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 PROFESSOR: All right. So today's task is going to be to outline some of the basic experimental facts that we will both have to deal with and that our aim should be to understand and model through the rest of the course. Physics doesn't tell you some abstract truth about why the universe is the way it is. Physics gives you models to understand how things work and predict what will happen next. And what we will be aiming to do is develop models that give us an intuition for the phenomena and allow us to make predictions. And these are going to be the experimental facts I would like to both explain, develop an intuition for, and be able to predict consequences of. So we'll start off with-- so let me just outline them. So, first fact, atoms exist. I'll go over some of the arguments for that. Randomness, definitely present in the world. Atomic spectre are discrete and structured. We have a photoelectric effect, which I'll describe in some detail. Electrons do some funny things. In particular electron diffraction. And sixth and finally, Bell's Inequality. Something that we will come back to at the very end of the class, which I like to think of as a sort of a frame for the entirety of 8.04. So... we'll stick with this for the moment. So everyone in here knows that atoms are made of electrons and nuclei. In particular, you know that electrons exist because you've seen a cathode ray tube. I used to be able to say you've seen a TV, but you all have flat panel TVs, so this is useless. So a cathode ray tube is a gun that shoots electrons at a phosphorescent screen. And every time the electron hits the screen it induces a little phosphorescence, a little glow. And that's how you see on a CRT. And so as was pithily stated long ago by a very famous physicist, if you can spray them, they exist. Pretty good argument. There's a better argument for the existence of electrons, which is that we can actually see them individually. And this is one of the most famous images in high-energy physics. It's from an experiment called Gargamelle, which was a 30-cubic meter tank of liquid freon pulsing just at its vapor pressure 60 times a second. And what this image is is, apart from all the schmut, you're watching a trail of bubbles in this de-pressurizing freon that wants to create bubbles but you have to nucleate bubbles. What you're seeing there in that central line that goes up and then curls around is a single electron that was nailed by a neutrino incident from a beam at CERN where currently the LHC is running. And this experiment revealed two things. First, to us it will reveal that you can see individual electrons and by studying the images of them moving through fluids and leaving a disturbing wake of bubbles behind them. We can study their properties in some considerable detail. The second thing it taught us is something new-- we're not going to talk about it in detail-- is that it's possible for a neutrino to hit an electron. And that process is called a weak neutral current for sort of stupid historical reasons. It's actually a really good name. And that was awesome and surprising and so this picture is both a monument to the technology of the experiment, but also to the physics of weak neutral currents and electrons. They exist if you can discover neutrinos by watching them. OK. Secondly, nuclei. We know that nuclei exist because you can shoot alpha particles, which come from radioactive decay, at atoms. And you have your atom which is some sort of vague thing, and I'm gonna make the-- I'm gonna find the atom by making a sheet of atoms. Maybe a foil. A very thin foil of stuff. And then I'm gonna shoot very high-energy alpha particles incident of this. Probably everyone has heard of this experiment, it was done by Rutherford and Geiger and Marsden, in particular his students at the time or post-docs. I don't recall-- and you shoot these alpha particles in. And if you think of these guys as some sort of jelly-ish lump then maybe they'll deflect a little bit, but if you shoot a bullet through Jello it just sort of maybe gets deflected a little bit. But Jello, I mean, come on. And I think what was shocking is that you should these alpha particles in and every once in a while, they bounce back at, you know, 180, 160 degrees. Rutherford likened this to rolling a bowling ball against a piece of paper and having it bounce back. Kind of surprising. And the explanation here that people eventually came up upon is that atoms are mostly zero density. Except they have very, very high density cores, which are many times smaller than the size of the atom but where most of the mass is concentrated. And as a consequence, most of the inertia. And so we know that atoms have substructure, and the picture we have is that well if you scrape this pile of metal, you can pull off the electrons, leaving behind nuclei which have positive charge because you've scraped off the electrons that have negative charge. So we have a picture from these experiments that there are electrons and there are nuclei-- which, I'll just write N and plus-- which are the constituents of atoms. Now this leads to a very natural picture of what an atom is. If you're a 19th-century physicist, or even an early 20th-century physicist, it's very natural to say, aha, well if I know if I have a positive charge and I have a negative charge, then they attract each other with a 1 over r minus q1 q2-- sorry, q1 q2 over r potential. This is just like gravity, right. The earth and the sun are attracted with an inverse-r potential. This leads to Keplerian orbits. And so maybe an atom is just some sort of orbiting classical combination of an electron and a nucleus, positively charged nucleus. The problem with this picture, as you explore in detail in your first problem on the problem set, is that it doesn't work. What happens when you accelerate a charge? It radiates. Exactly. So if it's radiating, it's gotta lose energy. It's dumping energy into this-- out of the system. So it's gotta fall lower into the potential. Well it falls lower, it speeds up. It radiates more. Because it's accelerating more to stay in a circular orbit. All right, it radiates more, it has to fall further down. So on the problem set you're going to calculate how long that takes. And it's not very long. And so the fact that we persist for more than a few picoseconds tells you that it's not that-- this is not a correct picture of an atom. OK. So in classical mechanics, atoms could not exist. And yet, atoms exist. So we have to explain that. That's gonna be our first challenge. Now interestingly Geiger who is this collaborator of Rutherford, a young junior collaborator of Rutherford, went on to develop a really neat instrument. So suppose you want to see radiation. We do this all the time. I'm looking at you and I'm seeing radiation, seeing light. But I'm not seeing ultra high energy radiation, I'm seeing energy radiation in the electromagnetic waves in the optical spectrum. Meanwhile I'm also not seeing alpha particles. So what Geiger wanted was a way to detect without using your eyes radiation that's hard to see. So the way he did this is he took a capacitor and he filled the-- surrounded the capacitor with some noble gas. It doesn't interact. There's no-- it's very hard to ionize. And if you crank up the potential across this capacitor plate high enough, what do you get? A spark. You all know this, if you crank up a capacitor it eventually breaks down because the dielectric in between breaks down, you get a spontaneous sparking. So what do you figure it would look if I take a capacitor plate and I charge it up, but not quite to breakdown. Just a good potential. And another charged particle comes flying through, like an alpha particle, which carries a charge of plus 2, that positive charge will disturb things and will add extra field effectively. And lead to the nucleation of a spark. So the presence of a spark when this potential is not strong enough to induce a spark spontaneously indicates the passage of a charged particle. Geiger worked later with-- Marsden? Muller. Heck. I don't even remember. And developed this into a device now known as the Geiger counter. And so you've probably all seen or heard Geiger counters going off in movies, right. They go ping ping ping ping ping ping ping ping ping, right. They bounce off randomly. This is an extremely important lesson, which is tantamount to the lesson of our second experiment yesterday. The 50-50, when we didn't expect it. The white electrons into the harness box then into a color box again, would come out 50-50, not 100 percent. And they come out in a way that's unpredictable. We have no ability to our knowledge-- and more than our knowledge, we'll come back to that with Bell's Inequality-- but we have no ability to predict which electron will come out of that third box, white or black, right. Similarly with a Geiger counter you hear that atoms decay, but they decay randomly. The radiation comes out of a pile of radioactive material totally at random. We know the probabilistic description of that. We're going to develop that, but we don't know exactly when. And that's a really powerful example-- both of those experiments are powerful examples of randomness. And so we're going to have to incorporate that into our laws of physics into our model of quantum phenomena as well. Questions? I usually have a Geiger counter at this point, which is totally awesome, so I'll try to produce the Geiger counter demo later. But the person with the Geiger counter turns out to have left the continent, so made it a little challenging. OK. Just sort of since we're at MIT, an interesting side note. This strategy of so-called hard scattering, of taking some object and sending it at very high velocity at some other object and looking for the rare events when they bounce off at some large angle, so-called hard scattering. Which is used to detect dense cores of objects. It didn't stop with Rutherford. People didn't just give up at that point. Similar experience in the '60s and '70s which are conducted at Slack, were involved not alpha particles incident on atoms but individual electrons incident on protons. So not shooting into the nucleus, but shooting and looking for the effect of hitting individual protons inside the nucleus. And through this process it was discovered that in fact-- so this was done in the '60s and '70s, that in fact the proton itself is also not a fundamental particle. The proton is itself composite. And in particular, it's made out of-- eventually people understood that it's made out of, morally speaking, and I'm gonna put this in quotation marks-- ask me about it in office hours-- three quarks, which are some particles. And the reason we-- all this tells you is that it's some object and we've given it the name quark. But indeed there are three point-like particles that in some sense make up a proton. It's actually much more complicated than that, but these quarks, among other things, have very strange properties. Like they have fractional charge. And this was discovered by a large group of people, in particular led by Kendall and Friedman and also Richard Taylor. Kendall and Friedman were at MIT, Richard Taylor was at Stanford. And in 1990 they shared the Nobel Prize for the discovery of the partonic structure out of the nucleons. So these sorts of techniques that people have been using for a very long time continue to be useful and awesome. And in particular the experiment, the experimental version of this that's currently going on, that I particularly love is something called the relativistic heavy ion collider, which is going on at Brookhaven. So here what you're doing is you take two protons and you blow them into each other at ultra high energy. Two protons, collide them and see what happens. And that's what happens. You get massive shrapnel coming flying out. So instead of having a simple thing where one of the protons just bounces because there's some hard quark, instead what happens is just shrapnel everywhere, right. So you might think, well, how do we interpret that at all. How do you make sense out of 14,000 particles coming out of two protons bouncing into each other. How does that make any sense? And the answer turns out to be kind of awesome. And so this touches on my research. So I want to make a quick comment on it just for color. The answer turns out to be really interesting. First off, the interior constituents of protons interact very strongly with each other. But at the brief moment when protons collide with each other, what you actually form is not a point-like quirk and another point-like quark. In fact, protons aren't made out of point-like quarks at all. Protons are big bags with quarks and gluons and all sorts of particles fluctuating in and out of existence in a complicated fashion. And what you actually get is, amazingly, a liquid. For a brief, brief moment of time the parts of those protons that overlap-- think of them as two spheres and they overlap in some sort of almond-shaped region. The parts of those protons that overlap form a liquid at ultra high temperature and at ultra high density. It's called the RHIC fireball or the quark-gluon plasma, although it's not actually a plasma. But it's a liquid like water. And what I mean by saying it's a liquid like water, if you push it, it spreads in waves. And like water, it's dissipative. Those waves dissipate. But it's a really funny bit of liquid. Imagine you take your cup of coffee. You drink it, you're drinking your coffee as I am wont to do, and it cools down over time. This is very frustrating. So you pour in a little bit of hot coffee and when you pour in that hot coffee, the system is out of equilibrium. It hasn't thermalized. So what you want is you want to wait for all of the system to wait until it's come to equilibrium so you don't get a swig of hot or swig of cold. You want some sort of Goldilocks-ean in between. So you can ask how long does it take for this coffee to come to thermal equilibrium. Well it takes a while. You know, a few seconds, a few minutes, depending on exactly how you mess with it. But let me ask you a quick question. How does that time scale compare to the time it takes for light to cross your mug? Much, much, much slower, right? By orders of magnitude. For this liquid that's formed in the ultra high energy collision of two protons, the time it takes for the system-- which starts out crazy out of equilibrium with all sorts of quarks here and gluons there and stuff flying about-- the time it takes for it to come to thermal equilibrium is of order the time it takes for light to cross the little puddle of liquid. This is a crazy liquid, it's called a quantum liquid. And it has all sorts of wonderful properties. And the best thing about it to my mind is that it's very well modeled by black holes. Which is totally separate issue, but it's a fun example. So from these sorts of collisions, we know a great deal about the existence of atoms and randomness, as you can see. That's a fairly random sorting. OK so moving on to more 8.04 things. Back to atoms. So let's look at specifics of that. I'm not kidding, they really are related to black holes. I get paid for this. So here's a nice fact, so let's get to atomic spectra. So to study atomic spectra, here's the experiment I want to run. The experiment I want to run starts out with some sort of power plant. And out of the power plant come two wires. And I'm going to run these wires across a spark gap, you know, a piece of metal here, a piece of metal here, and put them inside a container, which has some gas. Like H2 or neon or whatever you want. But some simple gas inside here. So we've got an electric potential established across it. Again, we don't want so much potential that it sparks, but we do want to excite the H2. So we can even make it spark, it doesn't really matter too much. The important thing is that we're going to excite the hydrogen, and in exciting the hydrogen the excited hydrogen is going to send out light. And then I'm going to take this light-- we take the light, and I'm gonna shine this on a prism, something I was taught to do by Newton. And-- metaphorically speaking-- and look at the image of this light having passed through the prism. And what you find is you find a very distinct set of patterns. You do not get a continuous band. In fact what you get-- I'm going to have a hard time drawing this so let me draw down here. I'm now going to draw the intensity of the light incident on the screen on this piece of paper-- people really used to use pieces of paper for this, which is kind of awesome-- as a function of the wavelength, and I'll measure it in angstroms. And what you discover is-- here's around 1,000 angstroms-- you get a bunch of lines. Get these spikes. And they start to spread out, and then there aren't so many. And then at around 3,000, you get another set. And then at around 10,000, you get another set. This is around 10,000. And here's the interesting thing about these. So the discovery of these lines-- these are named after a guy named Lyman, these are-- these are named after a guy named-- Ballmer. Thank you. Steve Ballmer. And these are passion, like passion fruit. So. Everyone needs a mnemonic, OK. And so these people identified these lines and explained various things about them. But here's an interesting fact. If you replace this nuclear power plant with a coal plant, it makes no difference. If you replace this prism by a different prism, it makes no difference to where the lines are. If you change this mechanism of exciting the hydrogen, it makes no difference. As long as it's hydrogen-- as long as it's hydrogen in here you get the same lines, mainly with different intensities depending upon how exactly you do the experiment. But you get the same position of the lines. And that's a really striking thing. Now if you use a different chemical, a different gas in here, like neon, you get a very different set of lines. And a very different effective color now when you eyeball this thing. So Ballmer, incidentally-- and I think this is actually why he got blamed for that particular series, although I don't know the history-- Ballmer noticed by being-- depending on which biography you read-- very clever or very obsessed that these guys, this particular set, could be-- they're wavelengths. If you wrote their wavelengths and labeled them by an integer n, where n ran from 3 to any positive integer above 3, could be written as 36. So this is pure numerology. 36, 46 angstroms times the function n squared over n squared minus 4, where N is equal to 3, 4, dot dot dot-- an integer. And it turns out if you just plug in these integers, you get a pretty good approximation to this series of lines. This is a hallowed tradition, a phenomenological fit to some data. Where did it come from? It came from his creative or obsessed mind. So this was Ballmer. And this is specifically for hydrogen gas, H2. So Rydberg and Ritz, R and R, said, well actually we can do one better. Now that they realized that this is true, they looked at the whole sequence. And they found a really neat little expression, which is that 1 over the wavelength is equal to a single constant parameter. Not just for all these, but for all of them. One single numerical coefficient times 1 over m squared minus 1 over n squared-- n is an integer greater than zero and greater in particular than m. And if you plug in any value of n and any value of m, for sufficiently reasonable-- I mean, if you put in 10 million integers you're not going to see it because it's way out there, but if you put in or-- rather, in here-- if you put any value of n and m, you will get one of these lines. So again, why? You know, as it's said, who ordered that. So this is experimental result three that we're going to have to deal with. When you look at atoms and you look at the specter of light coming off of them, their spectra are discrete. But they're not just stupidly discrete, they're discrete with real structure. Something that begs for an explanation. This is obviously more than numerology, because it explains with one tunable coefficient a tremendous number of spectral lines. And there's a difference-- and crucially, these both work specifically for hydrogen. For different atoms you need a totally different formula. But again, there's always some formula that nails those spectral lines. Why? Questions? OK. So speaking of atomic spectra-- whoops, I went one too far-- here's a different experiment. So people notice the following thing. People notice that if you take a piece of metal and you shine a light at it, by taking the sun or better yet, you know, these days we'd use a laser, but you shine light on this piece of metal. Something that is done all the time in condensed matter labs, it's a very useful technique. We really do use lasers not the sun, but still it continues to be useful in fact to this day. You shine light on a piece of metal and every once in a while what happens is electrons come flying off. And the more light and the stronger the light you shine, you see changes in the way that electrons bounce off. So we'd like to measure that. I'd like to make that precise. And this was done in a really lovely experiment. Here's the experiment. The basic idea of the experiment is I want to check to see, as I change the features of the light, the intensity, the frequency, whatever, I want to see how that changes the properties of the electrons that bounce off. Now one obvious way-- one obvious feature of an electron that flew off a piece of metal is how fast is it going, how much energy does it have. What's its kinetic energy. So I'd like to build an experiment that measures the kinetic energy of an electron that's been excited through this photoelectric effect. Through emission after shining light on a piece of metal. Cool? So I want to build that experiment. So here's how that experiment goes. Well if this electron comes flying off with some kinetic energy and I want to measure that kinetic energy, imagine the following circuit. OK first off imagine I just take a second piece of metal over here, and I'm going to put a little current meter here, an ammeter. And here's what this circuit does. When you shine light on this piece of metal-- we'll put a screen to protect the other piece of metal-- the electrons come flying off, they get over here. And now I've got a bunch of extra electrons over here and I'm missing electrons over here. So this is negative, this is positive. And the electrons will not flow along this wire back here to neutralize the system. The more light I shine, the more electrons will go through this circuit. And as a consequence, there will be a current running through this current meter. That cool with everyone? OK. So we haven't yet measured the kinetic energy, though. How do we measure the kinetic energy? I want to know how much energy, with how much energy, were these electrons ejected. Well I can do that by the following clever trick. I'm going to put now a voltage source here, which I can tune the voltage of, with the voltage V. And what that's going to do is set up a potential difference across these and the energy in that is the charge times the potential difference. So I know that the potential difference it takes, so the amount of energy it takes to overcome this potential difference, is q times V. That cool? So now imagine I send in an electron-- I send in light and it leads an electron to jump across, and it has kinetic energy, kE. Well if the kinetic energy is less than this, will it get across? Not so much. It'll just fall back. But if the kinetic energy is greater than the energy it takes to cross, it'll cross and induce a current. So the upshot is that, as a function of the voltage, what I should see is that there is some critical minimum voltage. And depending on how you set up the sign, the sign could be the other way, but there's some critical minimal voltage where, for less voltage, the electron doesn't get across. And for any greater voltage-- or, sorry, for any closer to zero voltage, the electron has enough kinetic energy to get across. And so the current should increase. So there's a critical voltage, V-critical, where the current running through the system runs to zero. You make it harder for the electrons by making the voltage in magnitude even larger. You make it harder for the electrons to get across. None will get across. Make it a little easier, more and more will get across. And the current will go up. So what you want to do to measure this kinetic energy is you want to measure the critical voltage at which the current goes to zero. So now the question is what do we expect to see. And remember that things we can tune in this experiment are the intensity of the light, which is like e squared plus b squared. And we can tune the frequency of the light. We can vary that. Now does the total energy, does that frequency show up in the total energy of a classical electromagnetic wave? No. If it's an electromagnetic wave, it cancels out. You just get the total intensity, which is a square of the fields. So this is just like a harmonic oscillator. The energy is in the amplitude. The frequency of the oscillator doesn't matter. You push the swing harder, it gets more kinetic energy. It's got more energy. OK. So what do we expect to see as we vary, for example, the intensity? So here's a natural gas. If you take-- so you can think about the light here as getting a person literally, like get the person next to you to take a bat and hit a piece of metal. If they hit it really lightly they're probably not going to excite electrons with a lot of energy. If they just whack the heck out of it, then it wouldn't be too surprising if you get much more energy in the particles that come flying off. Hit it hard enough, things are just gonna shrapnel and disintegrate. The expectation here is the following. That if you have a more intense beam, then you should get more-- the electrons coming off should be more energetic. Because you're hitting them harder. And remember that the potential, which I will call V0, the stopping voltage. So therefore V0 should be greater in magnitude. So this anticipates that the way this curve should look as we vary the current as a function of v, if we have a low voltage-- sorry, if we have a low-intensity beam-- it shouldn't take too much potential just to impede the motion. But if we have a-- so this is a low intensity. But if we have a high-intensity beam, it should take a really large voltage to impede the electric flow, the electric current, because high-intensity beam you're just whacking those electrons really hard and they're coming off with a lot of kinetic energy. So this is high intensity. Everyone down with that intuition? This is what you get from Maxwell's electrodynamics. This is what you'd expect. And in particular, as we vary-- so this is our predictions-- in particular as we vary-- so this is 1, 2, with greater intensity. And the second prediction is that V-naught should be independent of frequency. Because the energy density and electromagnetic wave is independent of the frequency. It just depends on the amplitude. And I will use nu to denote the frequency. So those are the predictions that come from 8.02 and 8.03. But this is 8.04. And here's what the experimental results actually look like. So here's the intensity, here's the potential. And if we look at high potential, it turns out that-- if we look, sorry, if we look at intermediate potentials, it's true that the high intensity leads to a larger current and the low intensity leads to a lower current. But here's the funny thing that happens. As you go down to the critical voltage, their critical voltages are the same. What that tells you is that the kinetic energy kicked out-- or the kinetic energy of an electron kicked out of this piece of metal by the light is independent of how intense that beam is. No matter how intense that beam is, no matter how strong the light you shine on the material, the electrons all come out with the same energy. This would be like taking a baseball and hitting it with a really powerful swing or a really weak swing and seeing that the electron dribbles away with the same amount of energy. This is very counter-intuitive. But more surprisingly, V-naught is actually independent of intensity. But here's the real shocker. V-naught varies linearly in the frequency. What does change V-naught is changing the frequency of the light in this incident. That means that if you take an incredibly diffuse light-- incredibly diffuse light, you can barely see it-- of a very high frequency, then it takes a lot of energy to impede the electrons that come popping off. The electrons that come popping off have a large energy. But if you take a low-frequency light with extremely high intensity, then those electrons are really easy to stop. Powerful beam but low frequency, it's easy to stop those electrons. Weak little tiny beam at high frequency, very hard to stop the electrons that do come off. So this is very counter-intuitive and it doesn't fit at all with the Maxwellian picture. Questions about that? So this led Einstein to make a prediction. This was his 1905 result. One of his many totally breathtaking papers of that year. And he didn't really propose a model or a detailed theoretical understanding of this, but he proposed a very simple idea. And he said, look, if you want to fit this-- if you want to fit this experiment with some simple equations, here's the way to explain it. I claim-- I here means Einstein, not me-- I claim that light comes in packets or chunks with definite energy. And the energy is linearly proportional to the frequency. And our energy is equal to something times nu, and we'll call the coefficient h. The intensity of light, or the amplitude squared, the intensity is like the number of packets. So if you have a more intense beam at the same frequency, the energy of each individual chunk of light is the same. There are just a lot more chunks flying around. And so to explain the photoelectric effect, Einstein observed the following. Look, he said, the electrons are stuck under the metal. And it takes some work to pull them off. So now what's the kinetic energy of an electron that comes flying off-- whoops, k3. Bart might have a laugh about that one. Kinetic, kE, not 3. So the kinetic energy of electron that comes flying off, well, it's the energy deposited by the photon, the chunk of light, h-nu well we have to subtract off the work it took. Minus the work to extract the electron from the material. And you can think of this as how much energy does it take to suck it off the surface. And the consequence of this is that the kinetic energy of an electron should be-- look, if h-nu is too small, if the frequency is too low, then the kinetic energy would be negative. But that doesn't make any sense. You can't have negative kinetic energy. It's a strictly positive quantity. So it just doesn't work until you have a critical value where the frequency times h-- this coefficient-- is equal to the work it takes to extract. And after that, the kinetic energy rises with the frequency with a slope equal to h. And that fits the data like a champ. So no matter-- let's think about what this is saying again. No matter what you do, if your light is very low-frequency and you pick some definite piece of metal that has a very definite work function, very definite amount of energy it takes to extract electrons from the surface. No matter how intense your beam, if the frequency is insufficiently high, no electrons come off. None. So it turns out none is maybe a little overstatement because what you can have is two photon processes, where two chunks hit one electron at the right, just at the same time. Roughly speaking the same time. And they have twice the energy, but you can imagine that the probability of two photon hitting one electron at the same time of pretty low. So the intensity has to be preposterously high. And you see those sorts of multi-photon effects. But as long as we're not talking about insanely high intensities, this is an absolutely fantastic probe of the physics. Now there's a whole long subsequent story in the development of quantum mechanics about this particular effect. And it turns out that the photoelectric effect is a little more complicated than this. But the story line is a very useful one for organizing your understanding of the photoelectric effect. And in particular, this relation that Einstein proposed out of the blue, with no other basis. No one else had ever seen this sort of statement that the electrons, or that the energy of a beam of light should be made up of some number of chunks, each of which has a definite minimum amount of energy. So you can take what you've learned from 8.02 and 8.03 and extract a little bit more information out of this. So here's something you learned from 8.02. In 8.02 you learned that the energy of an electromagnetic wave is equal to c times the momentum carried by that wave-- whoops, over two. And in 8.03 you should have learned that the wavelength of an electromagnetic wave times the frequency is equal to the speed of light, C. And we just had Einstein tell us-- or declare, without further evidence, just saying, look this fits-- that the energy of a chunk of light should be h times the frequency. So if you combine these together, you get another nice relation that's similar to this one, which says that the momentum of a chunk of light is equal to h over lambda. So these are two enormously influential expressions which come out of this argument from the photoelectric effect from Einstein. And they're going to be-- their legacy will be with us throughout the rest of the semester. Now this coefficient has a name, and it was named after Planck. It's called Planck's Constant. And the reason that it's called Planck's Constant has nothing to do with the photoelectric effect. It was first this idea that an electromagnetic wave, that light, has an energy which is linearly proportional not to its intensity squared, none of that, but just linearly proportional to the frequency. First came up an analysis of black body radiation by Planck. And you'll understand, you'll go through this in some detail in 8.044 later in the semester. So I'm not going to dwell on it now, but I do want to give you a little bit of perspective on it. So Planck ran across this idea that E is equal to h/nu. Through the process of trying to fit an experimental curve. There was a theory of how much energy should be emitted by an object that's hot and glowing as a function of frequency. And that theory turned out to be in total disagreement with experiment. Spectacular disagreement. The curve for the theory went up, the curve for the experiment went down. They were totally different. So Planck set about writing down a function that described the data. Literally curve-fitting, that's all he was doing. And this is the depths of desperation to which he was led, was curve-fitting. He's an adult. He shouldn't be doing this, but he was curve-fitting. And so he fits the curve, and in order to get it to fit the only thing that he can get to work even vaguely well is if he puts in this calculation of h/nu. He says, well, maybe when I sum over all the possible energies I should restrict the energies which were proportional to the frequency. And it was forced on him because it fit from the function. Just functional analysis. Hated it. Hated it, he completely hated it. He was really frustrated by this. It fit perfectly, he became very famous. He was already famous, but he became ridiculously famous. Just totally loathed this idea. OK. So it's now become a cornerstone of quantum mechanics. But he wasn't so happy about it. And to give you a sense for how bold and punchy this paper by Einstein was that said, look, seriously. Seriously guys. e equals h/nu. Here's what Planck had to say when he wrote a letter of recommendation to get Einstein into the Prussian Academy of Sciences in 1917, or 1913. So he said, there is hardly one among the great problems in physics to which Einstein has not made an important contribution. That he may sometimes have missed the target in his speculations as in his hypothesis of photons cannot really be held too much against him. It's not possible to introduce new ideas without occasionally taking a risk. Einstein who subsequently went on to develop special relativity and general relativity and prove the existence of atoms and the best measurement of Avogadro's Constant, subsequently got the Nobel Prize. Not for Avogadro's Constant, not for proving the existence of atoms, not for relativity, but for photons. Because of guys like Planck, right. This is crazy. So this was a pretty bold idea. And here, to get a sense for why-- we're gonna leave that up because it's just sort of fun to see these guys scowling and smiling-- there is, incidentally there's a great book about Einstein's years in Berlin by Tom Levenson, who's a professor here. A great writer and a sort of historian of science. You should take a class from him, which is really great. But I encourage you to read this book. It talks about why Planck is not looking so pleased right there, among many other things. It's a great story. So let's step back for a second. Why was Planck so upset by this, and why was in fact everyone so flustered by this idea that it led to the best prize you can give a physicist. Apart from a happy home and, you know. I've got that one. That's the one that matters to me. So why is this so surprising? And the answer is really simple. We know that it's false. We know empirically, we've known for two hundred and some years that light is a wave. Empirically. This isn't like people are like, oh I think it'd be nice if it was a wave. It's a wave. So how do we know that? So this goes back to the double-slit experiment from Young. Young's performance of this was in 1803. Intimations of it come much earlier. But this is really where it hits nails to the wall. And here's the experiment. So how many people in here have not seen a double-slit experiment described? Yeah, exactly. OK. So I'm just going to quickly remind you of how this goes. So we have a source for waves. We let the waves get big until they're basically plane waves. And then we take a barrier. And we poke two slits in it. And these plane waves induce-- they act like sources at the slits and we get nu. And you get crests and troughs. And you look at some distant screen and you look at the pattern, and the pattern you get has a maximum. But then it falls off, and it has these wiggles, these interference fringes. These interference fringes are, of course, extremely important. And what's going on here is that the waves sometimes add in-- so the amplitude of the wave, the height of the wave, sometimes adds constructively and sometimes destructively. So that sometimes you get twice the height and sometimes you get nothing. So just because it's fun to see this, here's Young's actual diagram from his original note on the double-slit experiment. So a and b are the slits, and c, d and f are the [INAUDIBLE] on the screen, the distant screen. He drew it by hand. It's pretty good. So we've known for a very long time that light, because of the double-slit experiment, light is clearly wavy, it behaves like a wave. And what are the senses in which it behaves like a wave? There are two important senses here. The first is answered by the question, where did the wave hit the screen? So when we send in a wave, you know, I drop a stone, one big pulsive wave comes out. It splits into-- it leads to new waves being instigated here and over here. Where did that wave hit the screen? Anyone? AUDIENCE: Everywhere. PROFESSOR: Yeah, exactly. It didn't hit this wave-- the screen in any one spot. But some amplitude shows up everywhere. The wave is a distributed object, it does not exist at one spot, and it's by virtue of the fact that it is not a localized object-- it is not a point-like object-- that it can interfere with itself. The wave is a big large phenomena in a liquid, in some thing. So it's sort of essential that it's not a localized object. So not localized. The answer is not localized. And let's contrast this with what happens if you take this double-slit experiment and you do it with, you know, I don't know, take-- who. Hmm. Tim Wakefield. Let's give some love to that guy. So, baseball player. And have him throw baseballs at a screen with two slits in it. OK? Now he's got pretty good-- well, he's got terrible accuracy, actually. So every once in a while he'll make it through the slits. So let's imagine first blocking off-- what, he's a knuckle-baller, right-- so every once in a while it goes, the baseball will go through the slit. And let's think about what happens, so let's cover one slit. And what we expect to happen is, well, it'll go through more or less straight, but sometimes it'll scrape the edge, it'll go off to the side, and sometimes it'll come over here. But if you take a whole bunch of baseballs, and-- so any one baseball, where does it hit? Some spot. Right? One spot. Not distributed. One spot. And as a consequence, you know, one goes here, one goes there, one goes there. And now, there's nothing like interference effects, but what happens is as it sort of doesn't-- you get some distribution if you look at where they all hit. Yeah? Everyone cool with that? And if we had covered over this slot, or slit, and let the baseballs go through this one, same thing would have happened. Now if we leave them both open, what happens is sometimes it goes here, sometimes it goes here. So now it's pretty useful that we've got a knuckle-baller. And what you actually get is the total distribution looks like this. It's the sum of the two. But at any given time, any one baseball, you say, aha, the baseball either went through the top slit, and more or less goes up here. Or it went through the bottom slit and more or less goes down here. So for chunks-- so this is for waves-- for chunks or localized particles, they are localized. And as a consequence, we get no interference. So for waves, they are not localized, and we do get interference. Yes, interference. OK. So on your problem set, you're going to deal with some calculations involving these interference effects. And I'm going to brush over them. Anyway the point of the double-slit experiment is that whatever else you want to say about baseballs or anything else, light, as we've learned since 1803 in Young's double-slit experiment, light behaves like a wave. It is not localized, it hits the screen over its entire extent. And as a consequence, we get interference. The amplitudes add. The intensity is the square of the amplitude. If the intensities add-- so sorry, if the amplitudes add-- amplitude total is equal to a1 plus a2, the intensity, which is the square of a1 plus a2 squared, has interference terms, the cross terms, from this square. So light, from this point of view, is an electromagnetic wave. It interferes with itself. It's made of chunks. And I can't help but think about it this way, this is literally the metaphor I use in my head-- light is creamy and smooth like a wave. Chunks are very different. But here's the funny thing. Light is both smooth like a wave, it is also chunky. It is super chunky, as we have learned from the photoelectric effect. So light is both at once. So it's the best of both worlds. Everyone will be satisfied, unless you're not from the US, in which case this is deeply disturbing. So of course the original Superchunk is a band. So we've learned now from Young that light is a wave. We've learned from the photoelectric effect that light is a bunch of chunks. OK. Most experimental results are true. So how does that work? Well, we're gonna have to deal with that. But enough about light. If this is true of light, if light, depending on what experiment you do and how you do the experiment, sometimes it seems like it's a wave, sometimes it seems like it's a chunk or particle, which is true? Which is the better description? So it's actually worthwhile to not think about light all the time. Let's think about something more general. Let's stick to electrons. So as we saw from yesterday's lecture, you probably want to be a little bit wary when thinking about individual electrons. Things could be a little bit different than your classical intuition. But here's a crucial thing. Before doing anything else, we can just think, which one of these two is more likely to describe electrons well. Well electrons are localized. When you throw an electron at a CRT, it does not hit the whole CRT with a wavy distribution. When you take a single electron and you throw it at a CRT, it goes ping and there's a little glowing spot. Electrons are localized. And we know that localized things don't lead to interference. Some guys at Hitachi, really good scientists and engineers, developed some really awesome technology a couple of decades ago. They were trying to figure out a good way to demonstrate their technology. And they decided that you know what would be really awesome, this thought experiment that people have always talked about that's never been done really well, of sending an electron through a two-slitted experiment. In this case it was like ten slits effectively, it was a grading. Send an electron, a bunch of electrons, one at a time, throw the electron, wait. Throw the electron, wait. Like our French guy with the boat. So do this experiment with our technology and let's see what happens. And this really is one of my favorite-- let's see, how we close these screens-- aha. OK. This is going to take a little bit of-- and it's broken. No, no. Oh that's so sad. AUDIENCE: [LAUGHTER] PROFESSOR: Come on. I'm just gonna let-- let's see if we can, we'll get part of the way. I don't want to destroy it. So what they actually did is they said, look, let's-- we want to see what happens. We want to actually do this experiment because we're so awesome at Hitachi Research Labs, so let's do it. So here's what they did. And I'm going to turn off the light. And I set this to some music because I like it. OK here's what's happening. One at a time, individual photons. [MUSIC PLAYING] PROFESSOR: So they look pretty localized. There's not a whole lot of structure. Now they're going to start speeding it up. It's 100 times the actual speed. [MUSIC PLAYING] PROFESSOR: Eh? Yeah. AUDIENCE: [APPLAUSE] PROFESSOR: So those guys know what they're doing. Let's-- there were go. So I think I don't know of a more vivid example of electron interference than that one. It's totally obvious. You see individual electrons. They run through the apparatus. You wait, they run through the apparatus. You wait. One at a time, single electron, like a baseball being pitched through two slits, and what you see is an interference effect. But you don't see the interference effect like you do from light, from waves on the sea. You see the interference effect by looking at the cumulative stacking up of all the electrons as they hit. Look at where all the electrons hit one at a time. So is an electron behaving like a wave in a pond? No. Does a wave in a pond at a spot? No. It's a distributed beast. OK yes, it interferes, but it's not localized. Well is it behaving like a baseball? Well it's localized. But on-- when I look at a whole bunch of electrons, they do that. They seem to interfere, but there's only one electron going through at a time. So in some sense it's interfering with itself. How does that work? Is an electron a wave? AUDIENCE: Yes. PROFESSOR: Does an electron hit at many spots at once? AUDIENCE: No. PROFESSOR: No. So is an electron a wave. No. Is an electron a baseball? No. It's an electron. So this is something you're going to have to deal with, that every once in awhile we have these wonderful moments where it's useful to think about an electron as behaving in a wave-like sense. Sometimes it's useful to think about it as behaving in a particle-like sense. But it is not a particle like you normally conceive of a baseball. And it is not a wave like you normally conceive of a wave on the surface of a pond. It's an electron. I like to think about this like an elephant. If you're closing your eyes and you walk up to an elephant, you might think like I've got a snake and I've got a tree trunk and, you know, there's a fan over here. And you wouldn't know, like, maybe it's a wave, maybe it's a particle, I can't really tell. But if you could just see the thing the way it is, not through the preconceived sort of notions you have, you'd see it's an elephant. Yes, that is the Stata Center. So-- look, everything has to happen sometime, right? AUDIENCE: [LAUGHTER] PROFESSOR: So Heisenberg-- it's often, people often give the false impression in popular books on physics, so I want to subvert this, that in the early days of quantum mechanics, the early people like Born and Oppenheimer and Heisenberg who invented quantum mechanics, they were really tortured about, you know, is it an electron, is it a wave. It's a wave-particle duality. It's both. And this is one of the best subversions of that sort of silliness that I know of. And so what Heisenberg says, the two mental pictures which experiments lead us to form, the one of particles the other waves, are both incomplete and have the validity of analogies, which are accurate only in limited cases. The apparent duality rises in the limitation of our language. And then he goes on to say, look, you developed your intuition by throwing rocks and, you know, swimming. And, duh, that's not going to be very good for atoms. So this will be posted, it's really wonderful. His whole lecture is really-- the lectures are really quite lovely. And by the way, that's him in the middle there, Pauley all the way on the right. I guess they were pleased. OK so that's the Hitachi thing. So now let's pick up on this, though. Let's pick up on this and think about what happens. I want to think in a little more detail about this Hitachi experiment. And I want to think about it in the context of a simple two-slit experiment. So here's our source of electrons. It's literally a gun, an electron gun. And it's firing off electrons. And here's our barrier, and it has two slits in it. And we know that any individual electron hits its own spot. But when we take many of them, we get an interference effect. We get interference fringes. And so the number that hit a given spot fill up, construct this distribution. So then here's the question I want to ask. When I take a single electron, I shoot one electron at a time through this experiment, one electron. It could go through the top slit, it could go through the bottom slit. While it's inside the apparatus, which path does it take? AUDIENCE: Superposition. PROFESSOR: Good. So did it take the top path? AUDIENCE: No. PROFESSOR: How do you know? [INTERPOSING VOICES] PROFESSOR: Good, let's block the bottom, OK, to force it to go through the top slit. So we'll block the bottom slit. Now the only electrons that make it through go through the top slit. Half of them don't make it through. But those that do make it through give you this distribution. No interference. But I didn't tell you these are hundreds of thousands of kilometers apart, the person who threw in the electron didn't know whether there was a barrier here. The electron, how could it possibly know whether there was a barrier here if you went through the top. This is exactly like our boxes. It's exactly like our box. Did it go through-- an electron, when the slits are both open and we know that ensemble average it will give us an interference effect, did the electron inside the apparatus go through the top path? No. Did it go through the bottom path? Did it go through both? Because we only see one electron. Did it go through neither? It is in a-- AUDIENCE: Superposition. PROFESSOR: --of having gone through the top and the bottom. Of being along the top half and being along the bottom path. This is a classic example of the two-box experiment. OK. So you want to tie that together. So let's nuance this just a little bit, though, because it's going to have an interesting implication for gravity. So here's the nuance I want to pull on this one. Let's cheat. OK. Suppose I want to measure which slit the electron actually did go through. How might I do that? Well I could do the course thing I've been doing which is I could block it and just catch the-- catch electrons that go through in that spot. But that's a little heavy-handed. Probably I can do something a little more delicate. And so here's the more delicate thing I'm going to do. I want to build a detector that uses very, very, very weak light, extremely weak light, to detect whether the particle went through here or here. And the way I can do that is I can sort of shine light through and-- I'm gonna, you know, bounce-- so here's my source of light. And I'll be able to tell whether the electron went through this slit or it went through this slit. Cool? So imagine I did that. So obviously I don't want to use some giant, huge, ultra high-energy laser because it would just blast the thing out of the way. It would destroy the experiment. So I wanna something very diffuse, very low energy, very low intensity electromagnetic wave. And the idea here is that, OK, it's true that when I bounce this light off an electron, let's say it bounces off an electron here, it's true it's going impart some momentum and the electron's gonna change its course. But if it's really, really weak, low energy light, then it's-- it's gonna deflect only a little tiny bit. So it will change the pattern I get over here. But it will change it in some relatively minor way because I've just thrown in very, very low energy light. Yeah? That make sense? So this is the experiment I want to do. This experiment doesn't work. Why. AUDIENCE: You know which slit it went through. PROFESSOR: No. It's true that it turns out that those are correlated facts, but here's the problem. I can run this experiment without anyone actually knowing what happens until long afterwards. So knowing doesn't seem to play any role in it. It's very tempting often to say, no, but it turns out that it's really not about what you know. It's really just about the experiment you're doing. So what principle that we've already run into today makes it impossible to make this work? If I want to shine really low-energy, really diffuse light through, and have it scatter weakly. Yeah. AUDIENCE: Um, light is chunky. PROFESSOR: Yeah, exactly. That's exactly right. So when I say really low-energy light, I don't-- I really can't mean, because we've already done this experiment, I cannot possibly mean low intensity. Because intensity doesn't control the energy imparted by the light. The thing that controls the energy imparted by a collision of the light with the electron is the frequency. The energy in a chunk of light is proportional to the frequency. So now if I want to make the effect the energy or the momentum, similarly-- the momentum, where did it go-- remember the momentum goes like h over lambda. If I want to make the energy really low, I need to make the frequency really low. Or if I want to make the momentum really low, I need to make the wavelength what? Really big. Right? So in order to make the momentum imparted by this photon really low, I need to make the wavelength really long. But now here's the problem. If I make the wavelength really long, so if I use a really long-wavelengthed wave, like this long of a wavelength, are you ever going to be able to tell which slit it went through? No, because the particle could have been anywhere. It could have scattered this light if it was here, if it was here, if it was here, right? In order to measure where the electron is to some reasonable precision-- so, for example, to this sort of wavelength, I need to be able to send in light with a wavelength that's comparable to the scale that I want to measure. And it turns out that if you run through and just do the calculation, suppose I send in-- and this is done in the books, in I think all four, but this is done in the books on the reading list-- if you send in a wave with a short enough wavelength to be able to distinguish between these two slits, which slit did it go through, the momentum that it imparts precisely watches-- washes out is just enough to wash out the interference effect, and break up these fringes so you don't see interference effects. It's not about what you know. It's about the particulate nature of light and the fact that the momentum of a chunk of light goes like h over lambda. OK? But this tells you something really interesting. Did I have to use light to do this measurement? I could have sent in anything, right? I didn't have to bounce light off these things. I could have bounced off gravitational waves. So if I had a gravitational wave detector, so-- Matt works on gravitational wave detectors, and so, I didn't tell you this but Matt gave me a pretty killer gravitational wave detector. It's, you know, here it is. There's my awesome gravitational wave detector. And I'm now going to build supernova. OK. And they are creeping under this black hole, and it's going to create giant gravitational waves. And we're gonna use those gravitational waves and detect them with the super advanced LIGO. And I'm gonna detect which slit it went through. But gravitational waves, those aren't photons. So I really can make a low-intensity gravitational wave, and then I can tell which slit it went through without destroying the interference effect. That would be awesome. What does that tell you about gravitational waves? They must come in chunks. In order for this all to fit together logically, you need all the interactions that you could scatter off this to satisfy these quantization properties. But the energy is proportional to the frequency. The line I just gave you is a heuristic. And making it precise is one of the great challenges of modern contemporary high-energy physics, of dealing with the quantum mechanics and gravity together. But this gives you a strong picture of why we need to treat all forces in all interactions quantum-mechanically in order for the world to be consistent. OK. Good. OK, questions at this point? OK. So-- oh, I forgot about this one-- so there are actually two more. So I want to just quickly show you-- well, OK. So, this is a gorgeous experiment. So remember I told you the story of the guy with the boat and the opaque wall and it turns out that's a cheat. It turns out that this opaque screen doesn't actually give you quantum mechanically isolated photons. They're still, in a very important way, classical. So this experiment was done truly with a source that gives you quantum mechanically isolated single photons, one at a time. So this is the analogue of the Hitachi experiment. And it was done by this pretty awesome Japanese group some number of years ago. And I just want to emphasize that it gives you exactly the same effects. We see that photons-- this should look essentially identical to what we saw at the end of the Hitachi video. And that's because it's exactly the same physics. It's a grating with something like 10 slits and individual particles going through one at a time and hitting the screen and going, bing. So what you see is the light going, bing, on a CCD. It's a pretty spectacular experience. So let's get back to electrons. I want another probe of whether electrons are really waves or not. So this other experiment-- again, you're going to study this on your problem set-- this other experiment was done by a couple of characters named Davisson and Germer. And in this experiment, what they did is they took a crystal, and a crystal is just a lattice of regularly-located ions, like diamond or something. Yeah? AUDIENCE: Before you go on I guess, I wanted to ask if the probability of a photon or an electron going through the 10 slits is about the same? PROFESSOR: Is what, sorry? AUDIENCE: Is exactly the same. PROFESSOR: You mean for different electrons? AUDIENCE: Yeah. PROFESSOR: Well they can be different if the initial conditions are different. But they could be-- if the initial conditions are the same, then the probabilities are identical. So every electron behaves identically to every other electron in that sense. Is that what you were asking? AUDIENCE: It is actually like through any [INAUDIBLE] the probability of it going like [INAUDIBLE]? PROFESSOR: Sure, absolutely. So the issue there is just a technological one of trying to build a beam that's perfectly columnated. And that's just not doable. So there's always some dispersion in your beam. So in practice it's very hard to make them identical, but in principle they could be if you were infinitely powerful as an experimentalist, which-- again, I was banned from the lab, so not me. So here's our crystal. You could think of this as diamond or nickel or whatever. I think they actually use nickel but I don't remember exactly. And they sent in a beam of electrons. So they send in a beam of electrons. and what they discover is that if you send in these electrons and watch how they scatter at various different angles-- I'm going to call the angle here of scattering theta-- what they discover is that the intensity of the reflected beam, as a function of theta, shows interference effects. And in particular they gave a whole calculation for this, which I'm not going to go through right now because it's not terribly germane for us-- you're going to go through it on your problem set, so that'll be good and it's a perfect thing for your recitation instructors to go through. But the important thing is the upshot. So if the distance between these crystal planes is L-- or, sorry, d-- let me call it d. If the distance between the crystal planes is d, what they discover is that the interference effects that they observed, these maxima and minima, are consistent with the wavelength of light. Or, sorry, with the electrons behaving as if they were waves with a definite wavelength, with a wavelength lambda being equal to some integer, n, over 2d sine theta. So this is the data-- these are the data they actually saw, data are plural. And these are the data they actually saw. And they infer from this that the electrons are behaving as if they were wave-like with this wavelength. And what they actually see are individual electrons hitting one by one. Although in their experiment, they couldn't resolve individual electrons. But that is what they see. And so in particular, plugging all of this back into the experiment, you send in the electrons with some energy, which corresponds to some definite momentum. This leads us back to the same expression as before, that the momentum is equal to h over lambda, with this lambda associated. So it turns out that this is correct. So the electrons diffract off the crystal as if they have a momentum which comes with a definite wavelength corresponding to its momentum. So that's experimental result-- oh, I forgot to check off four-- that's experimental result five, that electrons diffract. We already saw the electron diffraction. So something to emphasize is that-- so these experiments as we've described them were done with photons and with electrons, but you can imagine doing the experiments with soccer balls. This is of course hard. Quantum effects for macroscopic objects are usually insignificantly small. However, this experiment was done with Buckyballs, which are the same shape as soccer balls in some sense. But they're huge, they're gigantic objects. So here's the experiment in which this was actually done. So these guys are just totally amazing. So this is Zellinger's lab. And it doesn't look like all-- I mean it looks kind of, you know. It's hideous, right? I mean to a theorist it's like, come on, you've got to be kidding that that's-- But here's what a theorist is happy about. You know, because it looks simple. We really love lying to ourselves about that. So here's an over. We're going to cook up some Buckyballs and emit them with some definite known thermal energy. Known to some accuracy. We're going to columnate them by sending them through a single slit, and then we're going to send them through a diffraction grating which, again, is just a whole bunch of slits. And then we're going to image them using photo ionization and see where they pop through. So here is the horizontal position of this wave along the grating, and this is the number that come through. This is literally one by one counts because they're going bing, bing, bing, as a c60 molecule goes through. So without the grating, you just get a peek. But with the grating, you get the side bands. You get interference fringes. So these guys, again, they're going through one by one. A single Buckyball, 60 carbons, going through one by one is interfering with itself. This is a gigantic object by any sort of comparison to single electrons. And we're seeing these interference fringes. So this is a pretty tour de force experiment, but I just want to emphasize that if you could do this with your neighbor, it would work. You'd just have to isolate the system well enough. And that's a technological challenge but not an in-principle one. OK. So we have one last experimental facts to deal with. And this is Bell's Inequality, and this is my favorite one. So Bell's Inequality for many years languished in obscurity until someone realized that it could actually be done beautifully in an experiment that led to a very concrete experiment that they could actually do and that they wanted to do. And we now think of it as an enormously influential idea which nails the coffin closed for classical mechanics. And it starts with a very simple question. I claim that the following inequality is true: the number of undergraduate-- of the number of people in the room who are undergraduates, which I'll denote as U-- and not blonde, which I will denote as bar B-- so undergraduates who are not blonde-- actually let me write this out in English. It's gonna be easier. Number who are undergrads and not blonde plus the number of people in the room who are blonde but not from Massachusetts is strictly greater than or equal to the number of people in the room who are undergraduates and not from Massachusetts. I claim that this is true. I haven't checked in this room. But I claim that this is true. So let's check. How many people are undergraduates who are not blonde? OK this is going to-- jeez. OK that's-- so, lots. OK. How many people are blonde but not from Massachusetts? OK. A smattering. Oh God, this is actually going to be terrible. AUDIENCE: [LAUGHTER] PROFESSOR: Shoot. This is a really large class. OK. Small. And how many people are undergraduates who are not from Massachusetts? Yeah, this-- oh God. This counting is going to be-- so let's-- I'm going to do this just so I can do the counting with the first two rows here. OK. My life is going to be easier this way. So how many people in the first two rows, in the center section, are undergraduates but not blonde? One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen. We could dispute some of those, but we'll take it for the moment. So, fourteen. You're probably all undergraduates. So blonde and not from Massachusetts. One. Awesome. Undergraduates not from Massachusetts. One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen, fifteen. Equality. AUDIENCE: [LAUGHTER] PROFESSOR: OK. So that-- you might say well, look, you should have been nervous there. You know, and admittedly sometimes there's experimental error. But I want to convince you that I should never, ever ever be nervous about this moment in 8.04. And the reason is the following. I want to prove this for you. And the way I'm gonna prove it is slightly more general, in more generality. And I want to prove to you that the number-- if I have a set, or, sorry, if the number of people who are undergraduates and not blonde which, all right, is b bar plus the number who are blonde but not from Massachusetts is greater than or equal to the number that are undergraduates and not from Massachusetts. So how do I prove this? Well if you're an undergraduate and not blonde, you may or you may not be from Massachusetts. So this is equal to the number of undergraduates who are not blonde and are from Massachusetts plus the number of undergraduates who are not blonde and are not from Massachusetts. It could hardly be otherwise. You either are or you are not from Massachusetts. Not the sort of thing that you normally see in physics. So this is the number of people who are blonde and not from Massachusetts, number of people who are blonde, who are-- so if you're blonde and not from

Massachusetts, you may or may not be an undergraduate. So this is the number of people who are undergraduates, blonde, and not from Massachusetts plus the number of people who are not undergraduates, are blonde and are not from Massachusetts. And on the right hand side-- so, adding these two together gives us plus and plus. On the right hand side, the number of people that are undergraduates and not from Massachusetts, well each one could be either blonde or not blonde. So this is equal to the number that are undergraduates, blonde, and not from Massachusetts, plus-- remember that our undergraduates not blonde and not from Massachusetts. Agreed? I am now going to use the awesome power of-- and so this is what we want to prove, and I'm going to use the awesome power of subtraction. And note that U, B, M bar, these guys cancel. And U, B bar, M bar, these guys cancel. And we're left with the following proposition: the number of undergraduates who are not blonde but are from Massachusetts plus the number of undergrad-- of non-undergraduates who are blonde but not from Massachusetts must be greater than or equal to zero. Can you have a number of people in a room satisfying some condition be less than zero? Can minus 3 of you be blonde undergraduates not from Massachusetts? Not so much. This is a strictly positive number, because it's a numerative. It's a counting problem. How many are undergraduates not blonde and from Massachusetts. Yeah? Everyone cool with that? So it could hardly have been otherwise. It had to work out like this. And here's the more general statement. The more general statement is that the number of people, or the number of elements of any set where each element in that set has binary properties a b and c-- a or not a, b or not b, c or not c. Satisfies the following inequality. The number who are a but not b plus the number who are b but not c is greater than or equal to the number who are a but not c. And this is exactly the same argument. And this inequality which is a tautology, really, is called Bell's Inequality. And it's obviously true. What did I use to derive this? Logic and integers, right? I mean, that's bedrock stuff. Here's the problem. I didn't mention this last time, but in fact electrons have a third property in addition to-- electrons have a third property in addition to hardness and color. The third property is called whimsy, and you can either be whimsical or not whimsical. And every electron, when measured, is either whimsical or not whimsical. You never have a boring electron. You never have an ambiguous electron. Always whimsical or not whimsical. So we have hardness, we have color, we have whimsy. OK. And I can perform the following experiment. From a set of electrons, I can measure the number that are hard and not black, plus the number that are black but not whimsical. And I can measure the number that are hard and not whimsical. OK? And I want to just open up the case a little bit and tell you that the hardness here really is the angular momentum of the electron along the x-axis. Color is the angular momentum of the electron along the y-axis. And whimsy is the angular momentum of the electron along the z-axis. These are things I can measure because I can measure angular momentum. So I can perform this experiment with electrons and it needn't be satisfied. In particular, we will show that the number of electrons, just to be very precise, the number of electrons in a given set, which have positive angular momentum along the x-axis and down along the y-axis, plus up along the y-axis and down along the z-axis, is less than the number that are up. Actually let me do this in a very particular way. Up... zero down at theta. Up at theta, down at-- two theta is greater than the number that are up at zero and down at theta. Now here's the thing-- two theta. You can't at the moment understand what this equation means. But if I just tell you that these are three binary properties of the electron, OK, and that it violates this inequality, there is something deeply troubling about this result. Bell's Inequality, which we proved-- trivially, using integers, using logic-- is false in quantum mechanics. And it's not just false in quantum mechanics. We will at the end of the course derive the quantum mechanical prediction for this result and show that at least to a predicted violation of Bell's Inequality. This experiment has been done, and the real world violates Bell's Inequality. Logic and integers and adding probabilities, as we have done, is misguided. And our job, which we will begin with the next lecture, is to find a better way to add probabilities than classically. And that will be quantum mechanics See you on Tuesday. AUDIENCE: [APPLAUSE]