PROFESSOR: This will be qualitative insights on the wave function. It's qualitative, and it's partially quantitative of course, insights into, let's say, real energy eigenstates. So whenever you have a problem and a potential, we have what is called the total energy, the kinetic energy, and the potential energy. So you have the energy, which is total, equal kinetic energy plus the potential energy.

Now, the potential energy, as you've seen, sometimes depends on position. We did piecewise continuous end potentials, but they could be more complicated and do funny things. So this is a function of x. And classically speaking, we speak of the energy. You see in quantum mechanics, the energy is an observable and is the result of a measurement with a permission operator. Sometimes there could be uncertainty, sometimes not.

But in classical physics, which this intuition will come from, you have a total energy. It is conserved. It's equal to potential energy and kinetic energy. That will also depend on where the particle is in the potential. Let's do a very simple case. Coordinate x, a potential. v of x, this based the potential. v of x, it's a constant, nothing that complicated.

And suppose you have a total energy. Now, the total energy in classical mechanics is conserved. So when I draw a line, I'm not implying that is a function of x, that sometimes the energy is like that. No, it's just a number there that I fixed. Here is the energy. And then wherever you move, if the particle, the classical particle, is here, then it has some potential energy, v of x, and some kinetic energy, k of x, building up the total energy.

Classically, the kinetic energy determines the momentum. The kinetic energy is p squared over 2m. Now, the kinetic energy is p squared over 2m. In this case, the kinetic energy is a constant. The momentum will be a constant. And then what we really want to just say something about is the wave function. Well, but if we note the momentum classically it's a momentum p, we can infer the Broglie wavelength of the particle. And that the Broglie wavelength would be h over p. And that's for the wave function.

So we should expect a wave function that has a wave length equal to lambda. After all, that is what the Broglie did. And from the Broglie, you got the Schrodinger equation. The Schrodinger equation, in fact, says this, if you look at it again. So if you look at the wave function. Well, it must have wavelength lambda. And therefore, I'm talking about real wave function. So it could be a cosine or a sine that has that wavelength lambda.

Of course in quantum mechanics, a cosine or a side doesn't have exactly-- it's not an eigenstate of momentum. But it's an eigenstate state of energy. And we want to plug eigenstates of energy. So you will have something like that, with that lambda. And that's intuition. You go from the diagram to a kinetic energy, from a kinetic energy to a momentum, from a momentum to a wavelength, and that's the wavelength of your energy eigenstate. And maybe it's a good idea that you try to convince yourself this is true by looking again at the Schrodinger equation. For this simple case of a constant potential and an energy that is big, you will find this result very quickly.

But let's do now a more interesting case, in which here is x. And the potential is a growing function of x. And there's a total energy here still. So if you are at some point here, here is the potential of x. And now, this is k of x. And now comes the interesting thing. You see, as your particle, or whatever particle, is moving here, the kinetic energy is decreasing as you move to the right. So the kinetic energy [INAUDIBLE] velocity and slows down, slows down, slows down.

The kinetic energy is becoming smaller and smaller. Therefore the momentum is becoming smaller and smaller. And therefore the wavelength, the Broglie wavelength, must be becoming bigger and bigger. Now that is not exact because you really have to solve the Schrodinger equation to do this. But intuitively, you know that if a potential is constant, this is absolutely true.

The kinetic energy, and the momentum, and the Broglie waveform have related in this way. It will be sort of true, or approximately true, if the potential is not changing that fast. Because then it's approximately constant. So there's a notion the slowly changing potential, in which we can talk about the k of x that is decreasing as we move to the right, a p of x that is also decreasing, and a lambda of x that would be increasing, a wave with the Broglie wavelength that is increasing.

Now I should have written in here, maybe, k of x, p of x, lambda of x. This is decreasing, decreasing, increasing. So I can plot it here. And I would say, well, I don't know exactly how this goes. But maybe it's the wavelength is small. And then the wavelength is becoming bigger, something like that. Well, the wavelength's becoming bigger in the energy eigenstate that you will find is true. But there's also the question whether the amplitude of the wave will change or not. So we'll answer that in a couple of minutes.

But the Broglie wavelength now is becoming a function of position. Now, you know that solving the Schrodinger equation now with an arbitrary potential is a difficult thing. With a linear potentially it's a difficult problem, in which the exact solution exists in terms of Airy functions and things like that. So this can only be an approximate statement that the Broglie wavelength is becoming bigger and bigger, because the momentum is becoming smaller and smaller. But it's a very useful statement. And whenever you look at wave functions of potentials, you see that thing happening. Questions?

Let me draw another diagram that illustrates these issues.

[SIDE CONVERSATION]

So let's draw a general picture of a potential now, so we can make a few features here. So here it is. We'll have a potential that is like this, v of x, maybe some energy, e, and that's it. Now what happens classically, well, if your particle has some energy, you know already this part is v of x. This is k of x. There is a potential energy and kinetic energy. The kinetic energy cannot become negative classically. So the particle cannot go to the left of this point called xl, x to the left. So this region, x left, is the classically forbidden.

Similarly on the right, you cannot go beyond here. Because then you would have negative kinetic energy. So this is an x right. And everything to the right [INAUDIBLE], x right, is also classical forbidden. These points, x left and x right, are called turning points. Because those are the points where a particle, a classical particle, if it lives in this potential, has to bounce back and turn.

As we mentioned, at any general point, you have v of x and k of x. And this point, for example, is the point with maximum k of x or maximum velocity. This is the point where the particle is moving the fastest. And it always slows down as it reaches the turning point. Because the kinetic energy is becoming smaller and smaller. So as we said, if you had a constant potential, this would be the solution. It's constant p, constant lambda, nice, simple wave function. If it's not constant, well, nothing is guaranteed. But if it's sufficiently constant or slowly varied, then you're in good shape.

Now what is the meaning of slowly varying? The meaning of slowly varying has to be said in a precise way. And this is what leads eventually to the so-called WKB approximation of quantum mechanics. Because we're giving you the first results of this approximation that you can understand classically how they go. To mean that you have a slowly varying potential, is a potential whose percentage change is small in the relevant distances. So it's the change in the potential over the relevant distance must be small compared to the potential.

But what is a relevant distance? If we use intuition from quantum mechanics, it's at the Broglie wavelength at any point. That is what the quantum particle sees. So what we need is that the change in the potential over at the Broglie wavelength-- take the derivative multiply it by that. The Broglie wavelength must be much smaller than the potential itself. And notice, of course, the potential is a function of x. And even lambda is a function of x there at the Broglie wavelength.

Now, an exact solution will not be a sine or a cosine. So to say has a precise defined wavelength is an approximation. It is the approximation of slowly varying. But it's a nice approximation. And this lambda is the lambda that would come as h over p of x. And h over p of x is the square root of 2m times the kinetic energy over h squared-- no, it's just that, the square root of 2mk. Square root of 2mk of x.

So the idea is that you can roughly say that the Broglie wavelength here is of some value here. The momentum is small if the Broglie wavelength is large. And so when you draw things, you adjust that. You say, OK, here, the momentum is large. Therefore the Broglie wavelength is small. So you write a short wavelength thing. And then it becomes longer wavelength and then shorter. And you just tried to get some insight into how this thing looks.