Massachusetts Institute of Technology
Department of Physics
8.022 Fall 2004
Assignment 9: AC circuits; Filters; Quality Factor and Resonance
Due date: Monday, Nov 29th

1. Purcell 8.9: Effects of resistance on RLC frequency. Note, $Q = \infty$ corresponds to $R = 0$. Also the percentage change is just the fractional change,

$$\text{fractional change}=\frac{\omega_{Q=\infty}-\omega_{Q=1000}}{\omega_{Q=\infty}}$$

times 100.

2. Purcell 8.10 RL in parallel in C.

3. Purcell 8.16 An infinite L-C ladder.

4. Quality factor details.

(a) For the series RLC circuit, show that the frequencies ω_1 and ω_2 at which the average power provided by the source is half the maximum are given by:

$$\omega_{1,2} = \sqrt{\omega_0^2 + \frac{R^2}{4L^2}} \pm \frac{R}{2L} \qquad (1)$$

(b) Show that the AC quality factor

$$Q = \frac{\omega_0}{\omega_1 - \omega_2} \qquad (2)$$

is the same as the quality factor obtained for the dumped RLC circuit.

5. Two capacitors and inductor.

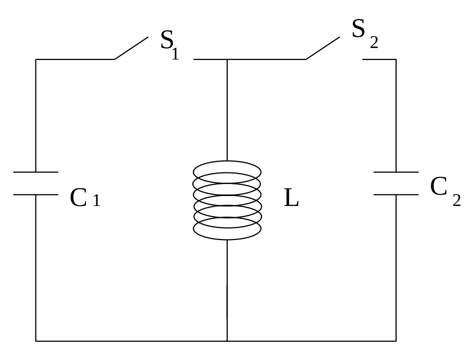


Figure 1: Two capacitors and an inductor.

Consider the following circuit where C_1 is initially charged to 75V. Supposed that C_1 is 10,000 μF, C2 is 3000 µF, and L is 15H.
Explain how to open and close the switches so as to discharge C_1 and charge C_2.
Starting at $t = 0$, you should give explicitly times for opening and closing each switch. What is the final voltage across C_2

6. Betatron accelerator.

A particle of mass M and charge Q is constrained by a magnetic field $B_0(R)$ to move on a circle of radius R which is perpendicular to the field. While $B_0(r)$ is constant in angle around a circle, the magnitude B_0 may depend on the radius. then one can define an average field B_{av} within the R-circle by;

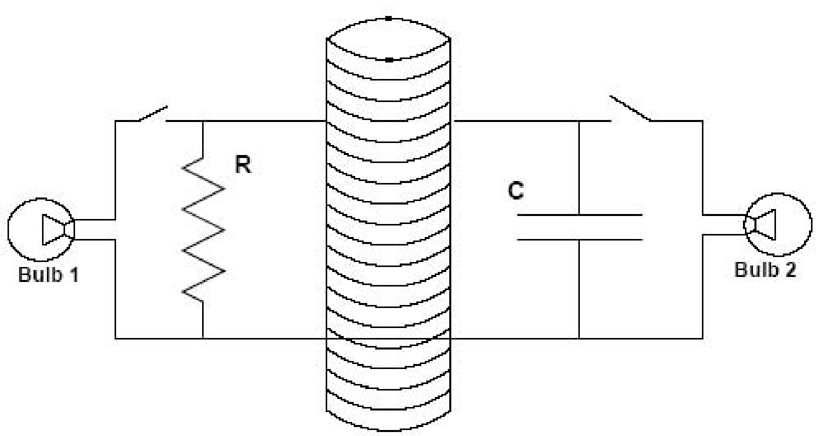
$$B_{av} = \frac{1}{\pi R^2} 2\pi \int_0^R r B_0(r) dr \tag{3}$$

In the betatron $B_0(R)$ can be increased with time. At t=0, $B_0(R)$ is very small and particles from an ion source are injected with a low velocity just sufficient to follow a circular path. Then both B_{av} and $B_0(R)$ are increased with time, accelerating the particle by Faraday induction and keeping it on a circular orbit by the Lorentz force.

(a) Find the force on the charge Q due to the changing field.
(b) Find the speed of the particle in its orbit as a function of $B_{av}(t)$
(c) The particle will stay in the orbit of radius R so long as $B_0(r)$ and the speed v have the correct relation. However for the accelerator to work at all speeds, B_{av} and $B_0(R)$ must have the proper relationship. What is this relationship? Can it be satisfied by a uniform B_0', that is, one which does not depend on r?
(This problem is not very closely related to this chapter, but it illustrated an important principal.)

7. **NB: the solenoid is positioned in such a way that its axis is perpendicular to the page and to the plane of the circuit. Not obvious from the picture…**

An infinite solenoid is enclosed by a series RC circuit. The solenoid has a radius b and N turns of wire per unit length. The resistor has resistance R and the capacitor has capacitance C. By closing a pair of switches, we can put a pair of lightbulbs in parallel with the resistor and the capacitor. In what follows, the solenoid is resistanceless and we may ignore the self inductance of the circuit around the solenoid.



The current through the solenoid is turned on at $t = 0$ and from that time has the form $I_{\text{sol}}(t) = \alpha t$; $\alpha > 0$. This current cycles counterclockwise (indicated by arrows). Both switches are open.

(a) What EMF $\mathcal{E}$ is induced in the RC circuit?

(b) Compute the charge across the capacitor, $Q(t)$, and the current through the resistor, $I(t)$, as functions of time. Which plate of the capacitor is positive and which is negative?

(c) Suppose that when we turn the solenoid on, the switches are both closed. Describe qualitatively the brightness of the bulbs as a function of time. (Assume the resistance of the bulbs is high enough that you do not need to worry about how the current through those bulbs will change your solution. The brightness is then proportional to the squared voltage in the bulb's circuit element.)

The current through the solenoid is now given by $I = I_0 \sin(\omega t)$ ($I_0 > 0$). The lightbulb switches are both closed, so that the lightbulbs are part of the circuit.

(d) Working in the complex representation, find the voltage across the resistor $\tilde{V}_R$ and the voltage across the capacitor $\tilde{V}_C$.

(e) Describe qualitatively the brightness of the bulbs as the frequency is varied from $\omega = 0$ to $\omega = \infty$.

(e) At what frequency is the brightness of the bulbs exactly equal?

(f) As the oscillation frequency is made very large, which bulb is most likely to burn out?