Massachusetts Institute of Technology
Department of Physics

Physics 8.033 | 17 October 2006

Quiz 1

Name: (Last, First) ______________________________ (please print).

Recitation number (circle one): 1 2 3

- Record all answers and show all work in this exam booklet. If you need extra space, use the back of the opposing page.
- All scratch paper must be handed in with the exam, but will not be graded.
- No materials besides pencils and erasers are allowed (no calculators, notes, books, pets, etc.)
- Whenever possible, try to solve problems using general analytic expressions. Plug in numbers only as a last step.
- Please make sure to answer all sub-questions.
- Good luck!

Problem	Max	Grade	Grader
1	25		
2	25		
3	25		
4	25		
Total	100		

Question 1: Quick answers `[25 Points]`

(a). **(4 pts)** Professor F. Ishy claims to have discovered a new force of magnitude $F = A|\mathbf{r}_1 + \mathbf{r}_2|$ between protons, where $\mathbf{r}_1$ and $\mathbf{r}_2$ are the position vectors of the protons and A is a constant that he's trying to get named in his honor.

(A) Is this force translationally invariant? YES / NO (circle one) No

(B) Is this force invariant under rotations around $\mathbf{r} = \mathbf{0}$? YES / NO (circle one) Yes

(b). **(4 pts)** Professor C. Rank claims that a charge at $(\mathbf{r}_1, t_1)$ will contribute to the air pressure at $(\mathbf{r}_2, t_2)$ by an amount $B \sin[C(|\mathbf{r}_2 - \mathbf{r}_1|^2 - c^2|t_2 - t_1|^2)]$, where B and C are constants.

(A) Is this effect Galilean invariant? YES / NO (circle one) No

(B) Is this effect Lorentz invariant? YES / NO (circle one) Yes

(c). **(1 pt)** To 1 significant figure, the speed of light in meters/second is ______________ $c = 3 \times 10^8$ m/s

(d). **(2 pts)** From which two postulates did Einstein derive special relativity?

(a) Laws of physics must be valid in all intertial frames (b) The speed of light is a constant, independent of observer

(e). **(11 pts)** Indicate whether each of the following statements are true of false.

(A) The proper length of a ruler is Lorentz invariant. TRUE / FALSE (circle one) True

(B) The wave equation is Galilean invariant. TRUE / FALSE (circle one) False

(C) The kinetic energy of a particle is Lorentz invariant. TRUE / FALSE (circle one) False

(D) The acceleration of a particle is Galilean invariant. TRUE / FALSE (circle one) True

(E) A ping-pong ball moving near the speed of light still looks spherical.
TRUE / FALSE (circle one) True

(F) X-rays travel faster than microwaves. TRUE / FALSE (circle one) False

(G) If you could send a signal faster than light, then there's a frame where you could send a signal backward in time. TRUE / FALSE (circle one) True

(H) If two twins are reunited, who is oldest may be frame-dependent.
TRUE / FALSE (circle one) False

(I) No experiment inside an isolated sealed lab in space can determine its orientation.
TRUE / FALSE (circle one) True

(J) No experiment inside an isolated sealed lab in space can determine its velocity.
TRUE / FALSE (circle one) True

(K) No experiment inside an isolated sealed lab in space can determine its acceleration.
TRUE / FALSE (circle one) False

(f). **(3 pts)** List three pieces of observational evidence supporting special relativity. Cosmic ray muons, atomic bombs, GPS measuring time slowdown

Question 2: Pole Vaulter Deluxe [25 Points]

Your goal in this problem is to completely fill out the table below.

	S (Chris)	S' (Zoe)	S'' (Train)
x_B	L/γ_1	L	$L\gamma_2/\gamma_1$
ct_B	0	$-\beta_1 L$	$-\gamma_2\beta_2 L/\gamma_1$
x_C	$\gamma_1 L$	L	$\gamma_1\gamma_2 L(1-\beta_1\beta_2)$
ct_C	$\gamma_1\beta_1 L$	0	$\gamma_1\gamma_2 L(\beta_1-\beta_2)$

To save time, note that all three frames have the same spacetime origin (at event A) and that there is no need to draw spacetime diagrams.

(a). **(2 pts)** Based on the text below, fill in the entries corresponding to $\mathbf{X}_B$ and $\mathbf{X}'_C$ in the table above (no calculation needed!). Chris is standing next to a barn taking measurements as Zoe runs through the barn in the x-direction holding a pole horizontally in the direction of motion. In Zoe's frame S', the pole has length L, the event when the rear of the pole is aligned with the entrance to the barn is $\mathbf{X}'_A = (x'_A, ct'_A) = (0, 0)$, and the location of the front of the pole at that same time is $\mathbf{X}'_C = (L, 0)$. In Chris' frame S, Zoe is running at speed β_1, the rear of the pole is aligned with the entrance to the barn (event A) at $\mathbf{X}_A = (x_A, ct_A) = (0, 0)$, and the front of the pole is aligned with the barn exit (event B) at $\mathbf{X}_B = (x_B, ct_B) = (L/\gamma_1, 0)$, where $\gamma_1 = 1/\sqrt{1-\beta_1^2}$.

(b). **(7 pts)** Compute $\mathbf{X}'_B$ and $\mathbf{X}_C$ and fill in the corresponding entries in the table above.

$x'_B = \gamma_1(x_B - \beta_1 ct_B) = \gamma_1 L/\gamma_1 = L$
$ct'_B = \gamma_1(ct_B - \beta_1 x_B) = -\gamma_1\beta_1 L/\gamma_1 = -\beta_1 L$
$x_C = \gamma_1(x'_C + \beta_1 ct'_C) = \gamma_1 L$ *(Note inverse transform from S' to S)*
$ct_C = \gamma_1(ct'_C + \beta_1 x'_C) = \gamma_1\beta_1 L$ *(Note inverse transform from S' to S)*

(c). **(8 pts)** A train with the rest of the 8.033 students passes by, and Chris measures its speed to be β_2 in the x-direction. The train's frame S'' is aligned such that $\mathbf{X}''_A = (0, 0)$. Compute $\mathbf{X}''_B$ and $\mathbf{X}''_C$ in terms of L, γ_1, β_1, γ_2, and β_2, and fill in the corresponding entries in the table above.

The trick here is to transform all events from the S frame to the S'' frame
$x''_B = \gamma_2(x_B - \beta_2 ct_B) = \gamma_2 L/\gamma_1$
$ct''_B = \gamma_2(ct_B - \beta_2 x_B) = -\gamma_2\beta_2 L/\gamma_1$
$x''_C = \gamma_2(x_C - \beta_2 ct_C) = \gamma_2(\gamma_1 L - \beta_2\gamma_1\beta_1 L) = \gamma_1\gamma_2 L(1-\beta_1\beta_2)$
$ct''_C = \gamma_2(ct_C - \beta_2 x_C) = \gamma_2(-\gamma_1\beta_1 L - \beta_2\gamma_1 L) = \gamma_1\gamma_2 L(\beta_1 - \beta_2)$

(d). **(4 pts)** Show that for $\beta_2 = 0$, the students on the train agree with Chris.

$\beta_2 = 0$, $\gamma_2 = 1$
$x''_B = L/\gamma_1$
$ct''_B = 0$
$x''_C = \gamma_1 L$
$ct''_C = \gamma_1\beta_1 L$

(e). **(4 pts)** Show that for $\beta_2 = \beta_1$ the students on the train agree with Zoe.

$\beta_2 = \beta_1$, $\gamma_2 = \gamma_1$
$x''_B = \gamma_1 L/\gamma_1 = L$
$ct''_B = -\gamma_1\beta_1 L/\gamma_1 = -\beta_1 L$
$x''_C = \gamma_1^2 L(1-\beta_1^2) = L$
$ct''_C = \gamma_1^2 L(\beta_1 - \beta_1) = 0$

Question 3: Variational calculus: Newton's second law modified `[25 Points]`

For a particle of rest mass m_0 in a potential $V(x)$, show that out of all trajectories $x(t)$ between two events A and B, the one maximizing the quantity

$$\int_{t_A}^{t_B} \left[\frac{1}{\gamma} + \frac{V(x)}{m_0 c^2}\right] dt$$

satisfies the relation

$$\frac{d}{dt}(m_0 \gamma \dot{x}) = -V'(x)$$

(the relativistic version of Newton's second law). Here $\gamma \equiv 1/\sqrt{1 - \dot{x}^2/c^2}$, $\dot{x} \equiv dx/dt$ and $V' = dV/dx$. Solution: $L(x) = \left[\frac{1}{\gamma} + \frac{V(x)}{m_0 c^2}\right]$. The Euler-Lagrange equation implies

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{x}}\right] - \frac{\partial L}{\partial x} = 0 \tag{3.1}$$

$$\Rightarrow \frac{d}{dt}\left[\frac{m_0 c^2}{\sqrt{1 - \dot{x}^2/c^2}} \frac{\dot{x}}{c^2}\right] + \frac{\partial V}{\partial x} = 0 \tag{3.2}$$

$$\Rightarrow \frac{d}{dt}\left[\frac{m_0 v}{\sqrt{1 - \dot{x}^2/c^2}}\right] = -\frac{dV}{dx} \tag{3.3}$$

Question 4: Relativistic Red Sox [25 Points]

In a parallel universe, the Boston team made the playoffs.

(a). **(6 pts)** Manny Relativirez hits the ball and starts running towards first base at speed β. How fast is he running, given that he sees third base 45° to his left (as opposed to straight to his left before he started running)? Assume that he is still very close to home plate.
Using the aberration formula with $\cos\theta' = -1/\sqrt{2}$, $\beta = 1/\sqrt{2}$

(b). **(7 pts)** A player standing on third base is wearing red socks emitting light of wavelenght λ_{red}. What wavelength does Manny see? What color are the socks according to Manny, in the approximation that $\lambda_{\text{green}} = \lambda_{\text{red}}/2^{1/4}$, $\lambda_{\text{blue}} = \lambda_{\text{red}}/\sqrt{2}$, $\lambda_{\text{violet}} = \lambda_{\text{red}}/\sqrt{3}$?

Using the doppler shift formula, $\lambda' = \lambda/\sqrt{2}$

(c). **(5 pts)** In Manny's frame, the ball is moving with speed $c/\sqrt{2}$ towards first base. How fast is it going in the rest frame of the stadium?
Using the relativistic velocity addition formula, $\beta = \sqrt{8/9}$

(d). **(5 pts)** Later in the game, a squabble erupts. An outfielder catches a fly ball at $(x_A, y_A, z_A, ct_A) = (50\text{m}, 40\text{m}, 2\text{m}, cT)$ (event A) and Manny starts to run from second base at $(x_B, y_B, z_B, ct_B) = (30\text{m}, 30\text{m}, 0, 0)$ (event B), where $cT = 30\text{m}$, which corresponds to $T \approx 90$ nanoseconds. The outfielder claims that Manny is out by virtue of running too soon, whereas Manny claims that B preceded A is his frame. You are are the umpire and must settle this. Compute the spacetime interval between A and B and indicate whether it is (circle one) TIMELIKE / SPACELIKE / NULL
The interval is timelike since $\Delta t^2 > \Delta r^2$.

(e). **(2 pts)** Is Manny correct? YES / NO / DEPENDS ON HIS VELOCITY (circle one)
In one sentence, why?
No, because there's no relativity of simulatneity for time like seperations.