Physics 8.03
Vibrations and Waves

Lecture 4

COMPLETE SOLUTION

to the Harmonically Driven Oscillator

Last time: Harmonically driven harmonic oscillator

- Equation of Motion

$$\ddot{x}+\gamma\dot{x}+\omega_0^2 x=\frac{F_0}{m}\cos(\omega t)$$

- Solutions
  - Oscillator oscillates at driving frequency
  - Amplitude and phase depend on driving frequency

$$x(t)=A(\omega)\cos(\omega t-\delta(\omega))$$

$$A(\omega)=\frac{F_0/m}{\sqrt{(\omega_0^2-\omega^2)^2+\gamma^2\omega^2}}$$

$$\delta(\omega)=\frac{\gamma\omega}{(\omega_0^2-\omega^2)}$$

- Resonance

$A(\omega)$ is maximum when $\omega \approx \omega_0$

- Correction

$$\omega_{\max}=\sqrt{\omega_0^2-\frac{\gamma^2}{2}}\approx\omega_0 \text{ when } \gamma<<1$$

Transient behavior

- What happens when driving force is first turned on? Transients
- We started with a second order diff. eqn. so we should get two constants of integration. Where are they?
- Complete solution to the diff. eqn. includes the a particular solution (we got that last time) AND the homogenous solution (that describes the transient behavior of the driven oscillator)