects displaced from an initial state A to a final state B,

$$\Delta K_{\text{sys}} = \Delta K_1 + \Delta K_2 = W_{\text{c}} = \int_A^B \vec{\mathbf{F}}_{2,1} \cdot d\,\vec{\mathbf{r}}_1 + \int_A^B \vec{\mathbf{F}}_{1,2} \cdot d\,\vec{\mathbf{r}}_2 . \qquad (14.3.2)$$

(In Equation (14.3.2), the labels "A" and "B" refer to initial and final states, not paths.)

From Newton's Third Law, Equation (14.3.1), the sum in Equation (14.3.2) becomes

$$\Delta K_{\text{sys}} = W_c = \int_A^B \vec{\mathbf{F}}_{2,1} \cdot d\vec{\mathbf{r}}_1 - \int_A^B \vec{\mathbf{F}}_{2,1} \cdot d\vec{\mathbf{r}}_2 = \int_A^B \vec{\mathbf{F}}_{2,1} \cdot (d\,\vec{\mathbf{r}}_1 - d\,\vec{\mathbf{r}}_2) = \int_A^B \vec{\mathbf{F}}_{2,1} \cdot d\,\vec{\mathbf{r}}_{2,1} \qquad (14.3.3)$$

where $d\,\vec{\mathbf{r}}_{2,1} = d\,\vec{\mathbf{r}}_1 - d\,\vec{\mathbf{r}}_2$ is the relative displacement of the two objects. Note that since $\vec{\mathbf{F}}_{2,1} = -\vec{\mathbf{F}}_{1,2}$ and $d\vec{\mathbf{r}}_{2,1} = -d\,\vec{\mathbf{r}}_{1,2}$, $\int_A^B \vec{\mathbf{F}}_{2,1} \cdot d\,\vec{\mathbf{r}}_{2,1} = \int_A^B \vec{\mathbf{F}}_{1,2} \cdot d\,\vec{\mathbf{r}}_{1,2}$.

Consider a system consisting of two objects interacting through a conservative force. Let $\vec{\mathbf{F}}_{2,1}$ *denote the force on object 1 due to the interaction with object 2 and let* $d\vec{\mathbf{r}}_{2,1} = d\vec{\mathbf{r}}_1 - d\vec{\mathbf{r}}_2$ *be the relative displacement of the two objects. The* ***change in internal potential energy of the system*** *is defined to be the negative of the work done by the conservative force when the objects undergo a relative displacement from the initial state* A *to the final state* B *along any displacement that changes the initial state* A *to the final state* B,

$$\Delta U_{\text{sys}} = -W_{\text{c}} = -\int_A^B \vec{\mathbf{F}}_{2,1} \cdot d\vec{\mathbf{r}}_{2,1} = -\int_A^B \vec{\mathbf{F}}_{1,2} \cdot d\vec{\mathbf{r}}_{1,2} . \tag{14.3.4}$$

Our definition of potential energy only holds for conservative forces, because the work done by a conservative force does not depend on the path but only on the initial and final positions. Because the work done by the conservative force is equal to the change in kinetic energy, we have that

$$\Delta U_{\text{sys}} = -\Delta K_{\text{sys}}, \text{ (closed system with no non-conservative forces)} . \tag{14.3.5}$$

Recall that the work done by a conservative force in going around a closed path is zero (Equation (14.2.16)); therefore the change in kinetic energy when a system returns to its initial state is zero. This means that the kinetic energy is completely recoverable.

In the *Appendix 13A: Work Done on a System of Two Particles,* we showed that the work done by an internal force in changing a system of two particles of masses m_1 and m_2 respectively from an initial state A to a final state B is equal to

$$W = \frac{1}{2}\mu (v_B^2 - v_A^2) = \Delta K_{\text{sys}} , \tag{14.3.6}$$

where v_B^2 is the square of the relative velocity in state B, v_A^2 is the square of the relative velocity in state A, and $\mu = m_1 m_2 / (m_1 + m_2)$ is a quantity known as the *reduced mass* of the system.

14.3.1 Change in Potential Energy for Several Conservative Forces

When there are several internal conservative forces acting on the system we define a separate change in potential energy for the work done by each conservative force,

$$\Delta U_{\text{sys}, i} = -W_{c,i} = -\int_A^B \vec{\mathbf{F}}_{\text{c}, i} \cdot d\vec{\mathbf{r}}_i . \tag{14.3.7}$$

where $\vec{\mathbf{F}}_{c,i}$ is a conservative internal force and $d\vec{\mathbf{r}}_i$ a change in the relative positions of the objects on which $\vec{\mathbf{F}}_{c,i}$ when the system is changed from state A to state B. The work done is the sum of the work done by the individual conservative forces,

$$W_c = W_{c,1} + W_{c,2} + \cdots . \tag{14.3.8}$$

Hence, the sum of the changes in potential energies for the system is the sum

$$\Delta U_{sys} = \Delta U_{sys,1} + \Delta U_{sys,2} + \cdots . \tag{14.3.9}$$

Therefore the change in potential energy of the system is equal to the negative of the work done

$$\Delta U_{sys} = -W_c = -\sum_i \int_A^B \vec{\mathbf{F}}_{c,i} \cdot d\vec{\mathbf{r}}_i . \tag{14.3.10}$$

If the system is closed (external forces do no work), and there are no non-conservative internal forces then Eq. (14.3.5) holds.