# Physics 8.03
# Vibrations and Waves

Lecture 6
Driven Coupled Oscillators

# Last time: Coupled oscillators

- Normal modes of oscillation
  - Harmonic motion at fixed (eigen)frequencies
  - Amplitude ratios for each mode (constant)
- "Any old motion"
  - All allowed motions are a superposition of all the normal modes

# External driving force

- Introduce harmonic external driving force in a coupled oscillator system
- N oscillators (N ⑦ 1)

# A Recipe'

- Find forces acting on each particle
- Coupled differential equations
  - No driving force → homogeneous
  - Driving force → at least one eqn. is inhomogenous
- Always solve homogeneous equation first
- Trial solution → $x_i(t) = C_i \cos(\omega t - \delta)$
- Coupled (simultaneous) algebraic equations

$$\vec{C} = \vec{D} \qquad \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_N \end{pmatrix}$$

# …The Recipe'

- "Normal" modes
  - Frequencies (eigenvalues): $\omega_i$ are the roots of 💣, calculate by solving for $\omega$ when det(💣) = 0
  - Ratios of amplitudes: Plug $\omega = \omega_i$ back into 💣 $\vec{C}$
- Any other motion → superposition of all normal modes
- Now turn on the harmonic driving force
- Solve inhomogenous set using Cramer's rule
  - For each $C_i$ replace the *i*-th column of 💣 with $\vec{D}$