

A Driven RCL Circuit

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1 Introduction

The presence of inductance in an electric circuit gives the current an “inertia” since inductors try to prevent changes in the flow of current. The presence of capacitance in a circuit means that charge can flow into one side of the capacitor to be stored there, and later on this charge can restore the electric current as the capacitor discharges. These two properties of inertia and energy storage are analogous to the inertia and energy storage of a mass-spring combination, which you studied in mechanics. In a mechanical system viscous friction causes damping and in electric circuits resistance causes the damping. If a mechanical system that has a natural frequency of oscillation is “driven” by a periodic external force whose frequency matches the natural frequency of oscillations, then the system is said to be “in resonance” with the driving force and the amplitude of oscillations can grow very large. An electric circuit driven by a periodic external voltage exhibits the same behavior. In this experiment you will study the properties of circuits consisting of an inductor, capacitor and resistor in series. You will observe the behavior near resonance and measure the resonant frequency.

2 Components Needed

In addition to the laptop, you will need the following components:

- 100μ F capacitor
- 750 Interface

- A/C electronics laboratory
- Data studio files:
 - 17-RLC.ds
 - 17-RLCcurve.ds
 - 17-RLCCurrentVsVoltage.ds

3 Experimental Setup

Take your AC/DC electronics laboratory and the 100 μF capacitor and connect the capacitor in series with the coil on the electronics board. Connect the Signal Generator of the Science Workshop 750 Interface into the electronics board using the banana plugs, Fig. 1, and connect the Signal Generator output in series with the capacitor and the coil. Connect the Science Workshop 750 Interface to the computer using the SCSI cable.

Connect the power supply to the 750 Interface and turn on the interface power. Always turn on the interface before powering up the computer. Turn on your computer. The Data Studio files (17-RLC.ds, 17-RLCcurve.ds and 17-RLCCurrentVsVoltage.ds) should be on the desktop. If any are missing, please contact the instructor. The file 17-RLC.ds has a Signal Generator Display, and a Scope Display 2.

In the Signal Generator dialog box choose a Sine Wave Function. If necessary, adjust the Amplitude to 2.0 V, the Frequency to 10 Hz and the Sampling Rate to 5000 Hz. The resistance of the coil is 5.4 ohms and its inductance is 8.5 mH.

1. Draw in the space provided below a circuit diagram showing the resistance of the coil (inductor), inductor, capacitor and frequency generator.

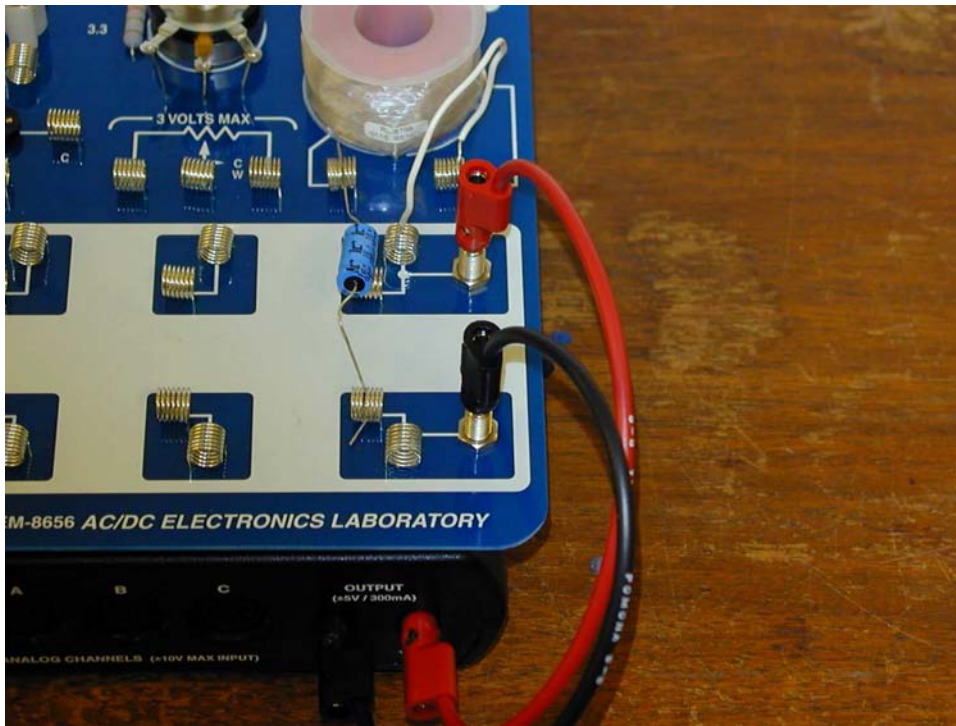


Figure 1: Experimental setup.

2. Calculate the resonant frequency you should expect for your experiment in Hertz (cycles per second) using the formulas $\omega_o = 1/\sqrt{LC}$ and $\nu_o =$

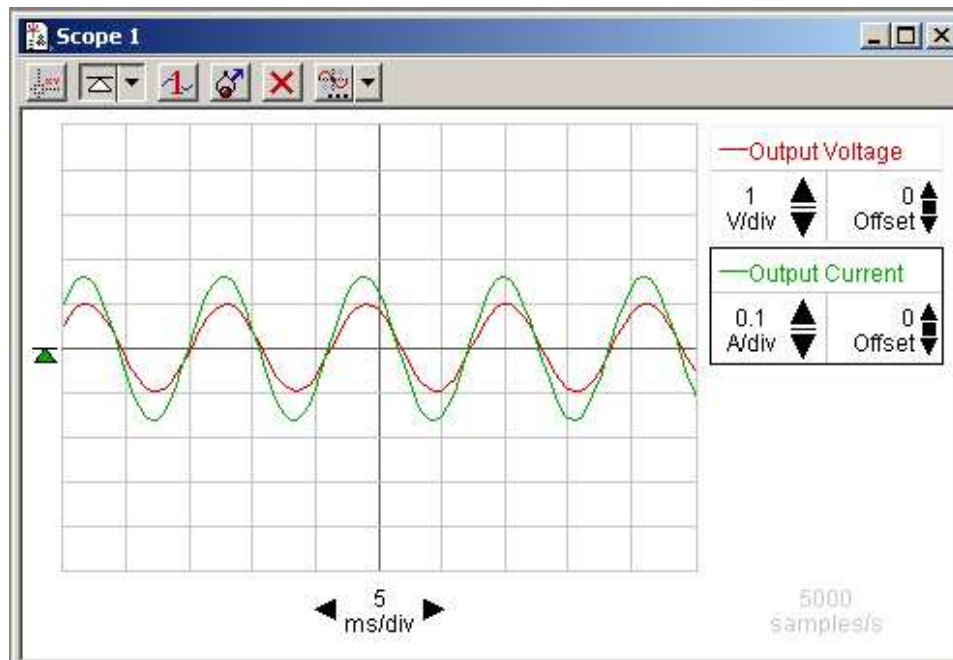


Figure 2: Data studio display showing the output voltage can current.

$\omega/2\pi$, with $L = 8.5 \text{ mH}$ and $C = 100 \mu\text{F}$.

Record your answer here: $\nu_o =$ _____ Hz

4 Measuring the resonant frequency

Click “Start” to begin collecting data. On the Scope display you should see two waveforms for the Output Voltage and the Output Current (if necessary, click the Trigger control on the Scope display to stabilize the picture). You can adjust the display using the controls. Increase the frequency of the sine wave in 10Hz steps (you change the frequency in the Signal Generator window). Identify a range of frequencies where the Output Current seems to have a maximum. Then use smaller frequency steps (say, 2Hz) in order to

locate the resonant frequency more precisely. Record the resonant frequency at which the Output Current reaches a maximum. Record your answer here

$\nu_{max} =$ Hz

Answer the following questions:

3. Does your value agree with the theoretical value you calculated above?
4. How does the phase relation between Output Current and Output Voltage change as you increase the frequency from below the resonant frequency to above it?
5. How could you use this information to help pinpoint the resonant frequency? ¹

Since the Output Voltage and Output Current are in phase with each other at resonance, the output power P varies as the square of a sinusoidal function. The maximum value of the output power is $P_{max} = I_{max}V_{max}$ and the *average* power supplied, $\langle P \rangle$ is one half of the maximum (because the average value of \sin^2 is 1/2.)

Carry out the following calculations:

6. Compute the average power consumed by your circuit at resonance.
7. The power lost in a resistor is I^2R . For a sinusoidal current, average value of I^2 is 1/2 of I_{max}^2 . Compute the average value of I^2R at the resonant frequency. Compare that with the average power consumed by the circuit.

¹Use the Smart Tool to measure the amplitude of the Output Current at the resonant frequency. NOTE on using the Smart Tool in the Scope Display: When you have more than one signal being displayed on the Scope, you can have a Smart Tool for EACH trace. You need separate Smart Tools because each signal can have a different scale. Look at the right side of the display. Click the name of the signal you want to make active and a box will appear around it. Then click on the Smart Tool button and a Smart Tool cursor will appear in the color of that trace. You can do the same for the other trace.

5 Unknown capacitors

You will be given two capacitors of unknown values. For each one, put it into the circuit and find the resonant frequency. Use this frequency to find the value of each capacitor.

$C_1 =$ F

$C_2 =$ F

6 The resonance curve and the Quality factor

The resonance curve and the “Quality factor” Now examine the behavior of the circuit near resonance in more detail. For this purpose it is more convenient to use a Graph display than the scope, so download and open the file 17-RLCcurve.ds, which should simplify data collection. Starting at 20Hz on the Signal Generator, click Start to obtain the maximum value of the output current for this frequency. As the data is taken, this maximum value is automatically calculated and placed in the box identifying the data run.

Record the maximum value of the current for this 20 Hz frequency in the Resonance Data Table on the right, in the first row of the column to the right of the frequency column. Then increase the frequency of the Signal Generator to the next value of the current in the Resonance Data Table and again record the maximum current in the data table. As you enter data in the table a curve will be drawn. Continue doing this until you have taken measurements at all of the listed frequencies in the Resonance Data Table. If you want to better define the shape of the curve around the resonance, take extra data points near the resonant frequency so you can see clearly what happens to the current there . What this curve tells you is that if a large number of different sine-wave voltage signals were all added together and connected to

this circuit, only those with frequencies near the resonant frequency will be able to produce significant currents, that is, the circuit would be tuned to “hear” only those signals and ignore the others. Obviously, something of this sort occurs in the tuner of a radio. The quality factor of a circuit is a measure of how sharp the resonance curve is. It is defined as

$$Q = \frac{\nu_o}{\nu_2 - \nu_1}$$

where ν_o is the resonant frequency and ν_1 and ν_2 are the frequencies for which the output power drops to half of its value at ν_o , Fig. 3. (In other words, these are the frequencies for which the output current drops to $1/\sqrt{2} \simeq 0.7$ of its value at ν_o .) In a radio tuner, Q may range from 10 to 100 or so. Compute Q for your circuit.

$Q =$

7 Phase relationship

An interesting way of exploring the phase relationships between current and voltage in a resonant circuit is to graph the Output Current versus the Output Voltage. You can either do this following the instructions below using the file 17-RLC.ds (re-open it), or you can download a different file named 17-RLCCurrentVsVoltage.ds for which the display is already set up. To amend the re-opened 17-RLC.ds file, go to the Scope display click on the box labelled Output Voltage and delete it from the display. Then go to the Data window on the left and click and drag the icon for Output Voltage into the scope window and drop it on the time setting at the bottom of the scope display. Now Start observing data, using either this amended 17-RLC.ds file, or the newly downloaded 17-RLCCurrentVsVoltage.ds file. Adjust the displayed signal so that it is wide enough and tall enough to be seen clearly. Change the frequency of the Signal Generator. Observe what happens at the resonance frequency of the circuit. When you are exactly at resonance, the scope should display a straight line. Can you explain this behavior?

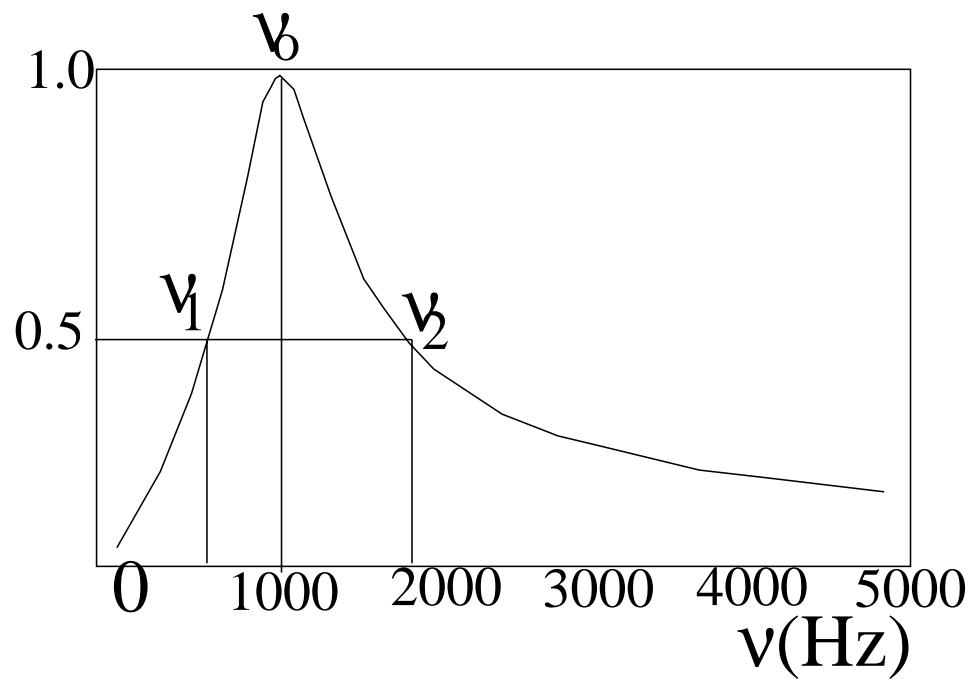


Figure 3: Resonance curve showing peak frequency ν_o and frequencies at half max, ν_1 and ν_2 .