PROFESSOR: Write w equals sum from k equals 0 to infinity a k rho k. And plug in-- I've suggested that usually the thing that you should know when you plug in those equations is to look for the power rho to the k in this equation. And just, since everything is equal to 0, the coefficient of rho to the k in this equation should be 0. And that's the easiest way to select the powers.

That's good practice. You should do it. I won't do it here. We've done it in a few cases.

So this will relate a k plus 1. To a k. So that's algebra. It's a good skill to be able to do it. But it would be not very good use of our time to do it right now.

So here is the answer. OK. This is more important. 2 to the k plus l plus 1 minus 1 over kappa over k plus 1 plus k plus 2l plus 2. OK.

We've got our recursion relation. And the issue is, again, what happens with this coefficient as k goes to infinity? So as k goes large, a k plus 1 over a k goes like what? Well, we have a k that is becoming large, and everything else doesn't matter. There's a k and a k, so there's going to be some cancellation. And this looks like roughly 2 to the k.

Now, you could change these numbers a little bit. I'm going to do a tiny trick to simplify it, but it's just a trick. Don't worry about it too much.

I'll put 2 to the k plus 1 here. And I will say, look, if the series diverges in this case, this coefficient is bigger than that one. So it will certainly diverge for this case, as well. So the coefficients here are smaller than those ones, this ratio. So if the ratio between coefficients here is such that the series diverges, then it will even diverge a little more in this case.

And the reason I put it here is because then this is kind of solvable, a k plus 1 nicely solvable, very nicely solvable, 2k plus 1 a k. And the solution of this is to say you can try with a 0, what a 1 is, what a 2. a k is 2 to the k a 0 over k factorial.

OK. With that we can reconstruct what kind of function this series would be building if the series doesn't terminate and will not be too surprising. So in this case, the sum over k of a k rho k, which is the function we're building, is roughly equal to this a k here, which is 2 to the k. a 0 can go out. k factorial rho k. So this is a 0 e to the 2 rho.

It's kind of fair of it to do that. It's kind of saying that if the w solution doesn't truncate, it's going to go like e to the 2 rho, which precisely with an e to the minus rho is going to give you the other possible behavior of the solutions. It happened for the harmonic oscillator.

So what this is saying is that then w, which is this, would go roughly like that. And that's bad. So the series must truncate. So we must truncate the series.

OK. So here comes the interesting part because there's lots of quantities, and that has to be done a little slowly so that nobody gets confused of what's going to happen. We have to terminate this series. So how are we going to do it?

I'm going to state it the following way. I'm going to say that let's assume that we want a polynomial of degree capital N. There will be lots of little constants, capital N, little n. I want a polynomial of degree capital N.

That means that a sub capital N is different from 0. And a capital N plus 1 is equal to 0. That's what should happen. If you have constants up to a capital N, you'll have rho to the capital N, and you'll have a polynomial of degree N. But that must happen that the next one must be 0.

I don't have to state that all of the rest are 0 because it's a one-step recursion relation. Once a 5 is 0, a 6, a 7, a 8, all of them are 0. That's it.

And we will have like even or odd solutions that we had for the harmonic oscillator because these are functions of r. And r and minus r you should not quite expect anything. Minus r doesn't exist.

So this is what should happen. But if that happens, think of this. You have a n plus 1 should be 0. So the numerator should have become 0 for k equals 2N. So you have one over kappa is equal to 2N plus l plus 1. And in a sense that's it. Whatever had to happen, happened.

Why? The energy got quantized already. Somehow it did because the energy is kappa. Remember, kappa squared actually was the ratio of the energy divided by the dimension [INAUDIBLE]. So here it's saying the energy is some number that has to do with an integer, which is the degree of the polynomial you're going to use, an l integer, and 1. So this is, of course, pretty important.

So what values happen here? What are the possible values of l? Well, l here, l can go 0, 1, 2, 3, all of them. All of them are possible.

And why is that? It's because of the physics of the problem. We assume we'll have a particle in a central potential. All values of angular momentum can exist. So we should be looking for l's that take all these values.

Moreover, N is the polynomial that you can choose. And N can also be all of those values. We can begin with 0. So a 0 would be a number. But then it dies. A polynomial of degree 0 would be just a constant. It's possible. 1, 2, 3, all of those are possible.

And for each combination will have some energy. But here you start to see degeneracies, multiple degeneracies, because if you have the number 100,000 here, it can be built in many, many ways, 100,000 and 1 ways or something like that, with two integers that have to add up to it. And all of them will have the same energy. So the hydrogen atom is going to have lots of degeneracy.

So here is a little bit of a definition that we follow. So all these are allowed, all allowed, all combinations allowed. So l can be anything, and capital N can be anything.

And let's define a slightly better version of this thing. So let's move the 2 down, 1 over 2 kappa. That's N plus l plus 1. And let's call all this n, or the principal quantum number.

So n is the principal quantum number. And in some sense, well, you know that n has to be greater or equal than 1. It's an integer. And has to be greater or equal than 1 because of this 1 here and because the other ones cannot be negative either.

So n is a principal quantum number, and it's a fundamental number because it immediately gives you the value of the energy, which we will write more physically shortly. But it hides within it a degeneracy that this allowed because of these differing numbers. So these different numbers have to do with the degree of the polynomial and the value of l that you are using.

So let's classify this and understand it a little better. So what do we have for the energy? Remember, the energy divided by the dimensionless factor-- well, to make it dimensionless, z squared e squared over a 0 kappa squared. We wrote actually that e divided by this quantity, which has units of energy, was kappa squared.

So kappa squared now, kappa is 1 over 2n. So when we substitute here, we get e is equal to minus z squared e squared over 2a 0 1 over n squared. It's probably the most famous formula that you certainly have studied in high school, the 1 over n squared of the energy levels of the hydrogen atom.

The units are nice. There's for z equals to 1, there's the e squared over 2a 0 that we mentioned a little while ago as giving you the characteristic energy. And e squared over a 0 was 27.2 EV. And therefore, e squared over 2 a 0 is the famous 13.6 EV.