8.022 - Class 2 - 9/7/2006

October 20, 2007

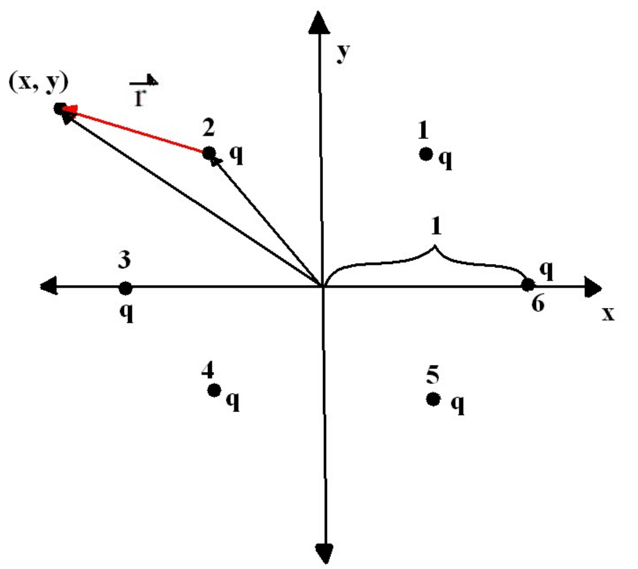


Figure 1: Six point charges of magnitude q in x-y plane.

1. Find $\vec{E}(x, y)$
2. Find $\vec{E}(x, 0)$
3. Find $\vec{E}(x, y)$ for $x >> 1$
4. Remove charge 6. Find $\vec{E}(0, 0)$

<u>Solutions</u>

1.

$$\vec{E}(x,y) = \frac{q}{4\pi\epsilon_0}\left(\sum_{k=1}^{6} \frac{(x-\cos(\frac{k\pi}{3}))\hat{i}+(y-\sin(\frac{k\pi}{3}))\hat{j}}{\sqrt{(x-\cos(\frac{k\pi}{3}))^2+(y-\sin(\frac{k\pi}{3}))^2}^3}\right)$$

$$= \frac{q}{4\pi\epsilon_0}\left(\sum_{k=1}^{6} \frac{(x-\cos(\frac{k\pi}{3}))\hat{i}+(y-\sin(\frac{k\pi}{3}))\hat{j}}{(x^2+y^2+1-2x\cos(\frac{k\pi}{3})-2y\sin(\frac{k\pi}{3}))^{3/2}}\right)$$

2.

$$\vec{E}(x,0) = \frac{q}{4\pi\epsilon_0}\left(\sum_{k=1}^{6} \frac{(x-\cos(\frac{k\pi}{3}))\hat{i}}{(1+x^2-2x\cos(\frac{k\pi}{3}))^{3/2}}\right)$$

3. For $x >> 1$,

$$\vec{E}(x,0) = \frac{q}{4\pi\epsilon_0}\sum_{k=1}^{6} \frac{x-\cos(\frac{k\pi}{3})}{(x^2-2x\cos(\frac{k\pi}{3})+1)^{3/2}}$$

$$= \frac{q}{4\pi\epsilon_0}\sum_{k=1}^{6} \frac{x(1-\frac{\cos(\frac{k\pi}{3})}{x})}{x^2(1-\frac{2\cos(\frac{k\pi}{3})}{x}+\frac{1}{x^2})^{3/2}}$$

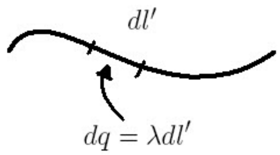
$$= \frac{q}{4\pi\epsilon_0}\sum_{k=1}^{6} \frac{1-\frac{\cos(\frac{k\pi}{3})}{x}}{(1-\frac{6\cos(\frac{k\pi}{3})}{x}+\frac{12\cos^2(\frac{k\pi}{3})+3}{x^2})^{1/2}}$$

<u>Expanding by Taylor Series:</u>

$$\sqrt{1+\frac{a}{x}} = \frac{f(1)}{0!}+\frac{f'(1)}{1!}(1+\frac{a}{x})+...$$

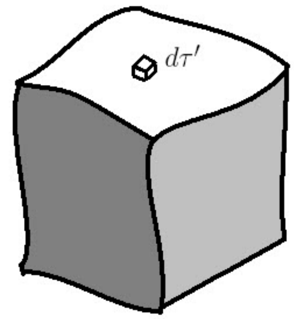
$$= 1+\frac{1}{2}(1+\frac{a}{x})+.....$$

Should show that terms $\frac{1}{x^3}$ and above don't matter



Line charge λ

Surface charge σ



Volume charge ρ

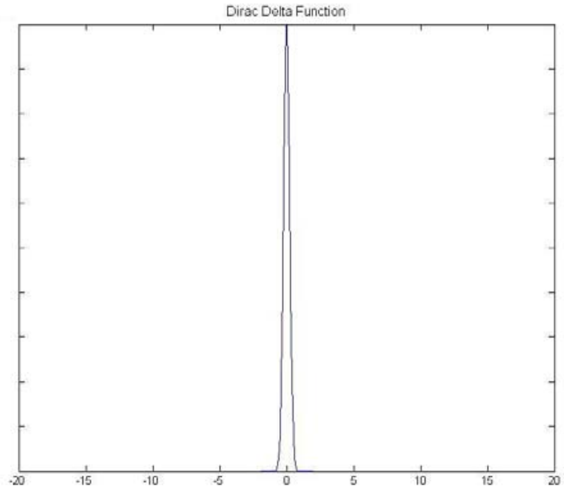
<u>For line charge:</u>

$$\begin{aligned} \vec{E}(\vec{r}) &= \frac{1}{4\pi\epsilon_0}\int \frac{1}{\zeta^2}\hat{\zeta}\,dq \\ &= \frac{1}{4\pi\epsilon_0}\int \frac{\tau}{\zeta^2}\hat{\zeta}\,dl' \end{aligned}$$

<u>For a volume charge:</u>

$$\vec{E}(\vec{r}) = \int \frac{l}{\zeta^2}\hat{\zeta}\,d\tau'$$

Can derive all other cases from this using Dirac delta function.



-Read 1.1.1-1.4.1 for Monday
Ch 1 in General