

**PROFESSOR:**  $E$  less than  $v_0$ . So you have an incident wave,  $e$  to the  $ikx$ , incident, and a reflected wave that you have,  $e$  to the minus  $ikx$ -- remember minus the other face-- and  $e$  to the  $2i$  delta of  $E$ . So this is the incident wave, and this is the reflected wave.

They correspond to your  $Ae$  to the  $ikx$  plus  $Be$  to the minus  $ikx$ . Remember, when the energy was less than  $v_0$ , the ratio of  $B$  over  $A$  was minus  $e$  to the  $2i$  delta. And since I take  $A$  equals 1, you get this thing.

So suppose I construct a  $\psi$  incident of  $x$  less than 0 and  $t$  as the sum 0 to infinity  $dk$   $f$  of  $k$   $e$  to the  $ikx$   $e$  to the minus  $iEt$  over  $\hbar$ . Whew. So I superimpose the incident thing here.

Then the reflected one should be superimposed too and would be 0 to infinity, a minus in front because there's a minus,  $dk$   $f$  of  $k$   $e$  to the minus  $ikx$   $e$  to the  $2i$  delta of  $E$   $e$  to the minus  $iEt$  over  $\hbar$ . Whew. That's the reflected wave superimposed.

So now you've constructed everything. Here the reflected wave is more interesting than the transmitted wave, because there's no real big transmitted wave. It just whistles out.

But the reflective thing is interesting. If you're doing the experiment, you send in a particle, you want to see what you get back. That's going to tell you what kind of potential you can expect it encountered.

So let's do the stationary phase for this one, for the reflected. Let's see how it moves. We know how the incident moves. The incident moves with  $x$  equals-- we've done it there--  $\hbar k_0$  over  $m$   $t$ .

But how about this one? Well, this one you would have to do  $d$   $dk$  of minus  $kx$  plus  $2$  delta of  $E$  minus  $Et$  over  $\hbar$ , all that at  $k_0$  equals 0. And you'll probably remember that this thing was in your midterm and your first test. You had this wave, and you had to analyze, what did stationary phase do? And it does that.

So what do you get? Well, when you take the derivative, you have to take the derivative of delta with respect to energy, that's delta prime, and then derivative of energy with respect to  $t$ . Let me save you a little time. The answer is minus  $\hbar k_0$   $m$   $t$  minus  $2\hbar$  delta prime of  $E$ . OK. That's what you get. That's how this packet moves.

And what does it do really? Remember, this is defined for  $x$  less than 0. So this is valid-- forget about this little term here-- this is valid for  $t$  positive. For  $t$  positive, you're going to get this to satisfy. So this is a big wave packet for  $t$  positive. It's the reflected wave. That's what you would expect. This is the reflected.

Now, if this factor was not here, it is as if, well, the incoming packet hit the origin at  $t$  equals 0. And this will be perfect bouncing in which the packet gets reflected. And at  $t$  equals 0, it starts to move to the left. And as  $t$  increases, it's moved more and more to the left. You see it there, because  $x$  must be negative.

But if there is this term, it really doesn't start to move to the left until  $t$  is bigger than that so that  $x$  is negative. So only at  $t$  equal to this amount of time the packet reflects. So there's a delay, and the delay is  $2\hbar \Delta' / E$ .

So this is a technology people use in scattering theory to figure out what kind of potential you have, figure out how much things get delayed from the bouncing. Now, this derivative-- we've plotted it there--  $\Delta' / E$ . You get a big delay for low energy, for energies near  $v_0$ , and in the middle it's not so big.