

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS
8.022 FALL 2004
ASSIGNMENT 3: ELECTRIC POTENTIAL AND CAPACITANCE
DUE DATE: FRIDAY, OCT 1ST

1. Purcell 3.1: Charges in a spherical conductor.
2. Purcell 3.4: Charges near a conducting plane.
3. Purcell 3.5: Work pulling a charge away from a conducting plane.
4. Purcell 3.9: More charges near a conducting plane.
5. Purcell 3.17: Design of a spherical capacitor. (Hint: you may want to look at 3.10 first)
6. Forces on conductors.

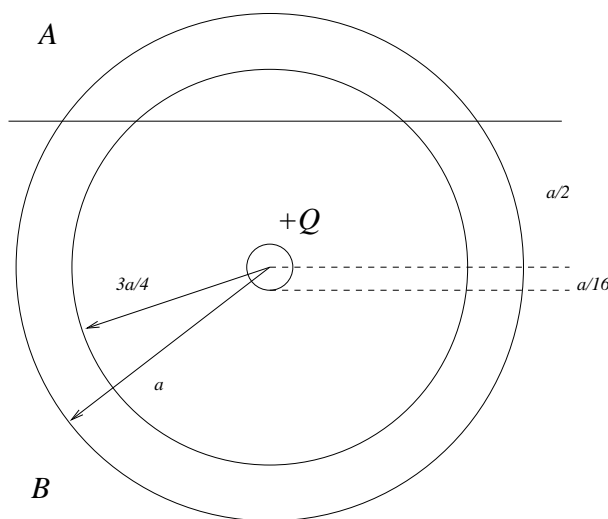


Figure 1: concentric conducting spheres.

A conducting spherical shell of outer radius a and inner radius $\frac{3a}{4}$ is cut in two pieces via a horizontal plane a distance $\frac{a}{2}$ above the center of the spherical shell as shown in figure 1. Let us label “A” the upper part of the shell and “B” the lower part of the shell. The shell is initially uncharged and the two pieces that result from the cutting procedure remain in perfect electrical contact. A new conducting sphere of radius $\frac{a}{16}$ and total charge $+Q$ is inserted in the shell and it is centered on the shell’s center as shown in the same figure.

- (a) Are there any charge densities on the inner ($r = \frac{3a}{4}$) and outer ($r = a$) surfaces of the shell as well as within it? If yes, derive them.
- (b) What (if any) is the force per unit area on the inner and outer surfaces of the shell?

- (c) From now on we focus only on the “A” part of the shell: Set up an integral that will yield the net force acting on the outer shell of the “A” part. What is its direction? Identify over which variables you are integrating and what are the limits of integration. (You are not asked to perform the integration!)
- (d) Do the same as (c) for the inner shell of the “A” part of the shell.
- (e) Do the same as (c) for the “A” part as a whole.

7. Parallel wires

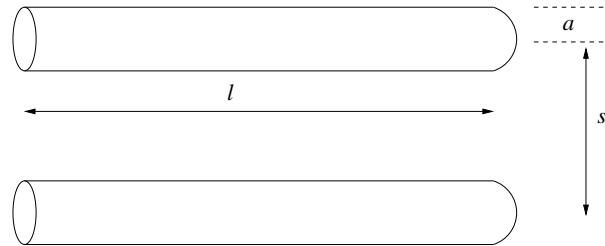


Figure 2: two long parallel wires.

Two long conducting wires of length l and radius a lie parallel a distance s apart. The upper wire carries charge Q and the lower charge $-Q$ as indicated in the figure. $l \gg s, a$ and $s \sim a$, so you may assume the wires are effectively infinitely long for purpose of finding the electric fields and potentials.

- (a) Find the electric field \vec{E} in the plane in between the wires.
- (b) Find the potential ϕ in the plane between the wires. Find V , the potential different between the wires.
- (c) Find the capacitance C of the two wire system.
- (d) Find the total electrical energy stored in the system. (You can do this part without integrating!)

8. Energy of a conductor in a capacitor.

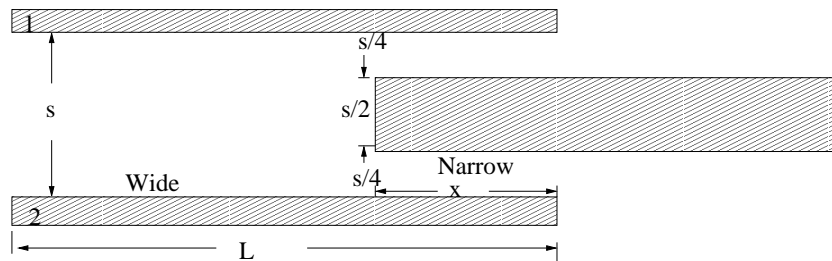


Figure 3: a capacitor of parallel square plates with a conducting square plate inserted.

Two flat square metal plates with sides of length L are arranged parallel to each other with a separation s ; $s \ll L$. A total charge Q is on the bottom plate, $-Q$ on the top. Now, a third uncharged conducting plate of the same size with thickness $s/2$ is slipped

between the original two plates. It is such placed that its distances to the top plate and the bottom plate are both $s/4$.

(a) When only a length x of the conducting plate is inside the metal plate, what are the surface charge densities σ_w and σ_n on the lower plate adjacent to the wide and narrow gaps? What is the electric field in the wide and narrow gaps? (Hint: think about $\oint \vec{E} \cdot d\vec{s}$ on certain paths.)

(b) What is the capacitance of this system?

(c) How much energy is stored in the electric field?

(d) What force must be exerted to keep the middle plate from moving?