Massachusetts Institute of Technology
Department of Physics
Physics 8.022 - Fall 2002

Assignment #12
Displacement Current, E/M Waves
Energy, Power, Momentum in E/M Waves
Transmission Lines

Reading *Purcell:* Chapter 9, Handouts on Electromagnetic (E/M) Waves, Polarization, Transmission Lines

Problem Set #12
Work on **all** problems. Not all problems receive equal points. Total points for this set is 100.

- **(15 points) [1]** *Purcell* Problem 9.13 (p.345): Lorentz invariance and waves.

- **(15 points) [2]** Electromagnetic (E/M) plane waves.

  For each of the following given $\vec{E}$ and $\vec{B}$ vectors (*assuming* they are describing an E/M plane wave) find (*if exist*) the accompanying $\vec{B}$ and $\vec{E}$ ones. Express your answer in terms of the given variables (E_0, B_0, k and ω are positive definite constants). For each case also draw a plot showing a right handed Cartesian coordinate system with x,y,z axes identified and with the vectors $\vec{E}$, $\vec{B}$ and $\vec{k}$ (propagation vector) shown on it. $\vec{E} = -E_0 cos(kx + \omega t)\hat{x}$

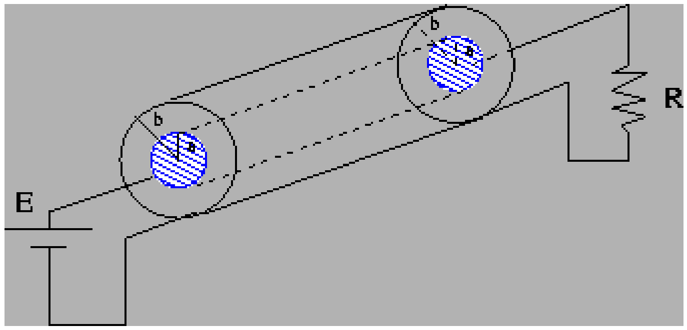
  $\vec{B} = B_0 cos(kz + \omega t)\hat{y}$

  $\vec{E} = E_0 sin(kx - \omega t)\hat{y}$

  $\vec{B} = B_0 sin(ky - \omega t)\hat{z}$

- **(20 points) [3]** Coaxial cable and Poynting vector.

  A coaxial cable ``delivers'' current I=E/R from the *emf* E to the resistor R as shown in the figure. The coaxial cable is resistanceless and it is made up of an inner metallic conductor of radius a and an outer metallic conductor of radius b. Our goal is to extend the definition of the Poynting vector to static fields and show that its physical significance remains the same, i.e., a measure of power flow.



- Find the $\vec{E}$ and $\vec{B}$ fields in the space in between the two conductors of the coax cable and construct the Poynting vector $\vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{B}$. Where does $\vec{S}$ ``poynt'' to?
- Convince yourself this is the only region of space where $\vec{S}$ is non-zero.
- Integrate $\vec{S}$ over the cross section of the cable and show that the total power flowing through the cable is E^2/R. Is this what you expected?
- The leads of the battery are now reversed. Does the direction of $\vec{S}$ change? Is this what you expected?

- **(15 points) [4]** Radiation from the Sun.

  At the top of the atmosphere the average radiant flux from the Sun is $N = 1.35 \times 10^3 W/m^2$.

  Although this radiation consists of a spectrum of frequencies, many of the interesting properties do not depend on frequency and can therefore be calculated by using the methods described for monochromatic waves.
  - What is the average energy density in the solar radiation at the top of the atmosphere?
  - What is the average momentum density?
  - What average force would the radiation exert on a completely absorbing surface with an area of $1 m^2$ oriented perpendicular to the Earth-Sun line?
  - What is the average value of E_0 in the wave?

- **(15 points) [5]** $\vec{E}$ and $\vec{B}$ fields in a capacitor.

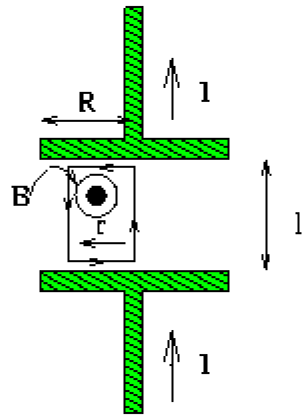
  We have worked in class on the $\vec{E}(\vec{r}, t)$ and $\vec{B}(\vec{r}, t)$ fields in a parallel circular plate capacitor

  (radius R, distance l) driven by an alternating current $I(t) = I_0 cos(\omega t)$. In doing this

  we have ignored fringing effects,

  assumed $\vec{E}$ spatially uniform and also assumed I being ``slowly varying'', i.e.,

$\omega R / 2\pi c << 1$.



- Show that our assumption of uniform $\vec{E}$ field is in violation of Faraday's law.
- Estimate the non-uniformity of $\vec{E}$ by calculating the circulation of $\vec{E}$ around the path shown in figure. Is $\vec{E}$ going to be decreasing or increasing with increasing *r*?
- Find the relative error $\Delta E / E$ and compare it with one of our assumptions (this saves us!).

- **(20 points) [6]** Wave Polarization.

An electromagnetic wave is the superposition of two linearly polarized wave along the $\hat{y}$ and $\hat{z}$ directions and is described by the following equation:

$$\vec{E} = \hat{y} E_0 sin(\omega t - \frac{\omega x}{c}) + \hat{z} E_0 cos(\omega t - \frac{\omega x}{c})$$

- What is the direction of propagation of the wave?
- What is the polarization status of this wave?
- Find the magnitude of the electric field at all points of space for all times.
- An observer stands at the origin of the coordinate system. Draw a diagram showing the vector $\vec{E}$ at $t = 0, t = \pi/2\omega, t = \pi/\omega, t = 3\pi/2\omega, t = 2\pi/\omega$.

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