8.022 Lecture Notes Class 46 - 12/13/2006

Schrodinger Equation

- matter waves
- probability wave

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\cdot\frac{\partial^2\psi}{\partial x^2} + V(x)\psi(x,t)$$

$$P(x,t) = |\psi(x,t)|^2 = \psi^*(x,t)\psi(x,t)$$

$$\begin{aligned}\frac{\partial}{\partial t}P(x,t) &= \tfrac{\partial\psi^*}{\partial t}\psi + \psi^*\tfrac{\partial\psi}{\partial t} \\ &= -\tfrac{\partial}{\partial x}[\tfrac{\hbar}{2im}(\psi^*\tfrac{\partial\psi}{\partial x} - \tfrac{\partial\psi^*}{\partial x}\psi)]\end{aligned}$$

Let $j(x,t) = \frac{\hbar}{2im}(\psi^*\frac{\partial\psi}{\partial x} - \frac{\partial\psi^*}{\partial x}\psi)$,

$$\frac{\partial}{\partial t}P + \frac{\partial}{\partial x}j = 0 \qquad \text{Conservation law for probability}$$

$$\frac{d}{dt}\int_a^b P(x,t)dx = j(b,t) - j(a,t)$$

Can write as:

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}\psi \text{ , H = Hamiltonian Operator}$$

Hamiltonian operator is an energy operator

$$H = \frac{p^2}{2m} + V(x) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)$$

First term is energy from momentum, second term is energy is potential energy

On operators :

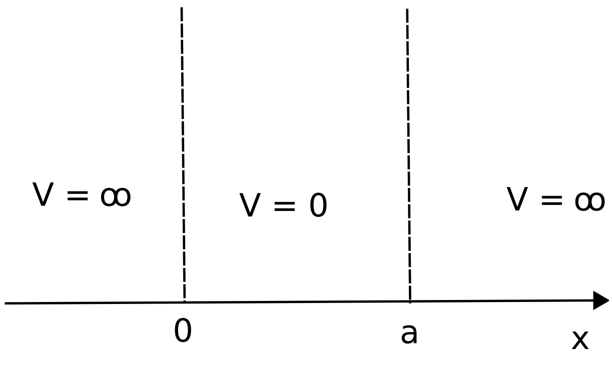
$$< \psi|\psi >= \int_{-\infty}^{\infty} dx\psi^*\psi \qquad \hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$$

$$< \psi|x|\psi >= \int_{-\infty}^{\infty} dx\psi^* x\psi \qquad \hat{x} = x$$

If H is constant, then let's call it E for certain t

$$-\frac{\hbar}{2m} \cdot (\frac{\partial}{\partial x})^2 \psi(x) + V(x)\psi(x) = E \cdot \psi(x)$$

Take $V(x)$ in the infinite square well case:



$E > 0$ in general, $E < 0$ unlikely

$$\frac{\partial^2}{\partial x^2}\psi + k^2\psi = 0$$

$$k^2 = \frac{2mE}{\hbar^2}$$

$$\psi = A\sin(kx)$$

$$ka = n\pi$$

$$E_n = \frac{h^2k^2}{2m} = \frac{\hbar^2\pi^2n^2}{2ma^2}$$

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin(\frac{n\pi x}{a})$$