8.022 Lecture Notes Class 44 - 12/11/2006

Relativity

$$x^\mu = (ct, x, y, z)$$

$$p^\mu = (\frac{E}{c}, p_x, p_y, p_z)$$

$$A^\mu = (V, A_x, A_y, A_z)$$

Faraday/EM Tensor

$$F^{\mu\nu} = \frac{\partial A^\mu}{\partial x_\mu} - \frac{\partial A^\nu}{\partial x_\nu}$$

Maxwell

$\partial_\nu F^{\mu\nu} \propto J^\mu$ electric half of Maxwell's

$\partial_\mu F^{\mu\nu} = 0$ magnetic half of Maxwell's

$$\nabla_\mu = (\frac{\partial}{\partial t}, -\frac{\partial}{\partial x}, -\frac{\partial}{\partial y}, -\frac{\partial}{\partial z})$$

$$\nabla_\mu b_\mu (g^\mu_\nu) = \nabla_\mu b^\mu g^\mu_\nu = \frac{\partial b^t}{\partial t} + \vec{\nabla} \cdot b^i$$

$$\nabla_\mu j_\mu = \frac{\partial p}{\partial t} + \vec{\nabla} \cdot j^i = 0$$

Conservation of charge

$$\nabla_\mu \nabla_\mu = \frac{\partial^2}{\partial t^2} - \nabla^2 = \Box^2$$

D'alembertion or "box" operator

$$\Box^2 A^\mu = \frac{1}{\epsilon_0} j_\mu \Leftarrow \quad \text{Maxwell's Equations}$$

iff $\frac{\partial V}{\partial t} + \vec{\nabla} \cdot \vec{A} = 0$ Lorentz Gauge (not $\vec{\nabla} \cdot \vec{A} = 0$)

Ex : Look at $\mu = 0$

$$A_0 = V, j_0 = \rho$$

$$\Box^2 V = \frac{\rho}{\epsilon_0}$$

$$\frac{\partial^2 V}{\partial t^2} - \nabla^2 V = \frac{\rho}{\epsilon_0}$$

$U = 0$

$U_\alpha = (F - m_a)^2$

General Relativity

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -c^2 d\tau^2$$

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \qquad \text{- Einstein field equation}$$

In General relativity,

$$ma = \vec{F_{em}} = q(\vec{E} + \vec{v} \times \vec{B}); \vec{F_{gravity}} = 0$$

$g_{\mu\nu} : G_{\mu\nu} : A^{\mu} : F^{\mu\nu}$

$g_{\mu\nu}$ is gravity potential and metric
$G_{\mu\nu}$ is gravity fiels and Einstein field tensor
A^{μ} are magnetic potentials

<u>Special</u>

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

<u>General</u>

$g_{\mu\nu}$ is arbitrarily nasty

two different concepts of mass:

- gravitational mass
- inertial mass

$$ds^2 = -c^2 d\tau^2$$

$$(c^2 dt^2 - dx^2 = d\tau^2)$$

Pick a $g_{\mu\nu}$ (Diagonal)

$$ds^2 = -(1 + 2\phi)c^2 dt^2 + (dx - \beta^x dt)^2 + (dy - \beta^y dt)^2 + (dz - \beta^z dt)^2$$

Weak field limit

Gravity is non-linear !

Photons have no charge so E/M happy. Gravitons have mass! Problem!

Affect themselves!

$$m\frac{d^2\vec{x}}{dt^2} = \vec{g}m + m \cdot \vec{v} \times \vec{H}$$

$$\vec{g} = -\vec{\nabla}\phi, \vec{H} = \vec{\nabla} \times \vec{\beta}$$

$$\nabla \cdot g = -4\pi G\rho_m \qquad \nabla \times g = 0$$

$$\nabla \cdot H = 0 \qquad \nabla \times \vec{H} = -\frac{16\pi G J_m}{c}$$