PROFESSOR: We have two equations now relating this number of states. And now you can say, oh, OK, so I look at the k line. And I look at the little piece of the k and say, oh, how many states were there with 0 potential, some number, first blackboard. How many states are there now with some potential, some other number? It has changed.

For every-- because these two equations, the n for equal dk, the n is not equal to the dn prime. In one case, the energy levels or the momentum levels are more compressed or more separated, but whatever it is, whatever the sign is, there is a little discrepancy. So both of them are giving me the total number of positive energy states in the little dk. Case So if I take the difference, I will get some information.

So I would say the following, if I want to calculate the number of positive energy solutions and now I think the following, I take the potential V equals 0 and slowly but surely deepen it, push it, and do things and create the potential V of x slowly, slow the formation. In this process, I can look at a little interval dk and tell how many states are positive energy states I lost. So if, for example, dn is bigger than dn prime, dn equal 5 and dn prime is equal to 3, I started with 5 positive energy states in this little interval and by the time I change the potential I ended up with 3. So I lost 2.

So let me write here the number of positive energy solutions lost in the interval dk as the potential is turned on is dn the original number minus the dn prime. If that's positive, I've lost state. If it's negative, I gained state, positive energy states. In this number, we can calculate the difference. This is minus 1 over pi d delta dk dk. I'll put it here.

We'll we're not far. We'll this is what you lost. The number of positive energy eigenstates that you lost in little dk. To see how many positive energy states you lost over all, you must integrate over all the dk's and see how much you lost in every little piece.

So the number of positive energy solutions lost, not in the dk, but lost as the potential is turned on is equal to the integral over k from 0 to infinity of minus 1 over pi d delta dk is in the way of that expression of that right coincide. But this is a total derivative. So this is minus 1 over pi delta of k evaluated between infinity and 0. And therefore, the number of states lost is 1 over pi, because of the sign down to 0 minus delta of infinity.

So we're almost there. This is the number of positive energy solutions lost. Now I want to emphasize that the situation is quite interesting. Let me make a little drawing here.

So suppose here is the case where you have the potential equal to 0 and here is energy equal to 0. Then you have all these states. Now even though we've put the wall, the wall allows us to count the states, but there are still going to be an infinite number of states. The infinite square wall has an infinite number of states.

So that thing really continues, but what happens by the time v is deferred from 0? Here is that the E equals 0 line and here is the E equals 0 line. As we've discussed, as you change the potential slowly, this are going to shift a little and some are going to go down here, are going to become bound states. They're going to be a number of bound states, N bound states, number of bound states equal N. And then there's going to be still sub states here that's also go to infinity.

So you cannot quite say so easily, well, the number of states here minus the number of states here is the number lost. That's not true, because that's infinite, that's infinite, and subtracting infinity is bad. But you know that you've lost a number of finite number of positive energy solutions.

So as you track here, the number of states must-- the states must go into each other. And therefore, if these four states are now here, before they were here, and those were the positive energy solutions that were lost, in going from here to here, you lost positive energy solutions. You lost a finite number of positive energy solutions. Even though there's infinite here and infinite here, you lost some. And you did that by keeping track at any place how much you lost.

And therefore the states lost are never really lost. They are the ones that became the band states here. So the positive energy states that got lost are the bound states. So the number bound states is equal to the number of positive energy solutions, because there are no lost states. So this is equal to a number of bound states, because there are overall no lost states.