Solutions to Exam #3 8.03 Spring 2014

1 a) $I = \frac{I_0}{2}$ $\quad E = E_0 \sin(kz-\omega t)\hat{x} + E_0 \cos(kz-\omega t)\hat{y}$

after polarizer only one component remains e.g. $E_0 \sin(kz-\omega t)\hat{x}$

b) $\Delta t \cdot \Delta\omega = 1$ $\quad \Delta\omega = 10^9\,\mathrm{Hz} \sim 1\,\mathrm{GHz}$

c) $V_p = \frac{\omega}{k} = \sqrt{\frac{g}{k}}$ $\quad V_g = \frac{d\omega}{dk} = \frac{\sqrt{g}}{2\sqrt{k}}$ $\quad V_p = 2V_g$

d) Width of the diffractive peak is $\approx \frac{\pi D}{\lambda}\sin\psi = \pi$

$\sin\psi = \frac{R}{L} = \frac{\lambda}{D}$; $R = \frac{\lambda}{D}\cdot L = \frac{5\cdot 10^{-7}}{1}\cdot 3.8\cdot 10^8 = 200\,\mathrm{m}$

More precise radius is obtained by analyzing circular sources $r = \frac{1.22\lambda}{D}\cdot R = 230\,\mathrm{m}$

e) Light reflected from water surface is predominantly polarized with $\vec{E}$ parallel to water surface. Polaroid glasses are filtering out this polarization. By rotating by 90° we let them in, negating sun-blocking effect.

f) Use $\frac{\sin^2\left(\frac{N\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)}$ $\quad \left.\begin{array}{l} N\cdot d = \text{const} \\ d \text{ smaller} \end{array}\right\} \Rightarrow$ Distance between principal maxima increases

Width smaller, height larger

λ larger $\quad$ Distance betw PM increases

(2) a) $\vec{B} \perp \vec{k}$, $\vec{\nabla} \cdot \vec{B} = 0$ $\quad \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$

$$\underset{0}{\underbrace{0 + \frac{\partial}{\partial y}\left(B_0 \cos(kz-\omega t)\right)}} + \underset{=0!}{U} \cdot \frac{\partial}{\partial z} \underbrace{\left(B_0 \cos(kz-\omega t)\right)}_{\neq 0} = 0$$

$\Rightarrow \quad U = 0$!

$$\vec{B}(\vec{r},t) = B_0 \hat{y} \cos(kz-\omega t)$$

b) $\vec{E}(\vec{r},t) = B_0 \cdot c \hat{x} \cos(kz-\omega t)$.

c) $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad \vec{a} = \frac{\vec{F}}{m}$

Force due to $\vec{B}$ field can be ignored since q is constrained to move along $\hat{x}$

$$\vec{a} = \frac{q\vec{E}}{m} = \frac{qB_0 c}{m} \hat{x} \cos(\omega t) \quad \text{since } z=0!$$

d) $\vec{E}_{rad} = -\frac{q \vec{a}_\perp (t - r/c)}{4\pi\varepsilon_0 r c^2}$

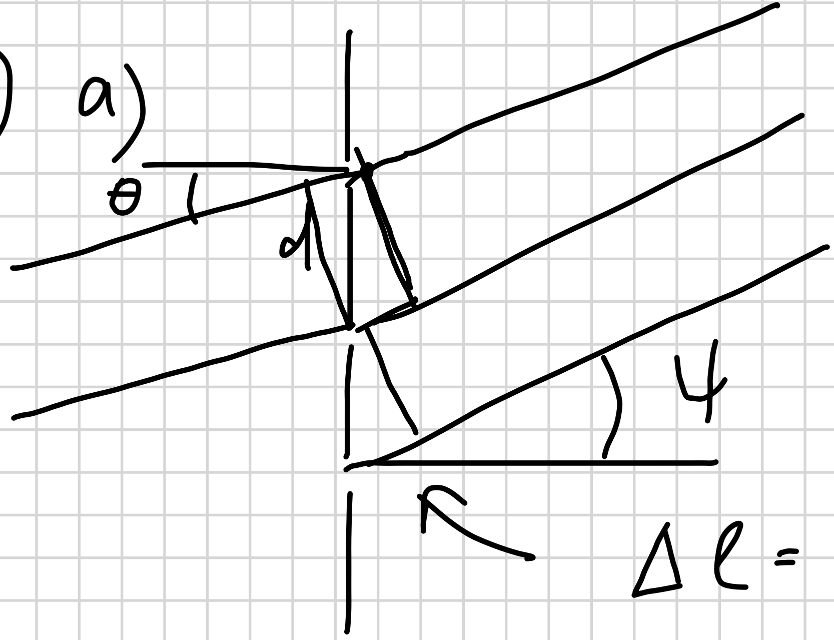
1) $\vec{r} = r \cdot \hat{x} \Rightarrow \vec{E}_{rad} = 0$, $\vec{a}$ is along $\hat{x}$

2) $\vec{r} = r\hat{y} \Rightarrow \vec{E}_{rad} = \frac{-q^2 B_0 c \cos(t - \frac{r}{c})}{4\pi\varepsilon_0 r c^2} \hat{x}$

Linear polarization

3) $\vec{r} = r\hat{z}$ SAME AS IN 2)

③ a)



$$\delta_{after} = \Delta l \cdot k = \frac{2\pi}{\lambda} \cdot d \sin\psi$$

$\Delta l = d \sin\psi$

b) Incoming wave will introduce add. phase delay. We defined θ and ψ such that phases cancel each other. In addition the presence of glass changes phase shift magnitude.

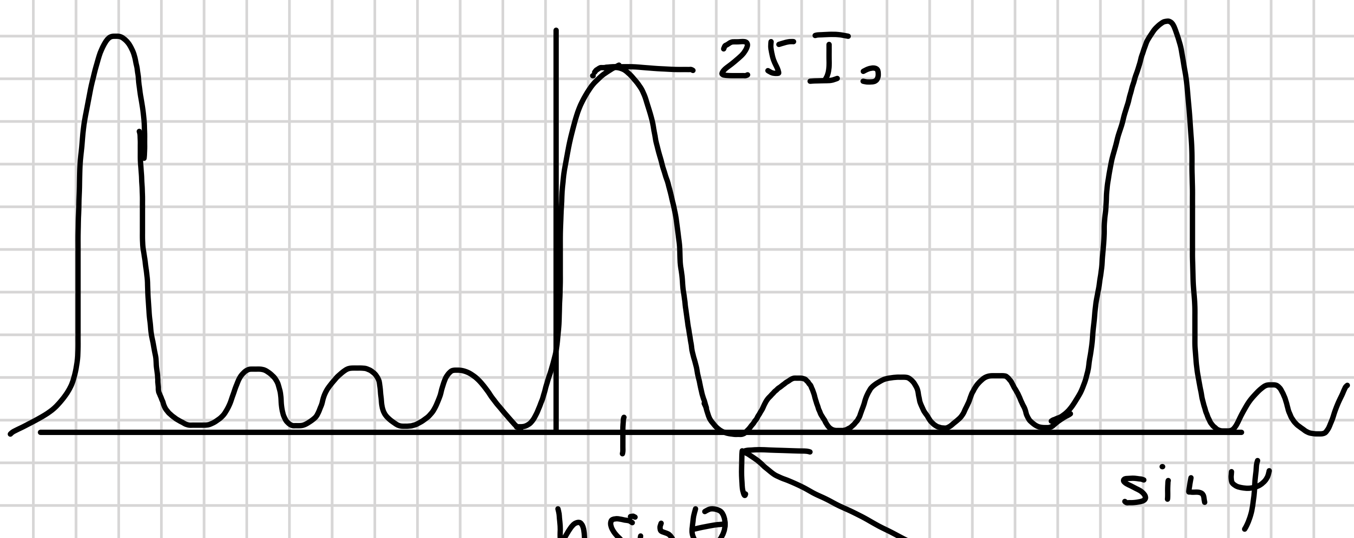
$$\Delta\delta_{before} = -\frac{2\pi d}{\lambda'} \sin\theta = -\frac{2\pi d n}{\lambda_0} \sin\theta \qquad \lambda' = \frac{1}{n}\lambda_0$$

$$\delta_{TOT} = \frac{2\pi d}{\lambda_0}\left(\sin\psi - n\sin\theta\right)$$

c) $$I = 25 I_0 \frac{\sin^2\left(\frac{5\pi d}{\lambda_0}(\sin\psi - n\sin\theta)\right)}{\sin^2\left(\frac{\pi d}{\lambda_0}(\sin\psi - n\sin\theta)\right)}$$

Where I_0 is the intensity from single slit.

d)



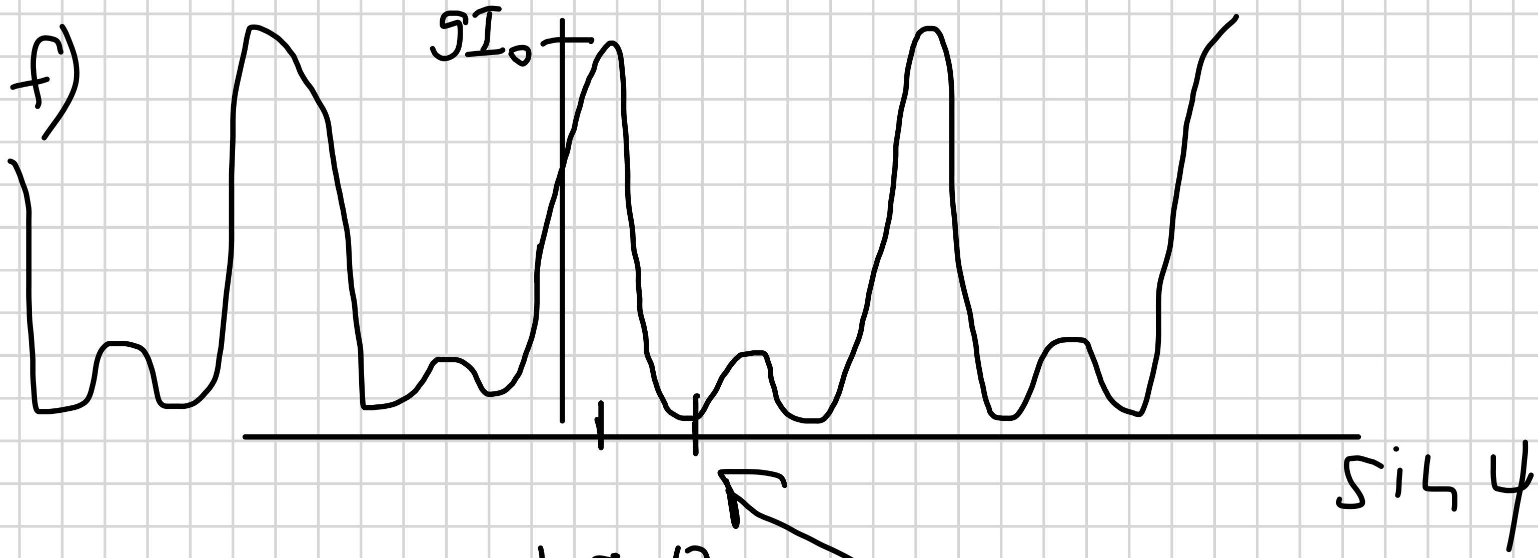
Primary max at $\delta = 0$

1st minimum at $\frac{5\delta}{2} = \pi$ $\qquad \sin\psi = n\sin\theta + \frac{\lambda}{5d}$

e) $N \to 3$

$$I = 9 I_0 \frac{\sin^2\left(\frac{3\pi d}{\lambda}(\sin\varphi - n\sin\theta)\right)}{\sin^2\left(\frac{\pi d}{\lambda}(\sin\varphi - n\sin\theta)\right)}$$

f)



Primary max at $\delta = 0$

1st minimum at $\frac{3\delta}{2} = \pi$ $\quad \sin\varphi = n\sin\theta + \frac{\lambda}{3d}$

Comparison: Max value $\quad 25 I_0 \leftrightarrow 9 I_0$

Primary max pos. $\quad$ same

1st minimum $\quad + \frac{\lambda}{5d} \quad + \frac{\lambda}{3d}$

further out for 3

P.M. wider for 3