ESG 8.022 Fall 2006 Exam 1

Instructor: Michael Shaw

Wednesday, October 11th, 2006 @ 1:00PM

1 Spherical Charge Distribution (35 points)

Consider the following charge distribution in 3-dimensional Euclidean space.

$$\rho = \rho(r) = \begin{cases} c_1 \cdot \frac{r}{r_1} & r \leq r_1 \\ c_2 \cdot \delta(r - r_2) \cdot \left(\frac{r}{r_2}\right)^2 & r > r_1 \end{cases}$$

where r is the usual spherical polar coordinate and $r_1 < r_2$.

a. What are the units of c_1 and c_2 ?

b. Find the electric field for $r < r_1$

c. Find the electric field for $r_1 < r < r_2$

d. Find the electric field for $r > r_2$

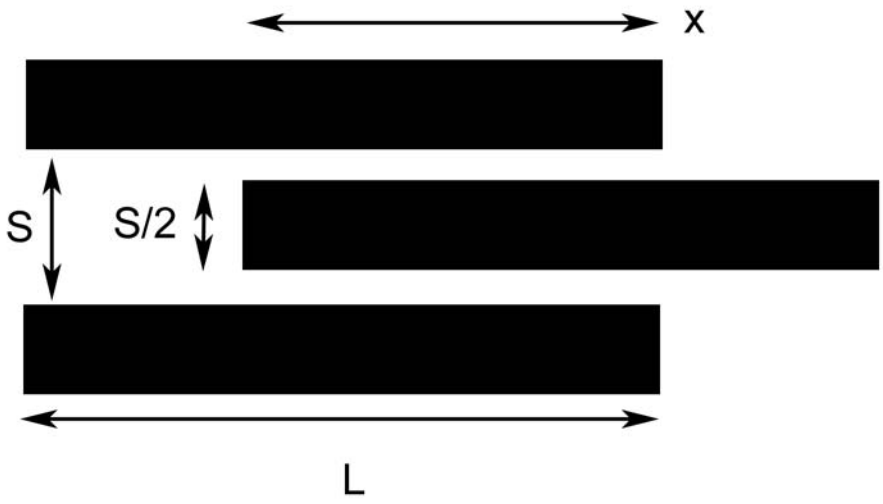
e. Graph the electric field as a function of r. Is the electric field continuous everywhere? Is this ok?

f. Find the electric potential in all space. Graph the electric potential as a function of r.

g. Is the electric potential continuous everywhere? Is this ok?

h. Find ρ_0 so that the total charge is Q.

2 A Parallel Plate Capacitor (35 points)



Two flat, square metal plates have sides of length L, and thickness $s/2$, are arranged parallel to each other with a separation of s, where $s << l$ so you may ignore fringing fields. A charge Q is moved from the upper plate to the lower plate. Now a force is applied to a third uncharged conducting plate of the same thickness so that it lies between the other two plates to a depth x, maintaining the same spacing $s/4$ between its surface and the surfaces of the other two.

a. Find the electric field E_w in the wide region.

b. Find the electric field E_n in the narrow region.

c. Find the surface charge in the wide region, σ_w and in the narrow region, σ_n.

d. What is the potential difference between the upper and lower plates?

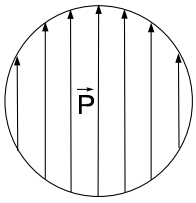
e. Find the capacitance of the system.

f. What is the energy stored in the electric field?

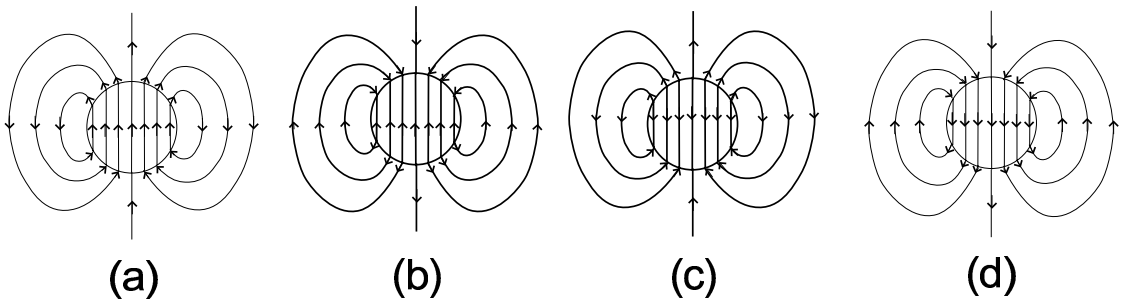
g. What force (magnitude and direction) must be exerted on the middle plate to keep it from moving?

h. If the force is no longer applied, in which direction does the middle plate move?

3 Multiple Choice Question (10 points)



3. A sphere is uniformly polarized as shown in the diagram above. Circle the case below that shows the correct electric field lines.



4 Electrostatic Equilibrium (20 points)

When can a point charge be in stable mechanical equilibrium in the electric field of other charges? That is, is there some arrangement of point charges so that if we fix all but one of them, the other one is in a stable equilibrium? If so, what configurations are possible? Please show sufficient reasoning for your answer.

Note 1: A mathematical proof is sufficient reasoning. So is a good proof by example. So is a sound physical argument. So is any other technique that shows you have a good grasp of the concept at hand.

Note 2: A stable equilibrium is one where if we move the charge in any direction, there is a restoring force directed opposite to the displacement.

Hint: Use Gauss's Law.