Physics 8.03
Vibrations and Waves

Lecture 7
The Wave Equation
Solutions to the Wave Equation

Last time:
External driving force

- Applied an external driving force to a coupled oscillator system
  - In steady-state coupled system takes on frequency of the driving force
  - When driving force is at a normal mode frequency $\rightarrow$ resonance

A Recipe' for coupled oscillators

- Find forces acting on each particle
- Coupled differential equations
  - No driving force → homogeneous
  - Driving force → at least one eqn. is inhomogenous
- Always solve homogeneous equation first
- Trial solution → $x_i(t) = C_i \cos(\omega t - \delta)$
- Coupled (simultaneous) algebraic equations

$$\vec{C} = \vec{D} \qquad \begin{pmatrix} C_1 \\ C_2 \\ \vdots \\ C_N \end{pmatrix}$$

A Recipe' for Coupled Oscillators …contd…

- "Normal" modes
  - Frequencies (eigenvalues): ω_i are the roots of 💣, calculate by solving for ω when det(💣) = 0
  - Ratios of amplitudes: Plug $\omega = \omega_i$ back into 💣 $\vec{C}$
- Any other motion → superposition of all normal modes
- Now turn on the harmonic driving force
- Solve inhomogenous set using Cramer's rule
  - For each C_i replace the *i*-th column of 💣 with $\vec{D}$

Last time: N coupled oscillators

- N identical oscillators (*N* beads on a string)
  - N normal modes
  - Frequency and amplitude of motion of the *p*-th depends on
    - Mode number, *n*
    - Location of particle in the array, *p*
- As N → ∞, we get a continuous system of oscillators

Wave Equation and its Solutions

- Waves → oscillations in space and time
  - $y\,(x, t\,)$
  - Transverse or longitudinal waves
  - Traveling or standing waves
- Solutions to wave equation
  - Pulses of arbitrary shape → $y\,(x, t\,) = f\,(x \pm v\,t\,)$
  - Harmonic pulses → $y\,(x, t\,) = y_0 \cos(k\,(x \pm v\,t\,) + \phi)$
  - Separable solutions → $y\,(x, t\,) = f\,(x) \cos(\omega\,t + \phi\,)$