PROFESSOR: Let's do a work check. So main check. If integral psi star x t0, psi x t0 dx is equal to 1 at t equal to t0, as we say there, then it must hold for later times, t greater than t0. This is what we want to check, or verify, or prove.

Now, to do it, we're going to take our time. So it's not going to happen in five minutes, not 10 minutes, maybe not even half an hour. Not because it's so difficult. It's because there's so many things that one can say in between that teach you a lot about quantum mechanics. So we're going to take our time here.

So we're going to first rewrite it with better notation. So we'll define rho of x and t, which is going to be called the probability density. And it's nothing else than what you would expect, psi star of x and t, psi of x and t. It's a probability density. You know that has the right interpretation, it's psi squared. And that's the kind of thing that integrated over space gives you the total probability. So this is a positive number given by this quantity is called the probability density. Fine.

What do we know about this probability density that we're trying to find about its integral? So define next N of t to be the integral of rho of x and t dx. Integrate this probability density throughout space, and that's going to give you N of t.

Now, what do we know? We know that N of t, or let's assume that N of t0 is equal to 1. N is that normalization. It's that total integral of the probability what had to be equal to 1. Well, let's assume N at t0 is equal to 1. That's good. The question is, will the Schrodinger equation guarantee that-- and here's the claim-- dN dt is equal to 0? Will the Schrodinger equation guarantee this?

If the Schrodinger equation guarantees that this derivative is, indeed, zero, then we're in good business. Because the derivative is zero, the value's 1, will remain 1 forever. Yes?

AUDIENCE: May I ask why you specified for t greater than t0?

Well, I don't have to specify for t greater than t naught. I could do it for all t different than t naught. But if I say this way, as imagining that somebody prepares the system at some time, t naught, and maybe the system didn't exist for other times below. Now, if a system existed for long time and you look at it at t naught, then certainly the Schrodinger equation should imply that it works later and it works before. So it's not really necessary, but no loss of generality.

OK, so that's it. Will it guarantee that? Well, that's our thing to do. So let's begin the work by doing a little bit of a calculation. And so what do we need to do? We need to find the derivative of this quantity. So what is this derivative of N dN dt will be the integral d dt of rho of x and t dx. So I went here and brought in the d dt, which became a partial derivative. Because this is just a function of t, but inside here, there's a function of t and a function of x. So I must make clear that I'm just differentiating t.

So is d dt of rho. And now we can write it as integral dx. What this rho? Psi star psi. So we would have d dt of psi star times psi plus psi star d dt of psi.

OK. And here you see, if you were waiting for that, that the Schrodinger equation has to be necessary. Because we have the psi dt. And that information is there with Schrodinger's equation. So let's do that.

So what do we have? ih bar d psi dt equal h psi. We'll write it like that for the time being without copying all what h is. That would take a lot of time. And from this equation, you can find immediately that d psi dt is minus i over h bar h hat psi.

Now we need to complex conjugate this equation, and that is always a little more scary. Actually, the way to do this in a way that you never get into scary or strange things. So let me take the complex conjugate of this equation. Here I would have i goes to minus i h bar, and now I would have-- we can go very slow-- d psi dt star equals, and then I'll be simple minded here. I think it's the best. I'll just start the right hand side. I start the left hand side and start the right hand side.

Now here, the complex conjugate of a derivative, in this case I want to clarify what it is. It's just the derivative of the complex conjugate. So this is minus ih bar d/dt of psi star equals h hat psi star, that's fine. And from here, if I multiply again by i divided by h bar, we get d psi star dt is equal to i over h star h hat psi star.

We obtain this useful formula and this useful formula, and both go into our calculation of dN dt. So what do we have here? dN dt equals integral dx, and I will put an i over h bar, I think, here. Yes. i over h bar. Look at this term first. We have i over h bar, h psi star psi. And the second term involves a d psi dt that comes with an opposite sign. Same factor of i over h bar, so minus psi star h psi.

So the virtue of what we've done so far is that it doesn't look so bad yet. And looks relatively clean, and it's very suggestive, actually. So what's happening? We want to show that dN dt is equal to 0. Now, are we going to be able to show that simply that to do a lot of algebra and say, oh, it's 0? Well, it's kind of going to work that way, but we're going to do the work and we're going to get to dN dt being an integral of something. And it's just not going to look like 0, but it will be manipulated in such a way that you can argue it's 0 using the boundary condition.

So it's kind of interesting how it's going to work. But here structurally, you see what must happen for this calculation to succeed. So we need for this to be 0. We need the following thing to happen. The integral of h hat psi star psi be equal to the integral of psi star h psi. And I should write the dx's. They are there.

So this would guarantee that dN dt is equal to 0. So that's a very nice statement, and it's kind of nice is that you have one function starred, one function non-starred. The h is where the function needs to be starred, but on the other side of the equation, the h is on the other side. So you've kind of moved the h from the complex conjugated function to the non-complex conjugated function. From the first function to this second function.

And that's a very nice thing to demand of the Hamiltonian. So actually what seems to be happening is that this conservation of probability will work if your Hamiltonian is good enough to do something like this. And this is a nice formula, it's a famous formula. This is true if H is a Hermitian operator.

It's a very interesting new name that shows up that an operator being Hermitian. So this is what I was promising you, that we're going to do this, and we're going to be learning all kinds of funny things as it happens. So what is it for a Hermitian operator? Well, a Hermitian operator, H, would actually satisfy the following. That the integral, H psi 1 star psi 2 is equal to the integral of psi 1 star H psi 2.

So an operator is said to be Hermitian if you can move it from the first part to the second part in this sense, and with two different functions. So this should be possible to do if an operator is to be called Hermitian. Now, of course, if it holds for two arbitrary functions, it holds when the two functions are the same, in this case.

So what we need is a particular case of the condition of hermiticity. Hermiticity simply means that the operator does this thing. Any two functions that you put here, this equality is true. Now if you ask yourself, how do I even understand that? What allows me to move the H from one side to the other? We'll see it very soon. But it's the fact that H has second derivatives, and maybe you can integrate them by parts and move the derivatives from the psi 1 to the psi 2, and do all kinds of things.

But you should try to think at this moment structurally, what kind of objects you have, what kind of properties you have. And the objects are this operator that controls the time evolution, called the Hamiltonian. And if I want probability interpretation to make sense, we need this equality, which is a consequence of hermiticity.

Now, I'll maybe use a little of this blackboard. I haven't used it much before. In terms of Hermitian operators, I'm almost there with a definition of a Hermitian operator. I haven't quite given it to you, but let's let state it, given that we're already in this discussion of hermiticity.

So this is what is called the Hermitian operator, does that. But in general, rho, given an operator T, one defines its hermitian conjugate P dagger as follows. So you have the integral of psi 1 star T psi 2, and that must be rearranged until it looks like T dagger psi 1 star psi 2.

Now, these things are the beginning of a whole set of ideas that are terribly important in quantum mechanics. Hermitian operators, or eigenvalues and eigenvectors. So it's going to take a little time for you to get accustomed to them. But this is the beginning. You will explore a little bit of these things in future homework, and start getting familiar. For now, it looks very strange and unmotivated. Maybe you will see that that will change soon, even throughout today's lecture.

So this is the Hermitian conjugate. So if you want to calculate the Hermitian conjugate, you must start with this thing, and start doing manipulations to clean up the psi 2, have nothing at the psi 2, everything acting on psi 1, and that thing is called the dagger.

And then finally, T is Hermitian if T dagger is equal to T. So its Hermitian conjugate is itself. It's almost like people say a real number is a number whose complex conjugate is equal to itself. So a Hermitian operator is one whose Hermitian conjugate is equal to itself, and you see if T is Hermitian, well then it's back to T and T in both places, which is what we've been saying here. This is a Hermitian operator.