Massachusetts Institute of Technology
Department of Physics
Physics 8.022 – Fall 2003
Quiz#1

- Total points in the quiz are 100. **ALL** problems receive **equal** points (25 each). Work on problems you are more comfortable with **first!**
- This is a closed book and closed notes exam. An equations table is given to you below.
- No programmable, plotting, integration/differentiation capable calculators are allowed.

Electrostatics Formulae for Quiz #1

Conservative Field: $\int_C \vec{F} \cdot d\vec{r} = 0$ for any closed path C, $W_{ab,C} = W_{ab,C'}$ for any C, C' connecting a *and* b, $\vec{F} = -\vec{\nabla}U$, $\vec{\nabla} \times \vec{F} = 0$

Coulomb Law: $\vec{F}_{21} = \frac{q_1 q_2}{r^2}\hat{r}_{21}$ for two point charges at distance r. $\vec{F}_{12} = -\vec{F}_{21}$, and for charges dq_1 and dq_2 that make part of continuous charge distributions 1 and 2, $d\vec{F}_{21} = \frac{dq_1 dq_2}{r^2}\hat{r}_{21}$

Electric Field: *at point 2* due to q_1 $\vec{E}_1 = \frac{q_1}{r^2}\hat{r}_{21}$. If q_1 is not a point charge but part of a continuous distribution, $d\vec{E} = \frac{dq}{r^2}\hat{r}$

Principle of Superposition: Two or more electric fields acting at a given point P add vectorially: $\vec{E}_P = \vec{E}_1 + \vec{E}_2 + ... + \vec{E}_n$

Electrostatic Field is Conservative: $\vec{\nabla} \times \vec{E} = 0$ and thus there exists scalar function ϕ such that $\vec{E} = -\vec{\nabla}\phi$ where $d\phi = \frac{dq}{r}$

Electrostatic potential: The potential at $\vec{x}$ with respect to a *ref* point is
$\phi(\vec{x}) - \phi(ref) = -\int_{ref}^{\vec{x}} \vec{E} \cdot d\vec{r} = -\frac{W_{ref \to \vec{x}}}{q}$

Gauss Law: $\int_S \vec{E} \cdot d\vec{a} = 4\pi \int_V \rho dV$ where S is a closed surface and V is its corresponding volume (integral form) or $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$ (differential form).

Poisson Eqn: $\nabla^2\phi = -4\pi\rho$, Laplace Eqn: $\nabla^2\phi = 0$

Energy: $U = \frac{1}{2}\int_V dV \int_{V'} dV' \frac{\rho(\vec{x})\rho(\vec{x}')}{|\vec{x}-\vec{x}'|} = \frac{1}{2}\int_V \rho\phi dV = \frac{1}{8\pi}\int_V E^2 dV$

Electric Force on Conductors: $\frac{dF}{da} = 2\pi\sigma^2 = \frac{E^2}{8\pi}$

Current Density: $\vec{J}(\vec{x}) = \rho(\vec{x})\vec{v}(\vec{x})$, Conservation Law/Continuity: $\vec{\nabla} \cdot \vec{J} = -\frac{\partial\rho}{\partial t}$

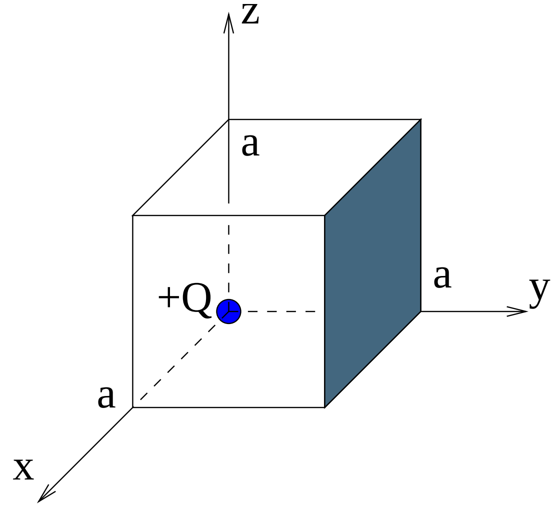
Capacitance: $Q = CV$, $U = \frac{1}{2}CV^2$

Gradient: in cartesian $\vec{\nabla}f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z}$, in cylindrical $\vec{\nabla}f = \frac{\partial f}{\partial \rho}\hat{\rho} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z}$, in spherical $\vec{\nabla}f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$

Divergence: in cartesian $\vec{\nabla}\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$, in cylindrical $\vec{\nabla}\cdot\vec{F} = \frac{F_\rho}{\rho} + \frac{\partial F_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$, in spherical $\vec{\nabla} \cdot \vec{F} = \frac{2F_r}{r} + \frac{\partial F_r}{\partial r} + \frac{F_\theta}{r}cot\theta + \frac{1}{r}\frac{\partial F_\theta}{\partial \theta} + \frac{1}{r sin\theta}\frac{\partial F_\phi}{\partial \phi}$

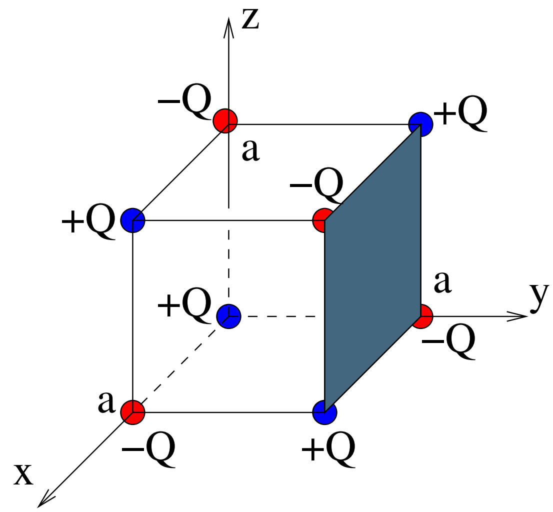
(25 points) [1] Charges on a Cube

An electric charge **+Q** sits at one of the corners of a cube of side **a** as shown in the figure below (the charge is fixed at that point).



- (a) What is the flux of the electric field $\vec{E}$ through the shaded region in the figure above, i.e, through the face of the cube that is described by $y = a$?

Seven additional charges are added at the corners of the cube as shown in the next figure so that the system now has a total of four positive electric charges of strength **+Q** each and four negative ones of strength **-Q** each distributed on the cube with alternating signs. All eight charges are fixed at these positions.



- (b) What is the potential energy of the system?
- (c) What is the potential (relative to infinity) at the center of the cube?
- (d) What is the electric field at the center of the cube?

An additional electric charge q is inserted at the center of the cube (the eight charges on the corners of the cube remain fixed at their positions).

- (e) Is the charge q in equilibrium? If not, for what charge q is the equilibrium reached? Is the equilibrium stable? Explain why.

(25 points) [2] Electrostatic Field

An *electrostatic field* measurement yielded the following results:

$$\vec{E} = A(3r + 4R)\vec{r} \qquad for \quad r \leq R$$
$$\vec{E} = \frac{7AR^4}{r^3}\vec{r} \qquad for \quad r \geq R$$

where $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$ and A is a constant with appropriate units.

- (a) Find the charge density ρ everywhere in space.
- (b) Find the total charge Q_t enclosed by a sphere of arbitrary radius r_o and with its center at the origin of the coordinate system.
- (c) Find the electrostatic potential ϕ everywhere in space.
- (d) Sketch ρ, Q_t and ϕ you just derived as a function of r_0.
- (e) Find the energy needed to assemble this charge distribution ρ.

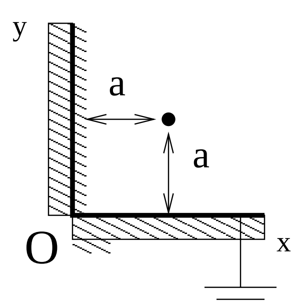
Hint: you will find helpful working in spherical coordinates.

(25 points) [3] Image(s) of a Charged Wire

An infinitely long wire carries static electric charge of line density λ.

- (a) Find the electric field $\vec{E}$ at a distance r from the wire.
- (b) Find the potential ϕ at a distance r from the wire with the constraint that $\phi(r = a) = 0$.

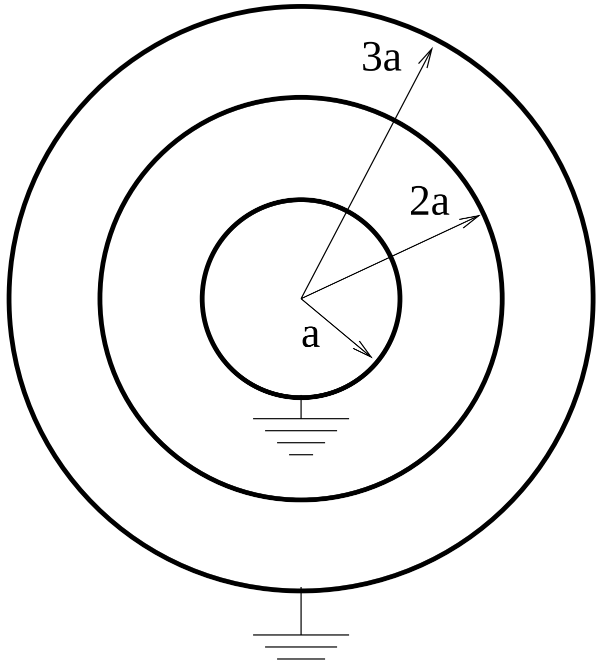
We now bring this wire in the vicinity of an L-shaped infinite conducting surface that is grounded (i.e., $\phi = 0$) as shown in the figure. The wire runs parallel to the z axis and all its points are equidistant to the planes. Let us name this distance a.



- (c) Find the images (or image, if it is just one) of the given wire.
- (d) Give an expression (other than Laplace's equation) for finding the potential at an arbitrary point in between the L-shaped planes.
- (e) What is the total induced charge per unit length ($\frac{dq}{dz}$) of the "x" part of the L-shaped conducting plane and what on its "y" part?

(25 points) [4] Triple Layer Capacitor System

A capacitor is made of three conducting concentric spherical shells of radii $a, 2a$ and $3a$ as shown in the figure below. In what follows we will assume that the shells are thick enough that we may distinguish the inner and outer surfaces, but thin enough that we do not actually need to know what their thickness is.



The inner and outer shells are *grounded:* their potentials are fixed to be zero ($\phi = 0$). The middle shell carries some net charge Q. This charge induces a charge Q_{in} on the outer edge of the inner shell and a charge Q_{out} on the inner edge of the outer shell. Note that these charges are taken *from ground,* so the inner and outer shells are *not* electrically neutral.

- (a) What is the electric field in the region $r < a$?
- (b) What is the electric field in the region $a < r < 2a$? Your answer should be expressed in terms of Q_{in}.
- (c) What is the electric field in the region $2a < r < 3a$? Your answer should be expressed in terms of Q_{in} and Q.
- (d) What is the electric field in the region $r > 3a$? Using this result, find Q_{out} in terms of Q_{in} and Q.
- (e) Find the potential of the middle shell with respect to the *ground* first by going from the inner shell to the middle shell and then by going from the outer shell to the middle shell. Leave your answers in terms of Q_{in} and Q.
- (f) Using the answer to part (e), find Q_{in} in terms of Q.
- (g) What is the capacitance of this system?
- (h) What is the potential energy of this system?