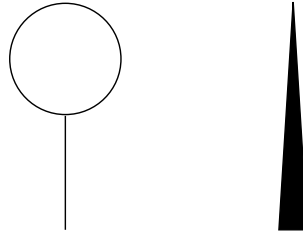


**PROBLEM 1** (8 pts)

Answer the following questions, justifying your answers with one short sentence.

- (a) A house has two lightning rods on its roof, one with a sphere on the end, the other with a pointy tip.



Which gets struck by lightning more often?

- (b) A charged rod is brought close to a balloon that is coated with a conducting foil. The balloon is neutral. Is the balloon attracted, repelled, or does it feel no force?
- (c) A soap bubble initially has a net positive charge smeared uniformly over its surface. Negative charge is slowly and uniformly added to the bubble's surface, reducing the charge until it passes through zero, and winds up with a net negative charge. Describe the bubble's behavior as the charge is added.
- (d) The door of a microwave oven contains a screen, which is made of a conducting material with lots of small holes. The size of these holes is roughly 1 mm. Given that you can look through this screen at your food without cooking your eyeballs, what can you conclude about the wavelength of radiation used in the oven?

**PROBLEM 2** (25 pts)

A spherically symmetric electric field

$$\vec{E}(r) = E_0 \left( \frac{r}{r_0} \right) \exp(-r^2/r_0^2) \hat{r}$$

fills all of space. The parameter  $E_0$  has the units of electric field;  $r_0$  has the units of length.

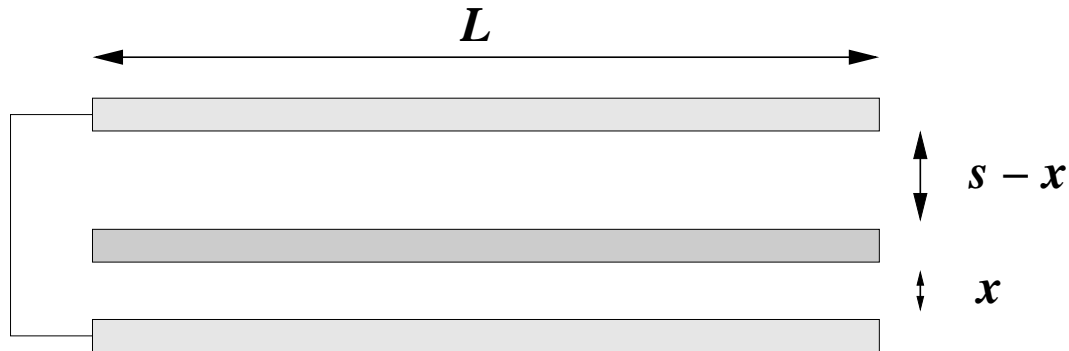
- (a) Find an expression for the charge density  $\rho(r)$ , valid everywhere.
- (b) Sketch this charge density versus radius. From this sketch, describe why the electric field falls to zero so quickly as a function of radius.
- (c) Find the radius  $r$  at which the charge density is zero (other than  $r = \infty$ ).
- (d) Find an expression for the electric potential  $\phi$  everywhere.
- (e) Find the total energy stored in the electric field.

The integral

$$\int_0^\infty r^4 \exp(-r^2/a^2) dr = \frac{3a^5}{8} \sqrt{\pi}$$

may prove very useful to you.

**PROBLEM 3** (20 pts)



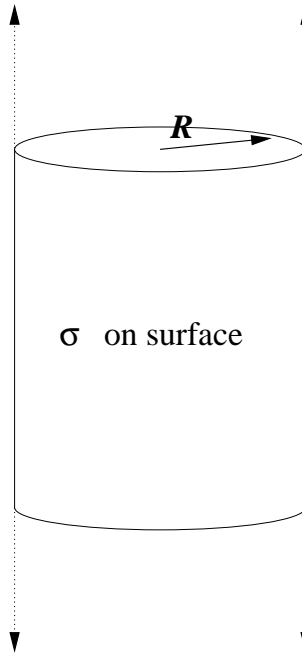
Three large, square conducting plates are placed parallel to one another as shown. The outer plates are connected by a conducting wire. A total charge  $Q$  is placed on the inner plate; this divides in an amount  $Q_T$  on the top of the inner plate and  $Q_B$  on the bottom of the inner plate.

The side length of the plates  $L$  is far larger than the separations ( $L \gg s$ ,  $L \gg x$ ), so you may ignore the effect of any edge effects in this problem.

- Write down expressions for the potential difference between the inner plate and the top plate, and between the inner plate and the bottom plate.
- Calculate  $Q_T$  and  $Q_B$  in terms of  $Q$ ,  $x$ , and  $s$ .
- Calculate the total energy stored in the electric field of this configuration as a function of  $x$ .
- Calculate the value of  $x$  for which the stored energy in this system is maximum. (Hint: you can do this with a straightforward but moderately tedious calculation. Or, you can look for a symmetry and think about what happens when you slightly disturb things from that symmetry.)

**PROBLEM 4** (20 pts)

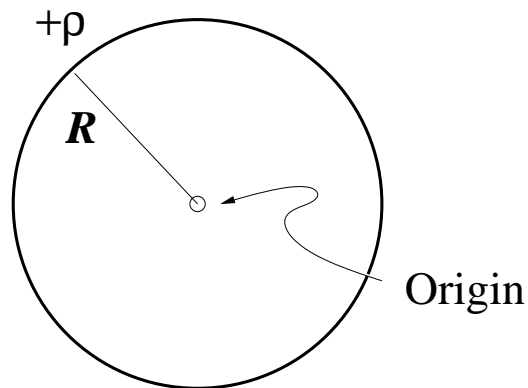
Consider the infinitely long cylinder below. It has a radius  $R$  and a thin layer of charge per unit area  $\sigma$  pasted to its surface. This means that amount of charge per unit length on the cylinder is  $\lambda = 2\pi R\sigma$ . In your responses below, please express your answers in terms of  $\lambda$ , not  $\sigma$ .



- (a) What is the electric field (magnitude and direction) outside of the cylinder?
- (b) What is the electric field (magnitude and direction) inside of the cylinder?
- (c) How much work does it take to squeeze the cylinder so that its radius shrinks by  $\delta R$ ?

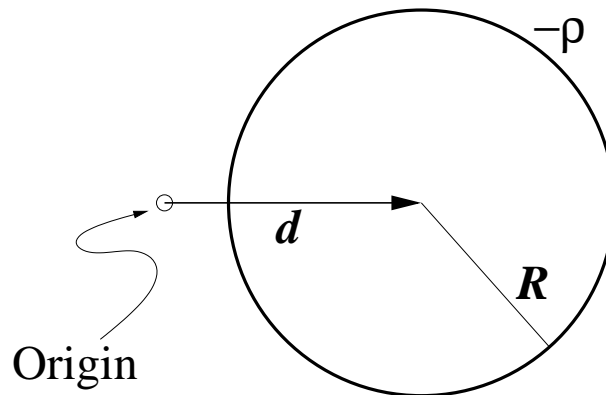
**PROBLEM 5** (27 pts)

A sphere of radius  $R$  is filled with charge density  $+\rho$  and is centered on the origin.



(a) Compute the electric field  $\vec{E}$  for any point inside the sphere. Write your answer in terms of the displacement vector from the origin  $\vec{r}$ .

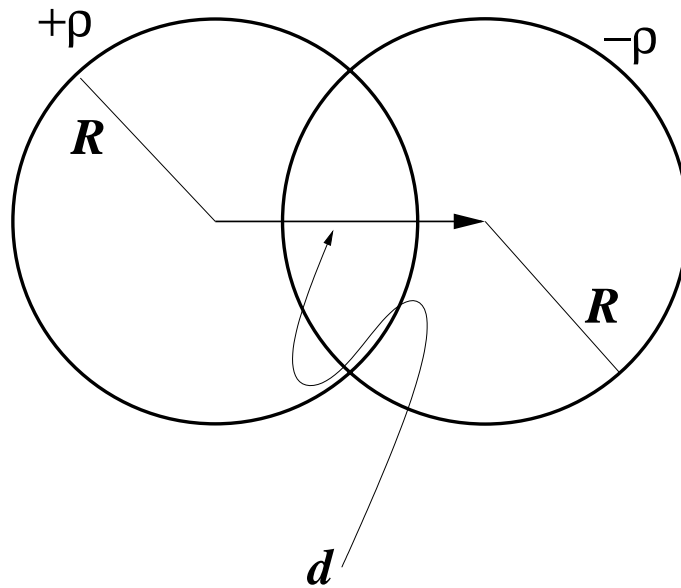
Consider now a sphere of radius  $R$  filled with charge density  $-\rho$ . The center of this sphere is displaced from the origin by the vector  $\vec{d} = d\hat{x}$ .



(b) Compute the electric field  $\vec{E}$  for any point inside this sphere. Write your answer in terms of the displacement vector from the origin  $\vec{r}$ .

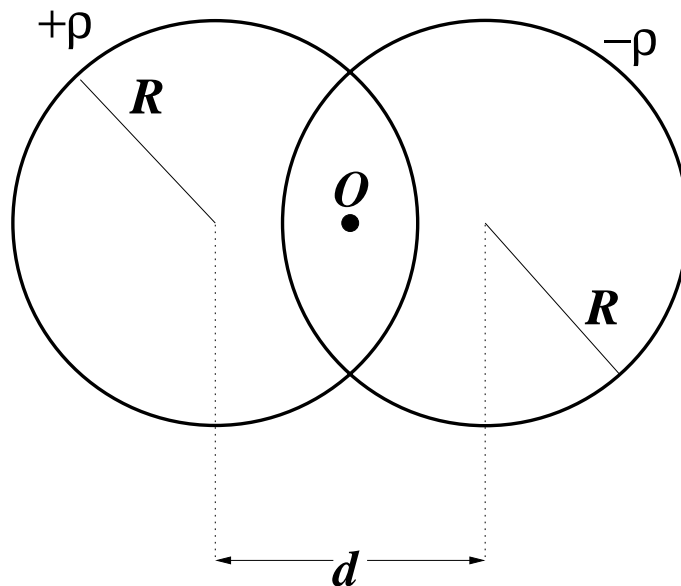
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Both spheres are now laid down such that they overlap:  $d \leq 2R$ .



(c) Compute the electric field  $\vec{E}$  in the overlap region.

(d) What is the potential difference  $\Delta V$  between the centers of the two spheres?



(e) Consider a point  $P$  on the  $x$  axis, a distance  $x$  away from the point  $O$ . What is the electric field for  $x \gg R$ ?