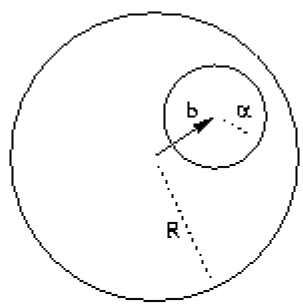
Massachusetts Institute of Technology
Department of Physics
Physics 8.022 - Fall 2002

Assignment #9
Biot-Savart and Ampere's Laws
Faraday's Law of Induction
Mutual and Self Inductance

Reading *Purcell:* Chapters 6 and 7.

Problem Set #9
Work on **all** problems. Not all problems receive equal points. Total points for this set is 100.

- **(15 points) [1]** Hollow wire.

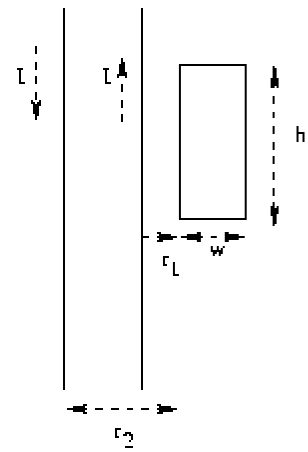


A straight wire (along the *z* axis) of radius *R* carries current density $\vec{J} = J_0 \hat{k}$. A cylindrical hole

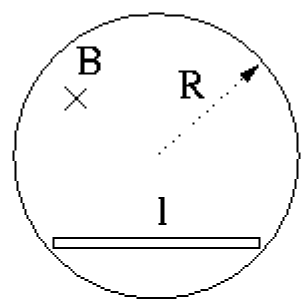
or radius α parallel to the axis of the wire is drilled at distance *b* from it as shown in figure (viewed from the top). Show that the field anywhere inside the hole is uniform and given by $\vec{B} = \frac{2\pi J_0}{c} \hat{k} \times \vec{b}$. If *I* is the total current flowing through the hollow wire, express *B* in terms of

I,*b*,*R* and α.

(15 points) [2] *Emf* in a loop.
A pair of parallel wires carries equal and opposite currents *I*. A closed rectangular wire loop of dimensions *h* and *w* is placed in the plane of them and as shown in the figure.
- Find the magnetic flux through the loop.
- Now allow *I* to vary with time at a slow enough rate *dI*/*dt*. Find the induced *Emf* in the loop.

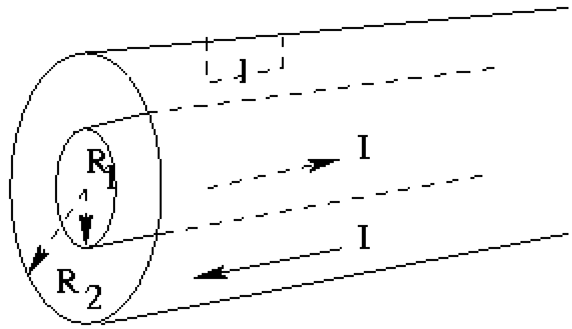


- **(10 points) [3]** *Emf* in a rod.



A uniform magnetic field *b* fills a cylindrical volume of radius *R*. A metal rod of length *l* is placed as shown. If *B* is changing at the rate $\frac{dB}{dt}$ show that the *emf* that is produced by the changing magnetic field and that acts between the ends of the rod is given by $\frac{dB}{dt}\frac{l}{2c}\sqrt{R^2-(l/2)^2}$.

- **(10 points) [4]** *Purcell* Problem 7.14 (p.289): Crossbar in a magnetic field.
- **(10 points) [5]** *Purcell* Problem 7.18 (p.290): Charge moved by electromotive force.
- **(15 points) [6]** *Purcell* Problem 7.22 (p.291): Angular momentum and electromagnetic fields.
- **(10 points) [7]** *Purcell* Problem 7.21 (p.291): Mutual inductance of coaxial solenoids.
- **(15 points) [8]** Coaxial conductors.
  Show that the self-inductance per unit length of a transmission line consisting of two concentric conducting tubes with radii R_1 and R_2 is $\frac{2}{c^2} ln \frac{R_2}{R_1}$. The current flows along one of the tubes and an equal and opposite current flows back along the other thus completing a circuit. The currents are uniformly distributed over the surfaces of each tube. Hint:calculate the magnetic flux coupling through a rectangle of length *l* 'hanging' from the top of the outer conductor as shown in the figure.



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