25 minutes Practice Quiz for Week #8

Work with p.16 of lecture notes #11 in hand.

Lorentz invariants (part III)

This is a hard derivation to pursue without the guidance provided by the steps in which the problem is described.

Solution

The rest mass m_0 is a Lorentz invariant quantity.
For all observers: $\frac{E^2}{c^2} - p^2 = {m_0}^2 c^2$
The RHS is a constant, thus the differential of the LHS will be zero:
$d(LHS) = dE\frac{\partial LHS}{\partial E} + dp_x\frac{\partial LHS}{\partial p_x} + dp_y\frac{\partial LHS}{\partial p_y} + dp_z\frac{\partial LHS}{\partial p_z} = 0$
If we calculate all partial derivatives and use the fact that $dp_y = dp_z = 0$ we find:
$dE(\frac{2E}{c^2}) - 2p_x dp_x = 0$
from which we readily then have:
$EdE = c^2 p_x dp_x$
Using the transformation for p_x and E we have (feel free to construct the differential using the partial derivatives the same way we did it above- it is rewarding!):
$dp'_x = \gamma(dp_x - \frac{v}{c^2}dE)$ and
$E' = \gamma(E - vp_x)$
Take the ratio of the above two: $\frac{dp'_x}{E'} = \frac{dp_x}{E}\frac{1-v\frac{dE}{c^2 dp_x}}{1-v\frac{p_x}{E}}$ and use $EdE = c^2 p_x dp_x$ or $\frac{dE}{c^2 dp_x} = \frac{p_x}{E}$ to identify that indeed $\frac{dp'_x}{E'} = \frac{dp_x}{E}$