Example 10.5 Exploding Projectile

An instrument-carrying projectile of mass m_1 accidentally explodes at the top of its trajectory. The horizontal distance between launch point and the explosion is x_i. The projectile breaks into two pieces that fly apart horizontally. The larger piece, m_3, has three times the mass of the smaller piece, m_2. To the surprise of the scientist in charge, the smaller piece returns to earth at the launching station. Neglect air resistance and effects due to the earth's curvature. How far away, $x_{3,f}$, from the original launching point does the larger piece land?
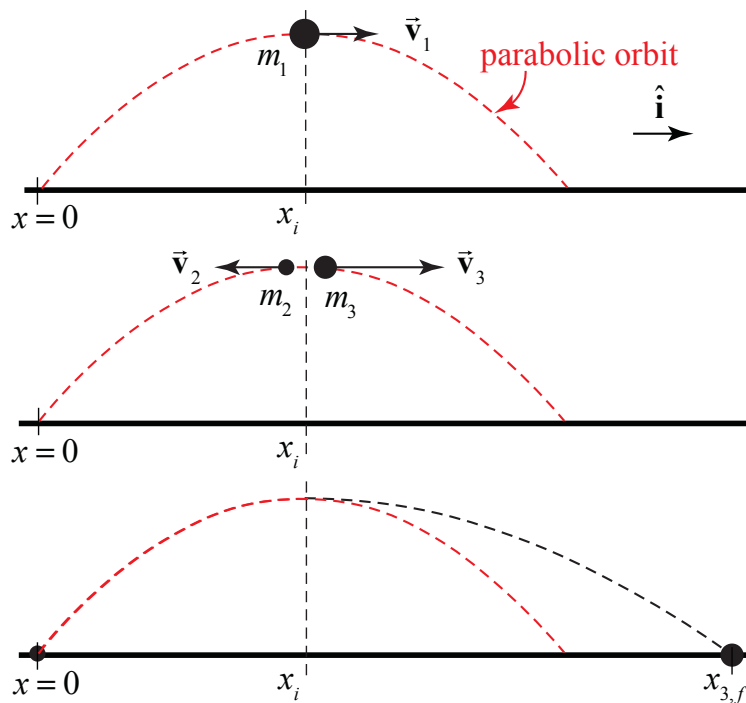

Figure 10.8 Exploding projectile trajectories

Solution: We can solve this problem two different ways. The easiest approach is utilizes the fact that the external force is the gravitational force and therefore the center of mass of the system follows a parabolic trajectory. From the information given in the problem $m_2 = m_1 / 4$ and $m_3 = 3m_1 / 4$. Thus when the two objects return to the ground the center of mass of the system has traveled a distance $R_{cm} = 2x_i$. We now use the definition of center of mass to find where the object with the greater mass hits the ground. Choose an origin at the starting point. The center of mass of the system is given by

$$\vec{R}_{cm} = \frac{m_2\vec{r}_2 + m_3\vec{r}_3}{m_2 + m_3} \ .$$

So when the objects hit the ground $\vec{R}_{cm} = 2x_i\,\hat{\mathbf{i}}$, the object with the smaller mass returns to the origin, $\vec{r}_2 = \vec{0}$, and the position vector of the other object is $\vec{r}_3 = x_{3,f}\,\hat{\mathbf{i}}$. So using the definition of the center of mass,

$$2x_i\,\hat{\mathbf{i}} = \frac{(3m_1/4)x_{3,f}\,\hat{\mathbf{i}}}{m_1/4 + 3m_1/4} = \frac{(3m_1/4)x_{3,f}\,\hat{\mathbf{i}}}{m_1} = \frac{3}{4}x_{3,f}\,\hat{\mathbf{i}} \ .$$

Therefore

$$x_{3,f} = \frac{8}{3}x_i \ .$$

Note that the neither the vertical height above ground nor the gravitational acceleration g entered into our solution.

Alternatively, we can use conservation of momentum and kinematics to find the distance traveled. Because the smaller piece returns to the starting point after the collision, the velocity of the smaller piece immediately after the explosion is equal to the negative of the velocity of original object immediately before the explosion. Because the collision is instantaneous, the horizontal component of the momentum is constant during the collision. We can use this to determine the speed of the larger piece after the collision. The larger piece takes the same amount of time to return to the ground as the projectile originally takes to reach the top of the flight. We can therefore determine how far the larger piece traveled horizontally.

We begin by identifying various states in the problem.

Initial state, time $t_0 = 0$: the projectile is launched.

State 1 time t_1 : the projectile is at the top of its flight trajectory immediately before the explosion. The mass is m_1 and the velocity of the projectile is $\vec{v}_1 = v_1\hat{\mathbf{i}}$.

State 2 time t_2: immediately after the explosion, the projectile has broken into two pieces, one of mass m_2 moving backwards (in the negative x-direction) with velocity $\vec{\mathbf{v}}_2 = -\vec{\mathbf{v}}_1$. The other piece of mass m_3 is moving in the positive x-direction with velocity $\vec{\mathbf{v}}_3 = v_3 \hat{\mathbf{i}}$, (Figure 10.8).

State 3: the two pieces strike the ground at time $t_f = 2t_1$, one at the original launch site and the other at a distance $x_{3,f}$ from the launch site, as indicated in Figure 10.8. The pieces take the same amount of time to reach the ground $\Delta t = t_1$ because both pieces are falling from the same height as the original piece reached at time t_1, and each has no component of velocity in the vertical direction immediately after the explosion. The momentum flow diagram with state 1 as the initial state and state 2 as the final state are shown in the upper two diagrams in Figure 10.8.

The initial momentum at time t_1 immediately before the explosion is

$$\vec{\mathbf{p}}^{sys}(t_1) = m_1 \vec{\mathbf{v}}_1 . \tag{10.9.9}$$

The momentum at time t_2 immediately after the explosion is

$$\vec{\mathbf{p}}^{sys}(t_2) = m_2 \vec{\mathbf{v}}_2 + m_3 \vec{\mathbf{v}}_3 = -\frac{1}{4} m_1 \vec{\mathbf{v}}_1 + \frac{3}{4} m_1 \vec{\mathbf{v}}_3 \tag{10.9.10}$$

During the duration of the instantaneous explosion, impulse due to the external gravitational force may be neglected and therefore the momentum of the system is constant. In the horizontal direction, we have that

$$m_1 \vec{\mathbf{v}}_1 = -\frac{1}{4} m_1 \vec{\mathbf{v}}_1 + \frac{3}{4} m_1 \vec{\mathbf{v}}_3 . \tag{10.9.11}$$

Equation (10.9.11) can now be solved for the velocity of the larger piece immediately after the collision,

$$\vec{\mathbf{v}}_3 = \frac{5}{3} \vec{\mathbf{v}}_1 . \tag{10.9.12}$$

The larger piece travels a distance

$$x_{3,f} = v_3 t_1 = \frac{5}{3} v_1 t_1 = \frac{5}{3} x_i .$$ (10.9.13)

Therefore the total distance the larger piece traveled from the launching station is

$$x_f = x_i + \frac{5}{3} x_i = \frac{8}{3} x_i ,$$ (10.9.14)

in agreement with our previous approach.

Example 10.6 Landing Plane and Sandbag

Figure 10.9 Plane and sandbag

A light plane of mass 1000 kg makes an emergency landing on a short runway. With its engine off, it lands on the runway at a speed of $40 \text{ m} \cdot \text{s}^{-1}$. A hook on the plane snags a cable attached to a 120 kg sandbag and drags the sandbag along. If the coefficient of friction between the sandbag and the runway is $\mu_k = 0.4$, and if the plane's brakes give an additional retarding force of magnitude 1400 N, how far does the plane go before it comes to a stop?

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