PROFESSOR: Here is x. And here is a. Various copies of the x-axis. For the ground state, what is the lowest energy state? It's not zero energy, because n begins with 1. So it's this. The lowest energy state is a sine. So the wave function looks like this. This corresponds to sine. 1. Or n equals 1.

The next one corresponds to n equals 2, and begins as a sine. And it just goes up like this. You add half a wave each time. Remember, we quantize k a with n pi. So each time that you increase n, you're adding pi to k a. So the phase, you see, you have sine of kx. So you have sine that goes from 0 up to k a. And k a is equal to n pi. So you go from 0 to pi, from 0 to 2 pi, then from 0 to 3 pi. So it would be one up, one down, like that. And I could do one more. This one would be with four cycles, two, three, four. So this is psi 1, psi 2, psi 3, and psi 4. Wave functions do more and more things.

So what can we learn from this wave function? There are several things that we need to understand. So one important thing is that the wave function, these are all normalizable wave functions. The ground state-- this is the ground state-- has no nodes. A node, in a wave function, is called the point where the wave function vanishes. But it's not the endpoints or the points at infinity, if you could have a range that goes up to infinity. It's an interior point that vanishes. And the ground state has no nodes.

So a node, node, so zero of the wave function, not at the end of the domain. And of the domain. Because if we included that, I would have to say that the ground states has two nodes already, you'll see, around 0. But the 0 at the end of the domain should not be counted as a node. Nodes are the zeros inside. And look. This has no nodes, and it's a general fact about states of potentials. The next excited state has one node. It's here. The next has two nodes, and then the next is three nodes.

So the number of nodes of the wave function increases in potential. You have more and more wave functions with higher and higher excited states, and the number of nodes increases one by one on each solution. That's actually a theorem that is valid for general potentials that have bound states. Bound states are states that are normalizable. So the decay at infinity. You see, a state that is not normalizable, like a plane, where it is not a bound state. It exists all over.

And it's a general theorem that this phase, this one-dimensional potentials, whenever you have bound states, the number of nodes increases with the energy of the eigenstate. We will see a lot of evidence for this as we move along the course, and a little bit of a proof. Not a very rigorous proof.

The other thing I want to comment on this thing that is extremely important is the issue of symmetry. This potential for simplicity, to write everything nicely, was written from 0 to a. So all the wave functions are sine of n pi x over a. But in some ways, it perhaps would have been better to put the 0 here. And you say, why? What difference does it make? Well, you have a 0 at the middle of the interval, the potential and the domain of the wave function are symmetric with respect to x going to minus x.

So actually, when you look at the wave functions thinking you can rethink this as an infinite box from a over 2 to minus a over 2, and the solutions, you just copy them, and you see now, this line that I drew in the middle, the ground state is symmetric. The next state is anti-symmetric with respect to the midpoint. The next state is now symmetric. And the following one, anti-symmetric.

So this is also a true fact. If you have bound states of a symmetric potential-- I will prove this one, probably on Wednesday. A symmetric potential is a potential for which V of minus x is V of x. Bound states of a symmetric potential are either odd or even. This is not a completely simple thing to prove. We will prove it, but you need, in fact, another result. It will be in the homework. Not this week's homework, but next week's homework. In fact, homework that is due this week is due on Friday.

So the bound states of a symmetric potential, a potential that satisfies this, are either odd or even. And that's exactly what you see here. That's not a coincidence. It's a true fact. The number of nodes increase. And the other fact that is very important, of bound states, of one-dimensional potentials-- supremely important fact. No degeneracies. If you have a bound state of a potential that is either localized like this or goes to infinity, there are no degenerate energy eigenstates. Each energy eigenstate here, there was no degeneracy.

Now, that is violated by our particle in a circle. The particle in the circle did have degenerate energy eigenstates. But as you will see, when you have a particle in a circle, you cannot prove that theorem. This theorem is valid for particles in infinitely-- not in a circle. For x's that go from minus infinity to infinity or x's with vanishing conditions at some hard walls. In those cases, it's true.

So, look, you're seeing at this moment the beginning of very important general results, of very fundamental general results that allow you to understand the structure of the wave function in general. We're illustrating it here, but they are very much, truly now, general potential. So what are they? For one-dimensional potential unbound states, no degeneracy, number of nodes increasing one by one. If the potential is symmetric, the wave functions are either even or odd.