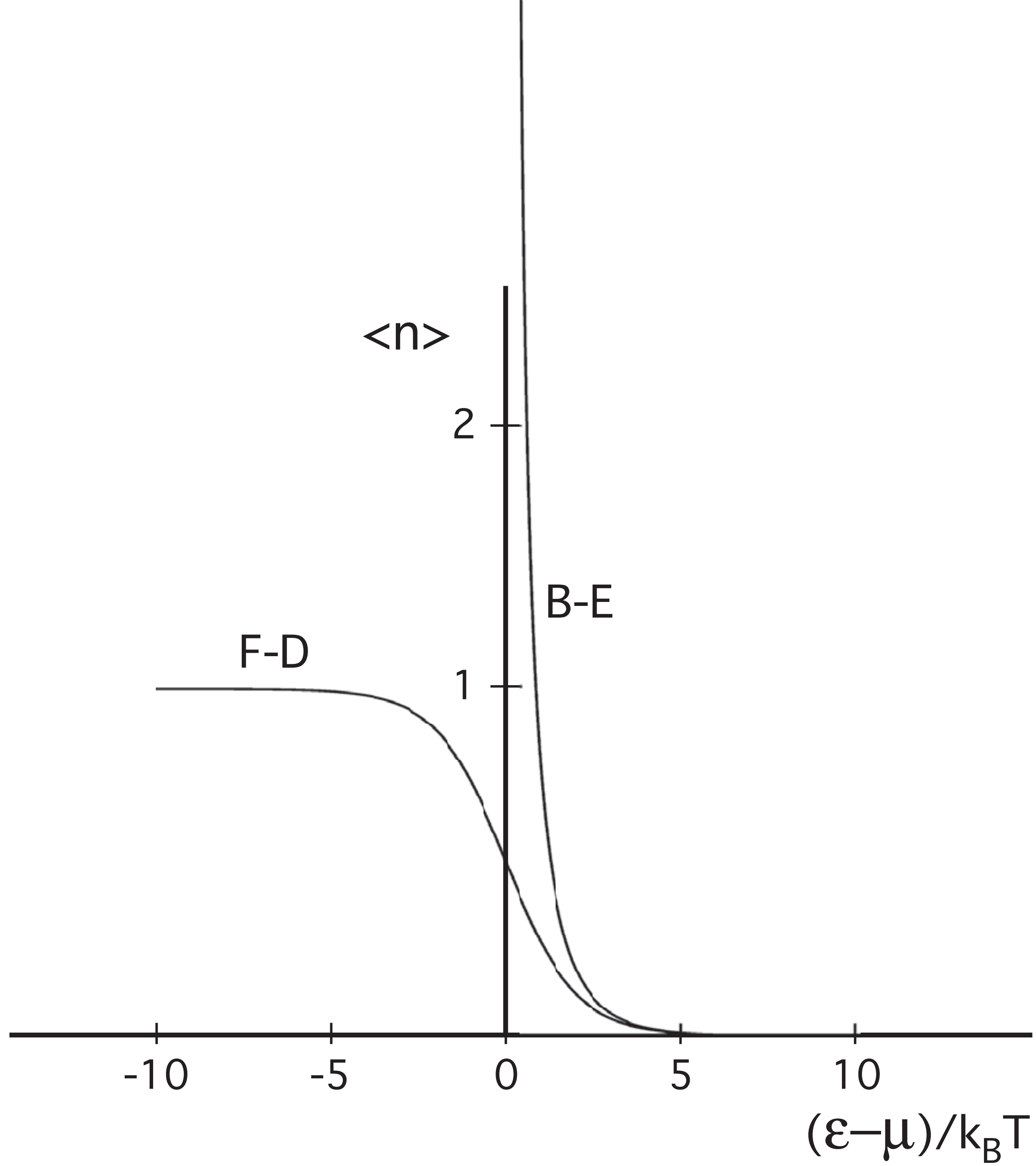
$$N = \int_0^\infty < n(\epsilon, \mu(T), T) > D(\epsilon)\, d\epsilon$$

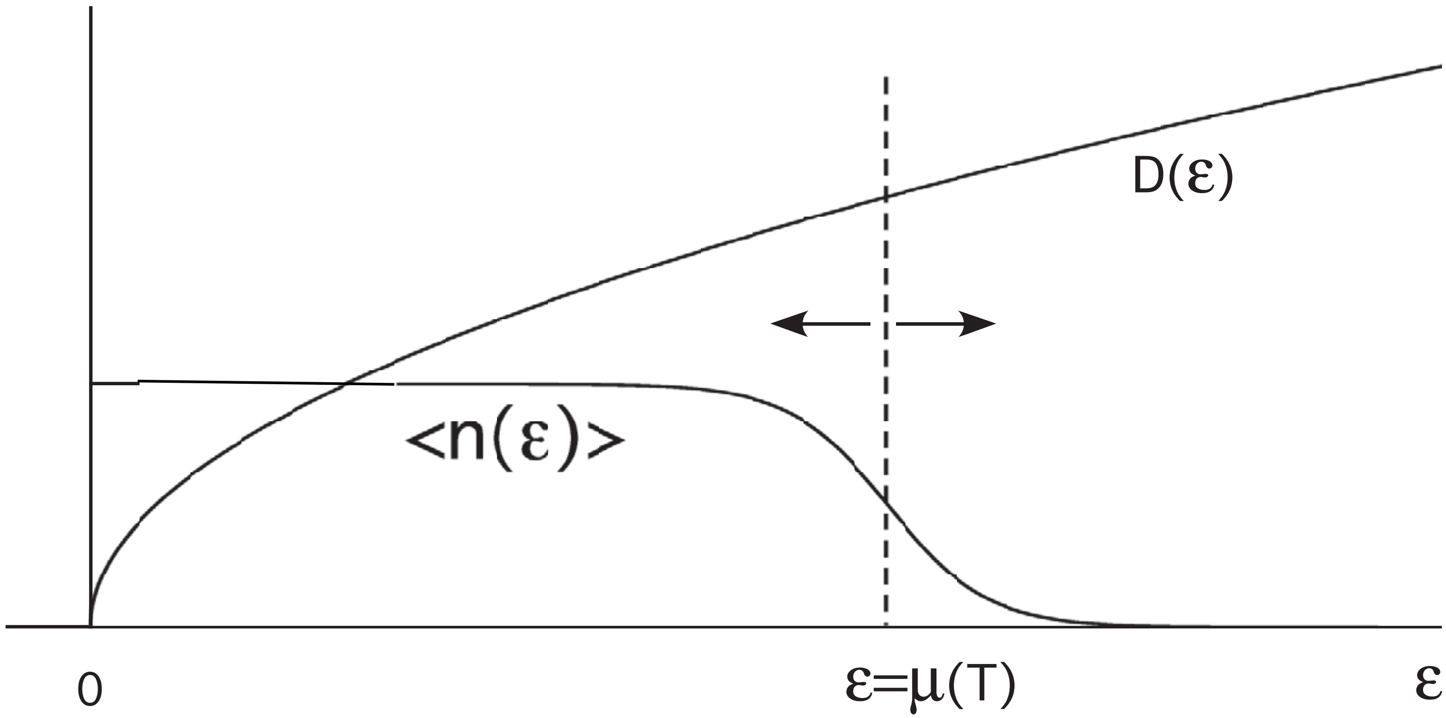
This expression implicitly determines $\mu = \mu(T)$.

$$U = \int_0^\infty < n(\epsilon, \mu(T), T) > \epsilon\, D(\epsilon)\, d\epsilon$$

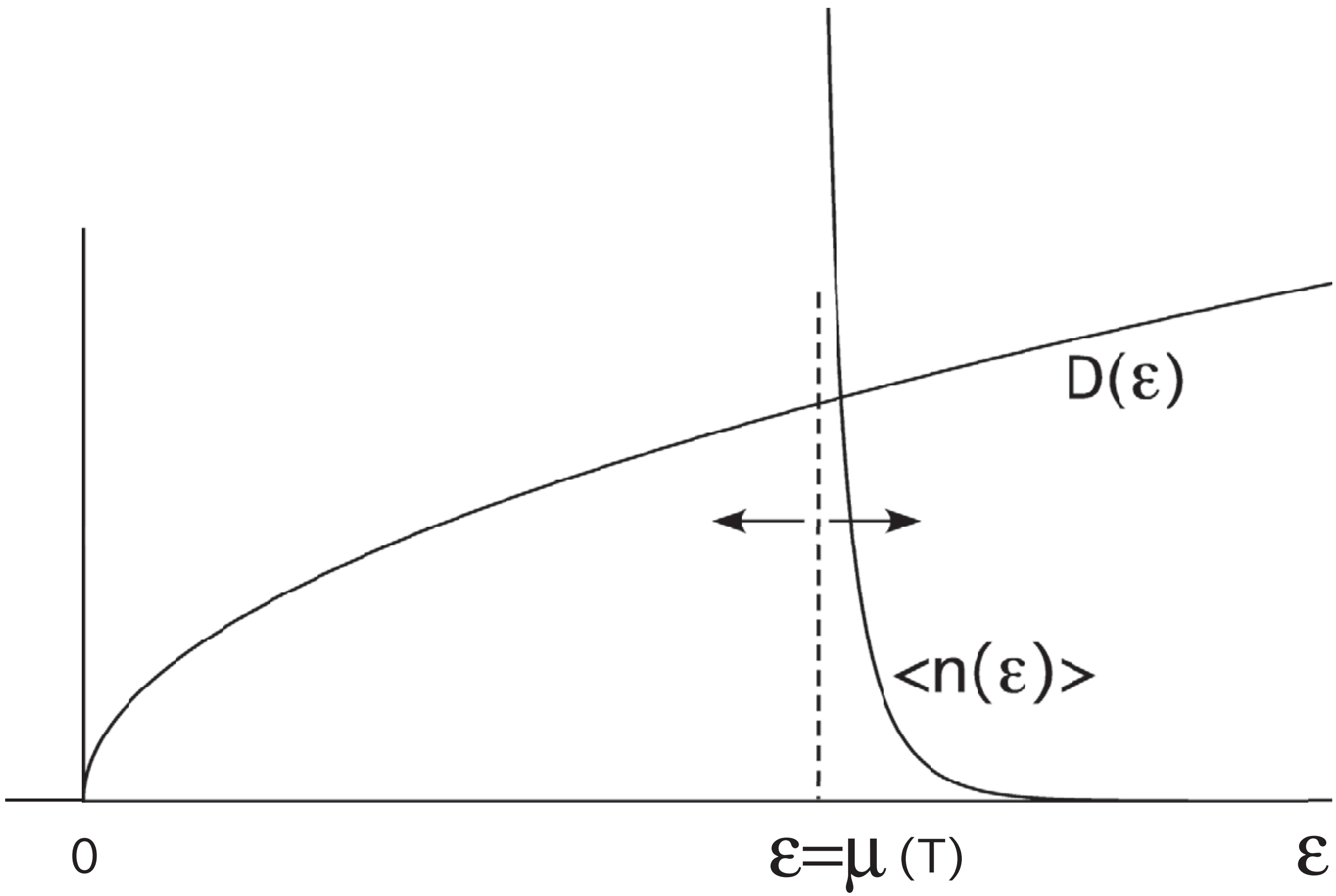
To determine C_V from this expression one must take into account the temperature dependence of μ in addition to the explicit dependence of $< n >$ on T.



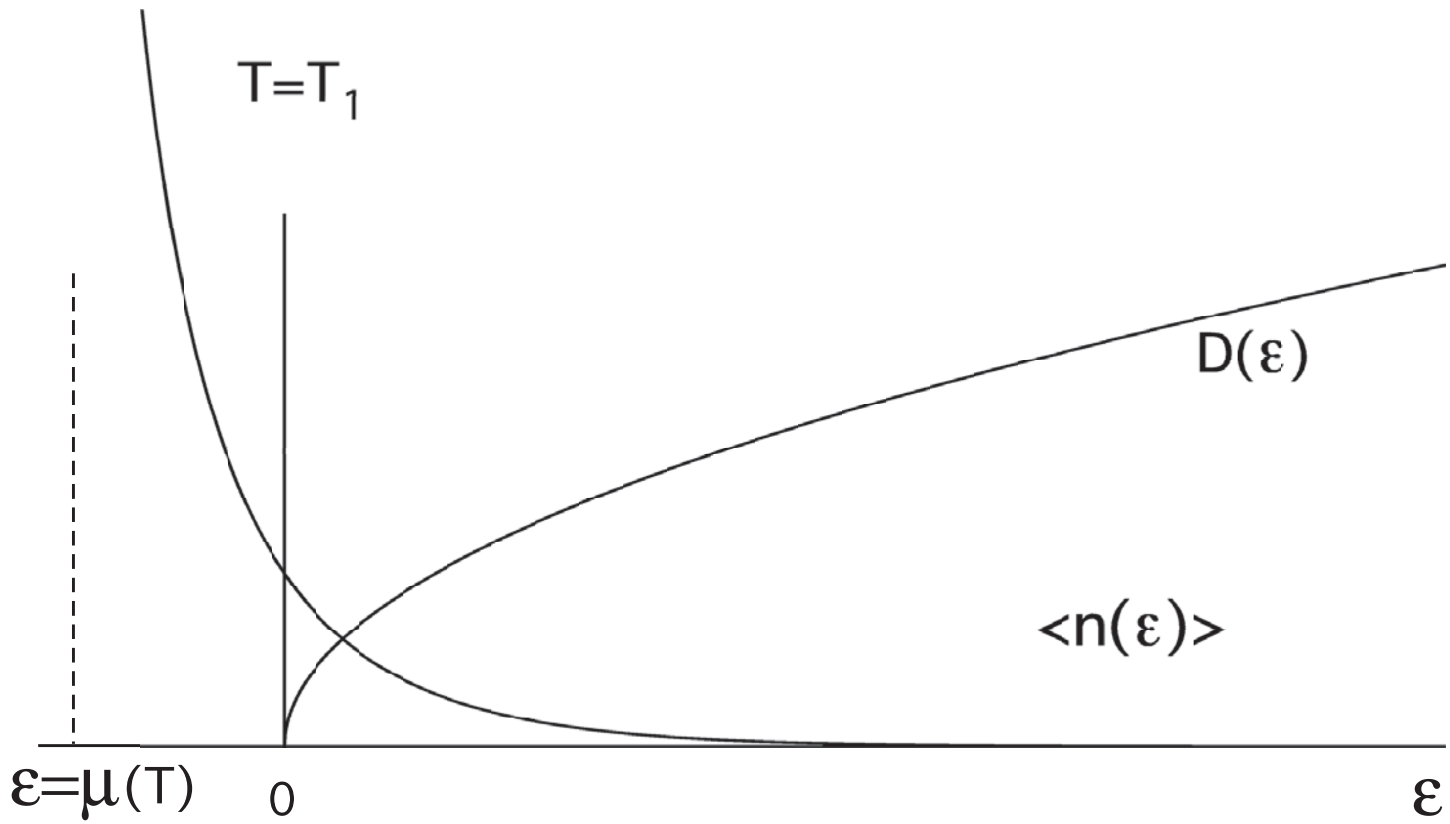
8.044 L22B1

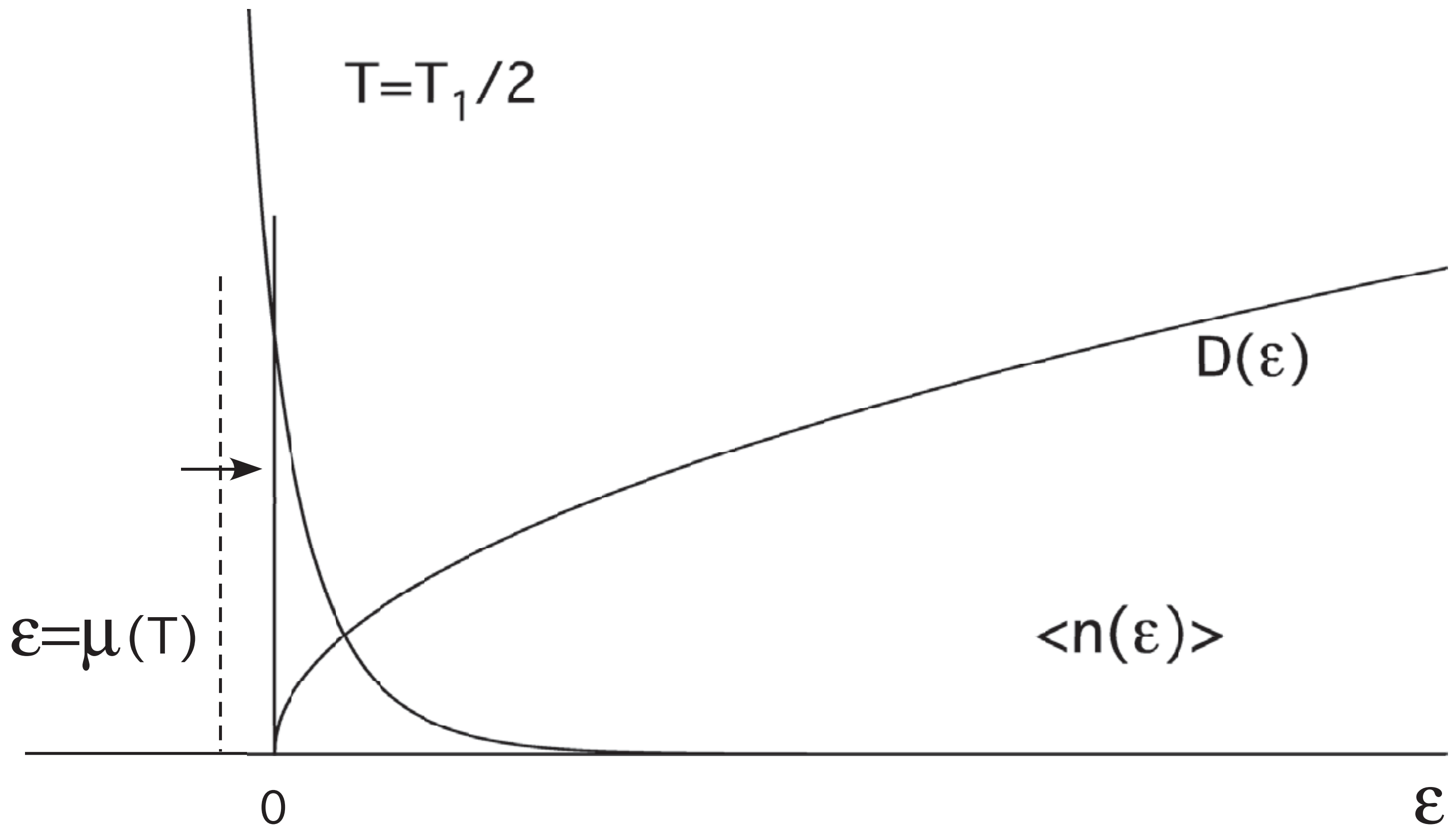


8.044 L22B2

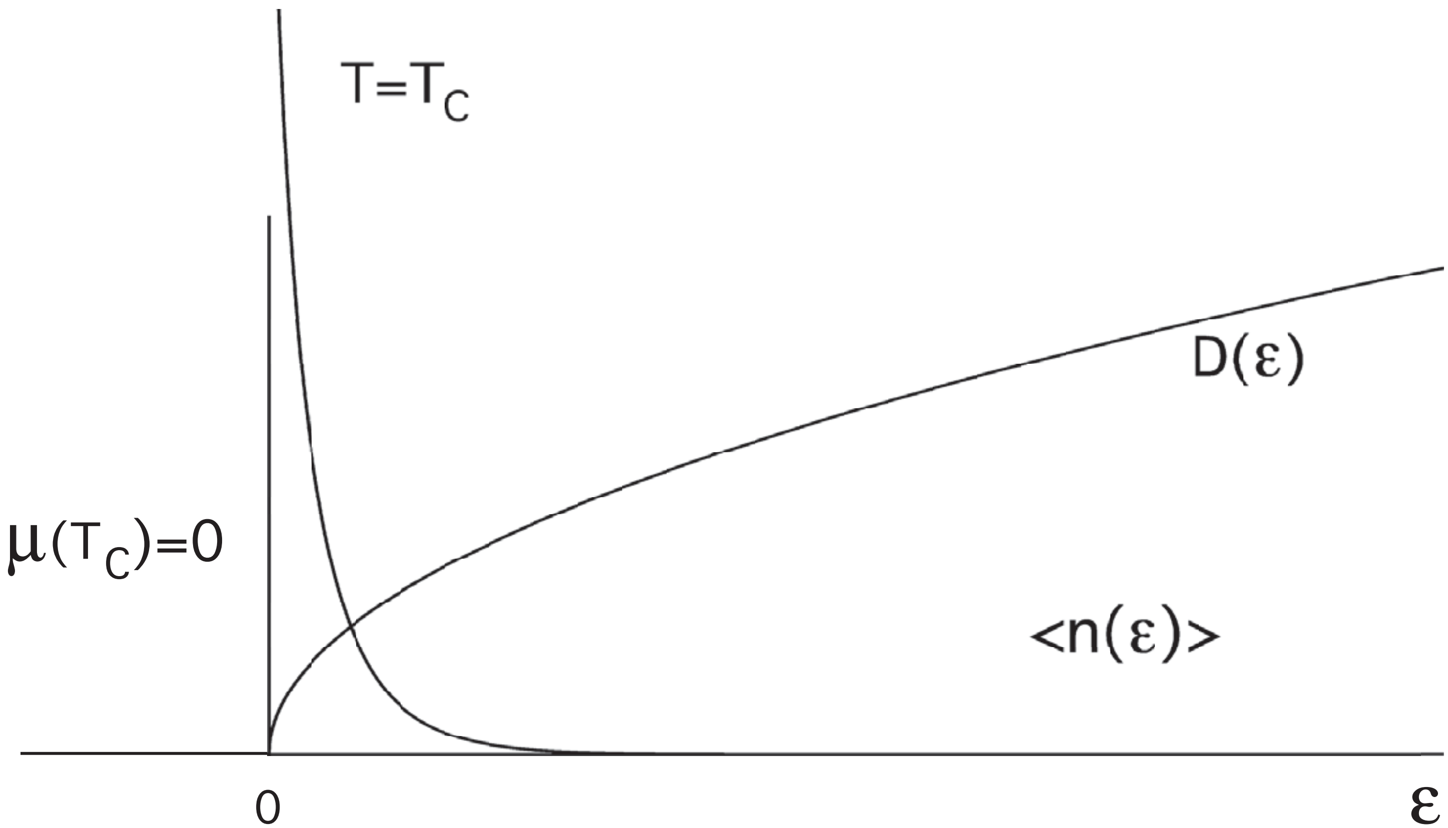


8.044 L22B3


T=T1
D(ε)
<n(ε)>
ε=μ(T)
0
ε



T=T1/2
D(ε)
ε=μ(T)
<n(ε)>
0
ε


T=T_C
D(ε)
μ(T_C)=0
<n(ε)>
0
ε

$$
\begin{aligned}
N &= \int_0^\infty < n > D(\boldsymbol{\epsilon})\, d\epsilon \\
&= \int_0^\infty \frac{1}{e^{\epsilon/k_B T} - 1} \left[\frac{(2S+1)V}{4\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \sqrt{\epsilon} \right] d\epsilon \\
&= \frac{(2S+1)V}{4\pi^2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \int_0^\infty \frac{\sqrt{\epsilon}}{e^{\epsilon/k_B T} - 1}\, d\epsilon \\
&= \frac{(2S+1)V}{4\pi^2} \left(\frac{2mk_B T}{\hbar^2}\right)^{3/2} \underbrace{\int_0^\infty \frac{\sqrt{x}}{e^x - 1}\, dx}_{(\sqrt{\pi}/2)\zeta(3/2)} \\
&= (2S+1)\zeta(3/2) \left(\frac{V}{\lambda^3(T)}\right)
\end{aligned}
$$

$\zeta(3/2) = 2.612\ldots$ and $\lambda(T) = h/\sqrt{2\pi m k_B T}$.

For a fixed number density $n = N/V$ the critical temperature for $S = 0$ Bosons is given by

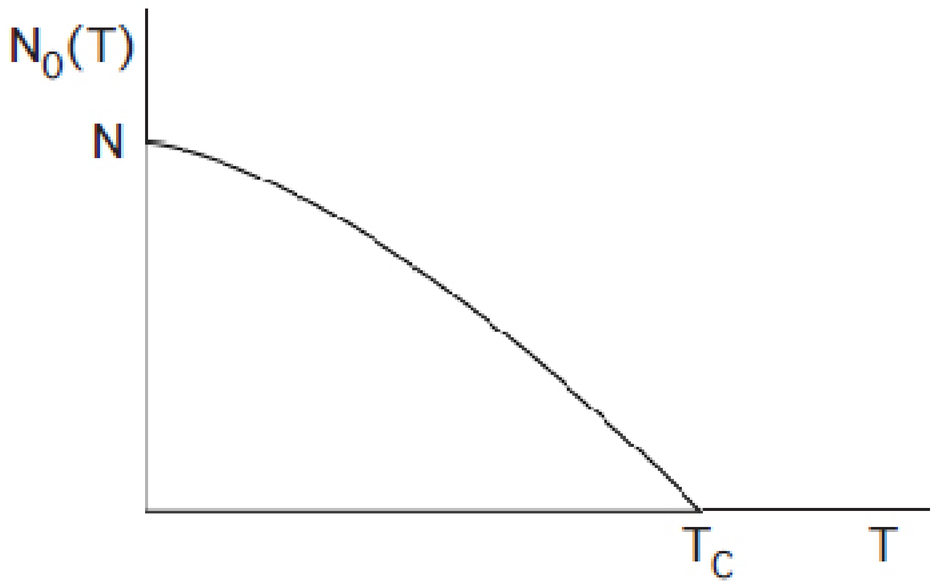
$$T_c = \frac{h^2}{2\pi m k_B}\left(\frac{n}{\zeta(3/2)}\right)^{2/3} \propto n^{2/3}$$

At a fixed temperature the critical number density is given by

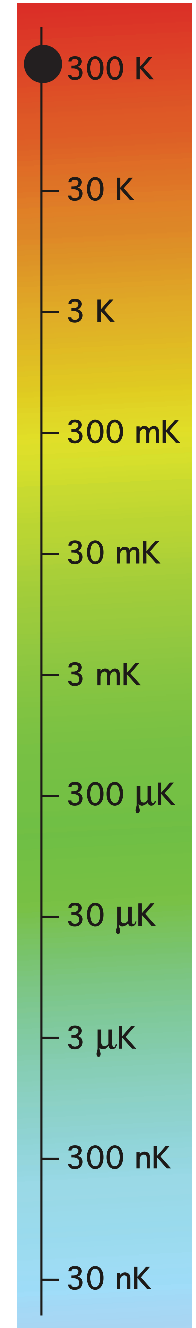
$$n_c = \zeta(3/2)\,\lambda^{-3}(T) \propto T^{3/2}$$

For a fixed number of atoms N, once T falls below T_c those atoms that can not be accommodated in states with finite ϵ begin accumulating in the ground state where $\epsilon = 0$.

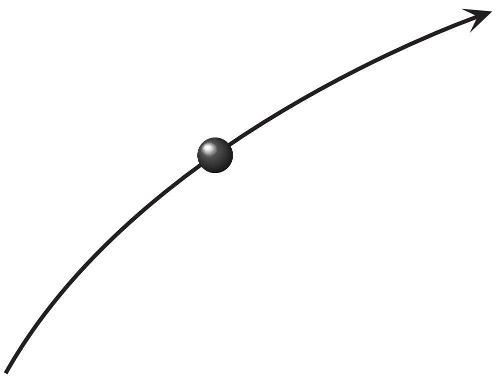
$$N_0 = N - \underbrace{\int_0^\infty < n > D(\epsilon)\, d\epsilon}_{\propto T^{3/2}} = N(1 - (T/T_c)^{3/2})$$



BOSE-EINSTEIN CONDENSATION IS A QUANUM MECHANICAL EFFECT

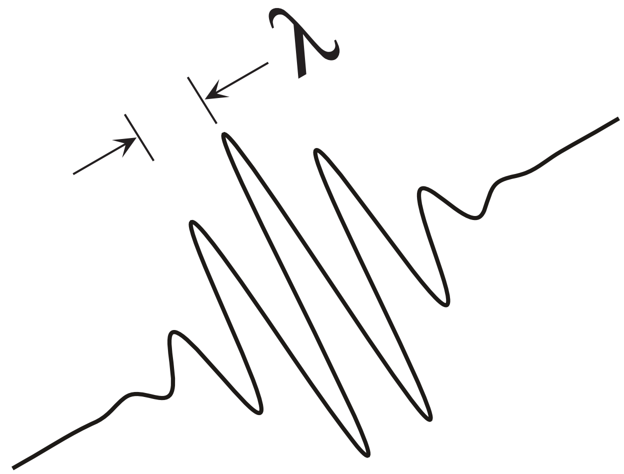


CLASSICAL MODEL

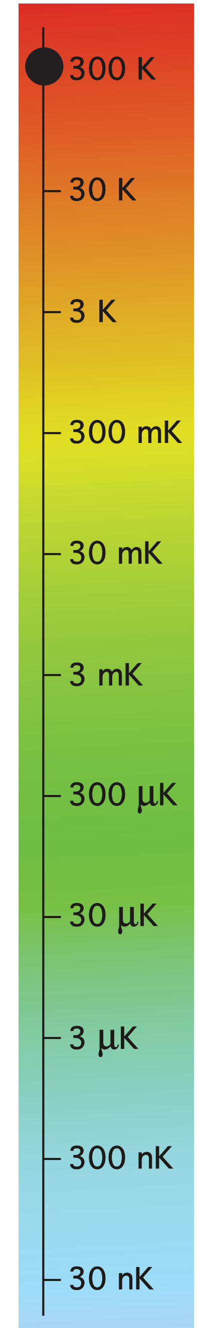


POINT-LIKE
PARTICLES
FOLLOWING
TRAJECTORIES

QUANTUM REALITY



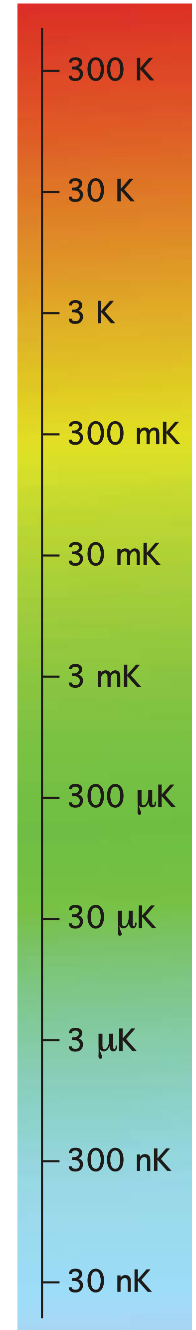
WAVES
PROPAGATING
THROUGH
SPACE

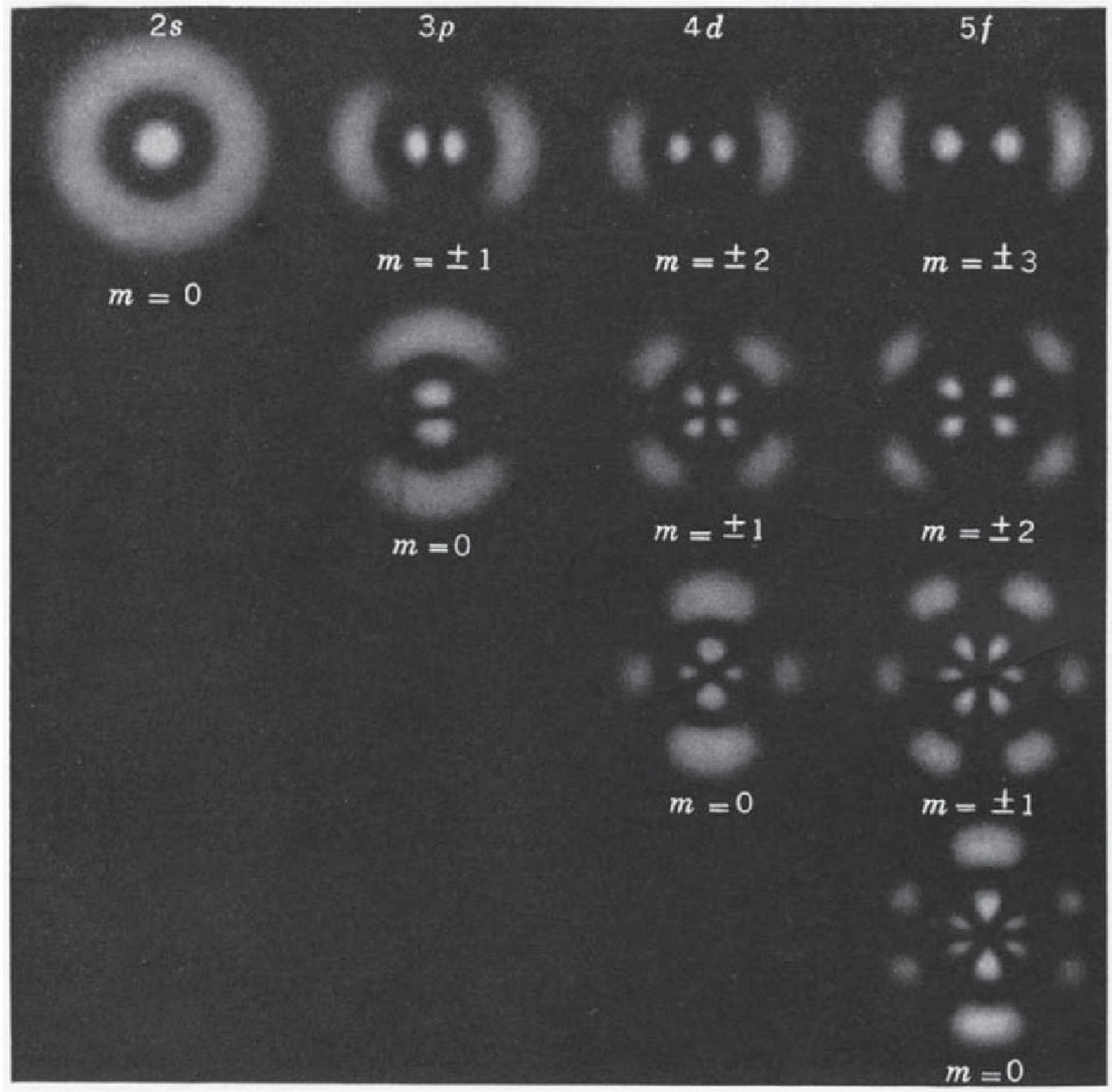


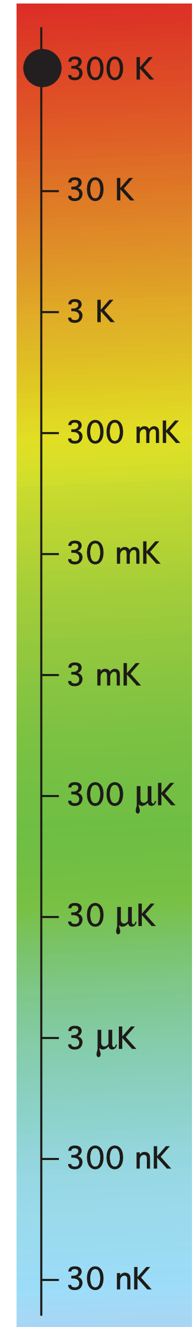
$$\lambda \propto \frac{1}{m \times V}$$

FOR ATOMS MOVING AT THERMAL VELOCITY
AT ROOM TEMPERATURE (300K),
$\lambda <$ THEIR PHYSICAL SIZE.

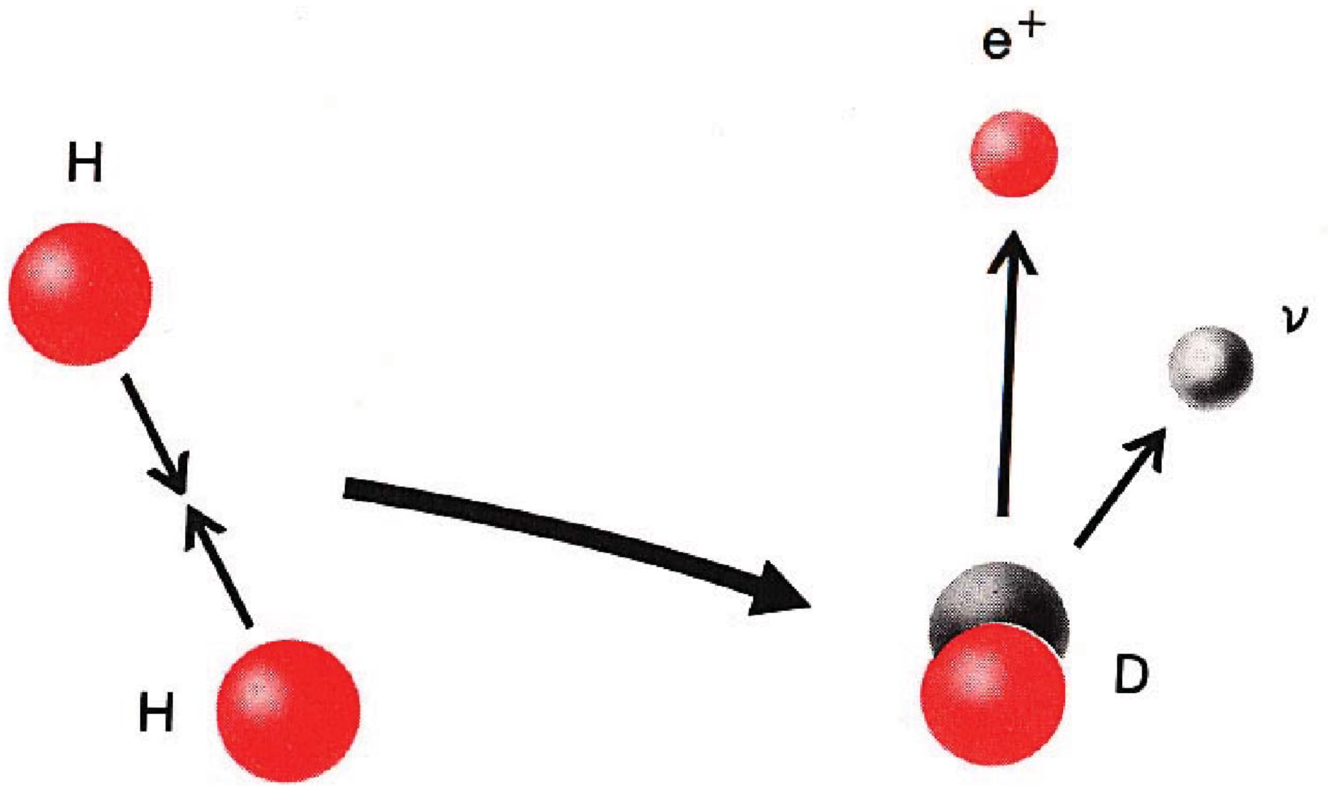
FOR THE ELECTRONS MOVING AROUND THE NUCLEI
IN THOSE ATOMS, $\lambda \approx 1$ ANGSTROM.

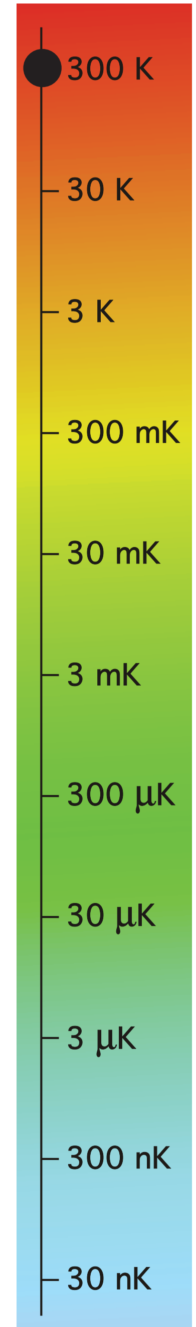




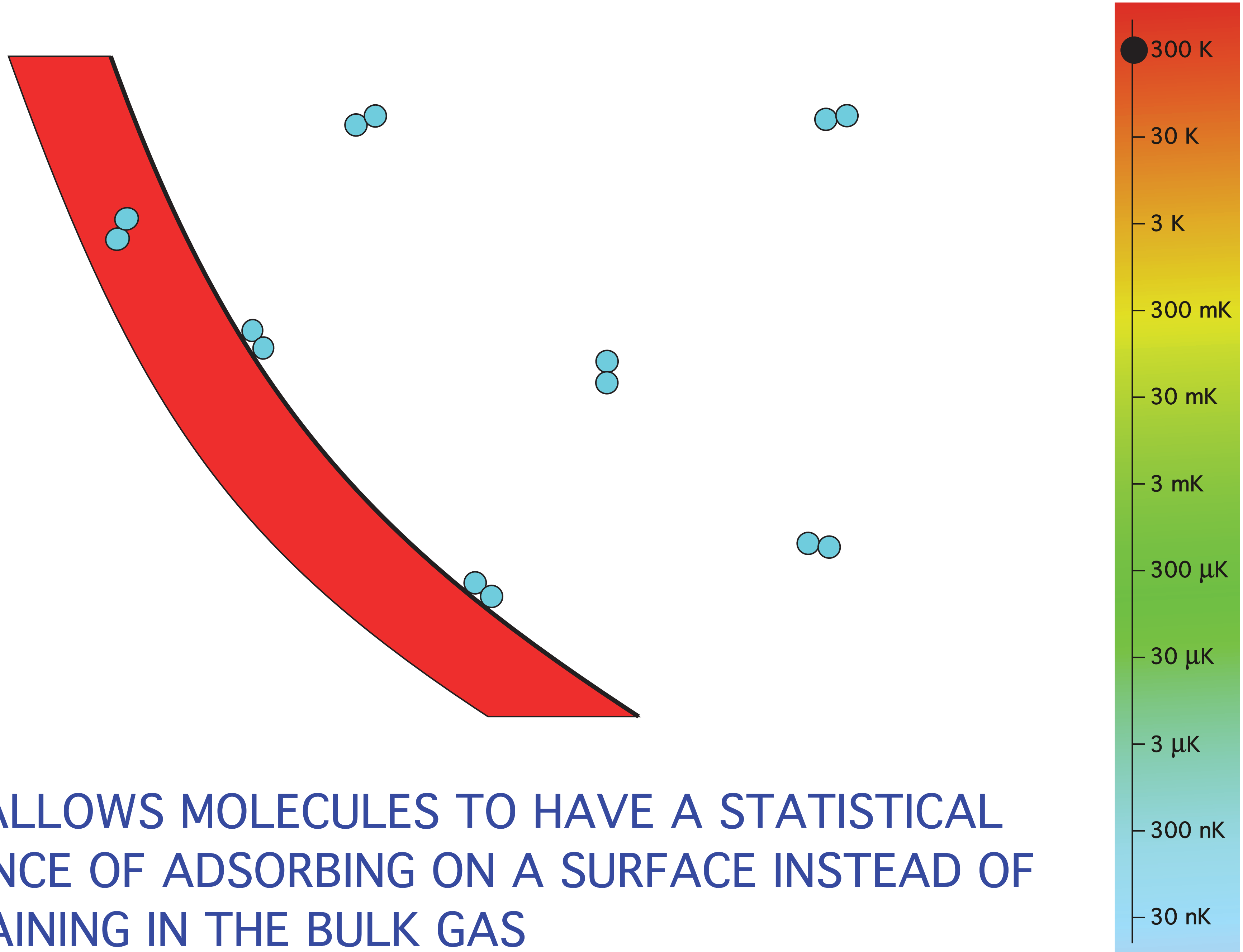


THE WAVE NATURE OF THE ELECTRONS STABILIZES THEM AGAINST LOSING ENERGY AND FALLING INTO THE NUCLEUS.



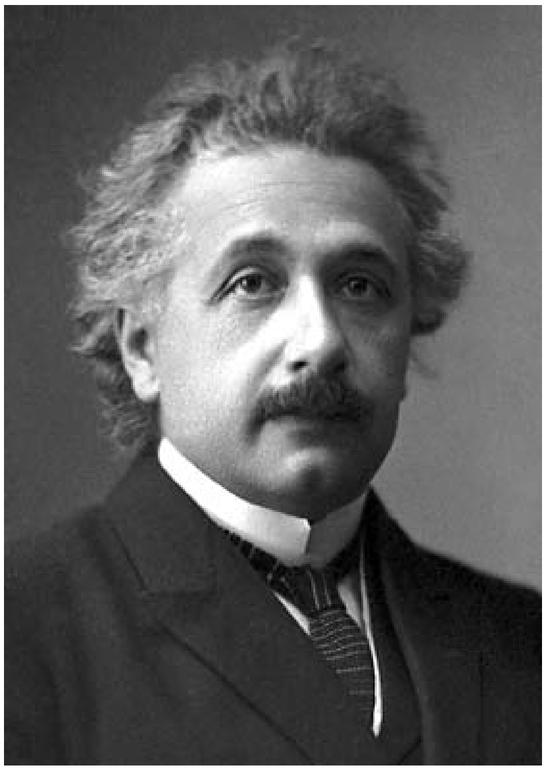


THE WAVE NATURE OF PROTONS ALLOWS THEM TO GET CLOSE ENOUGH DURING COLLISIONS IN THE SUN TO INITIATE FUSION.

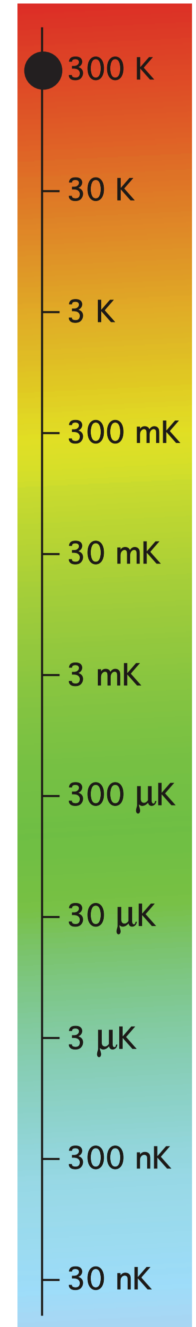


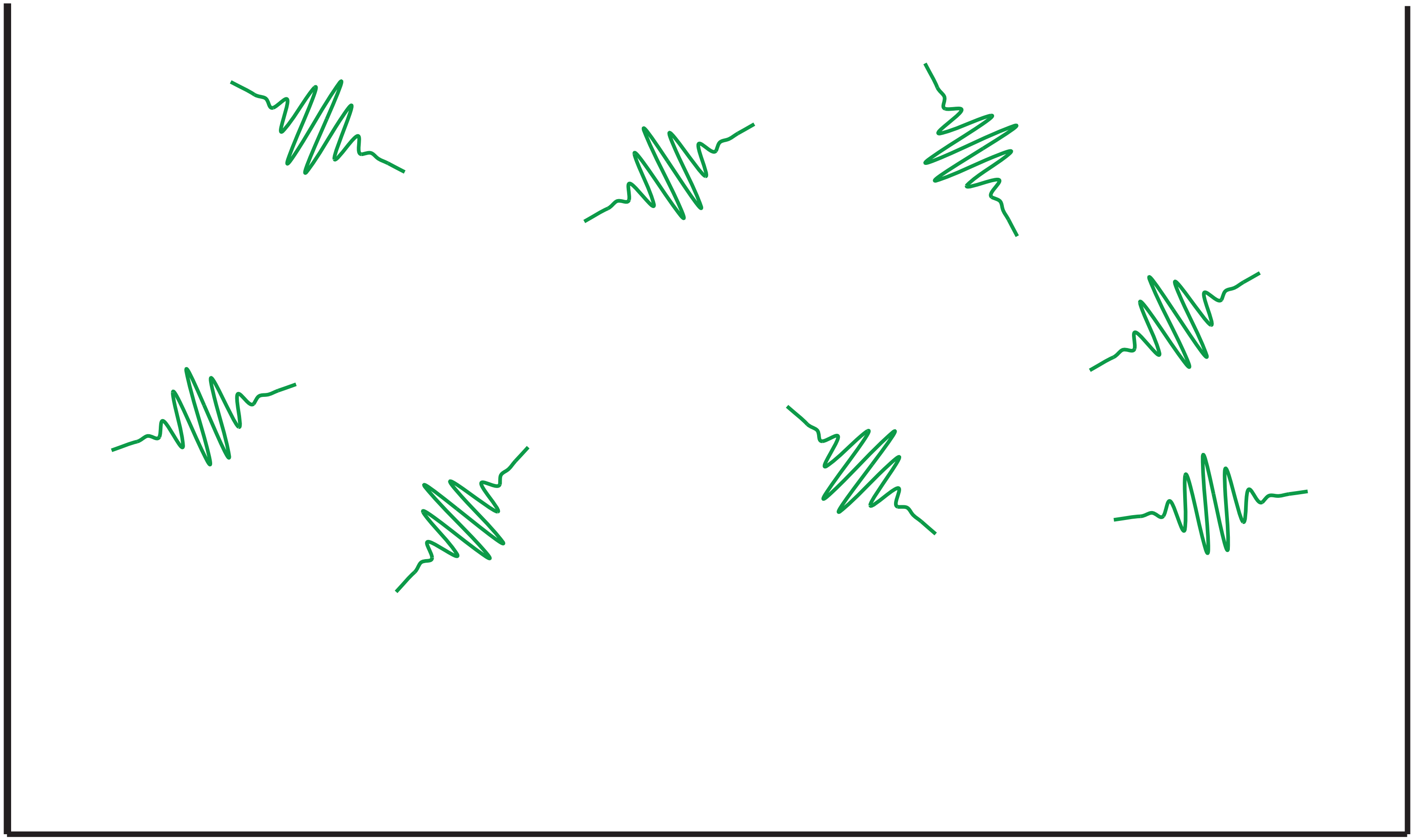
QM ALLOWS MOLECULES TO HAVE A STATISTICAL CHANCE OF ADSORBING ON A SURFACE INSTEAD OF REMAINING IN THE BULK GAS

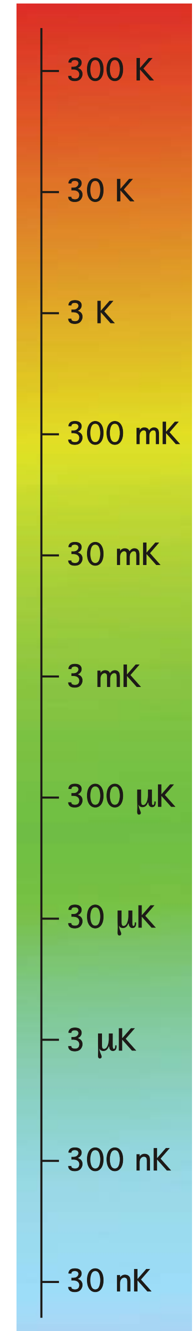
IN 1924 AND 1925 SATYENDRA BOSE AND ALBERT EINSTEIN INVESTIGATED THE INFLUENCE OF QM ON THE COLLECTIVE BEHAVIOR OF PARTICLES.



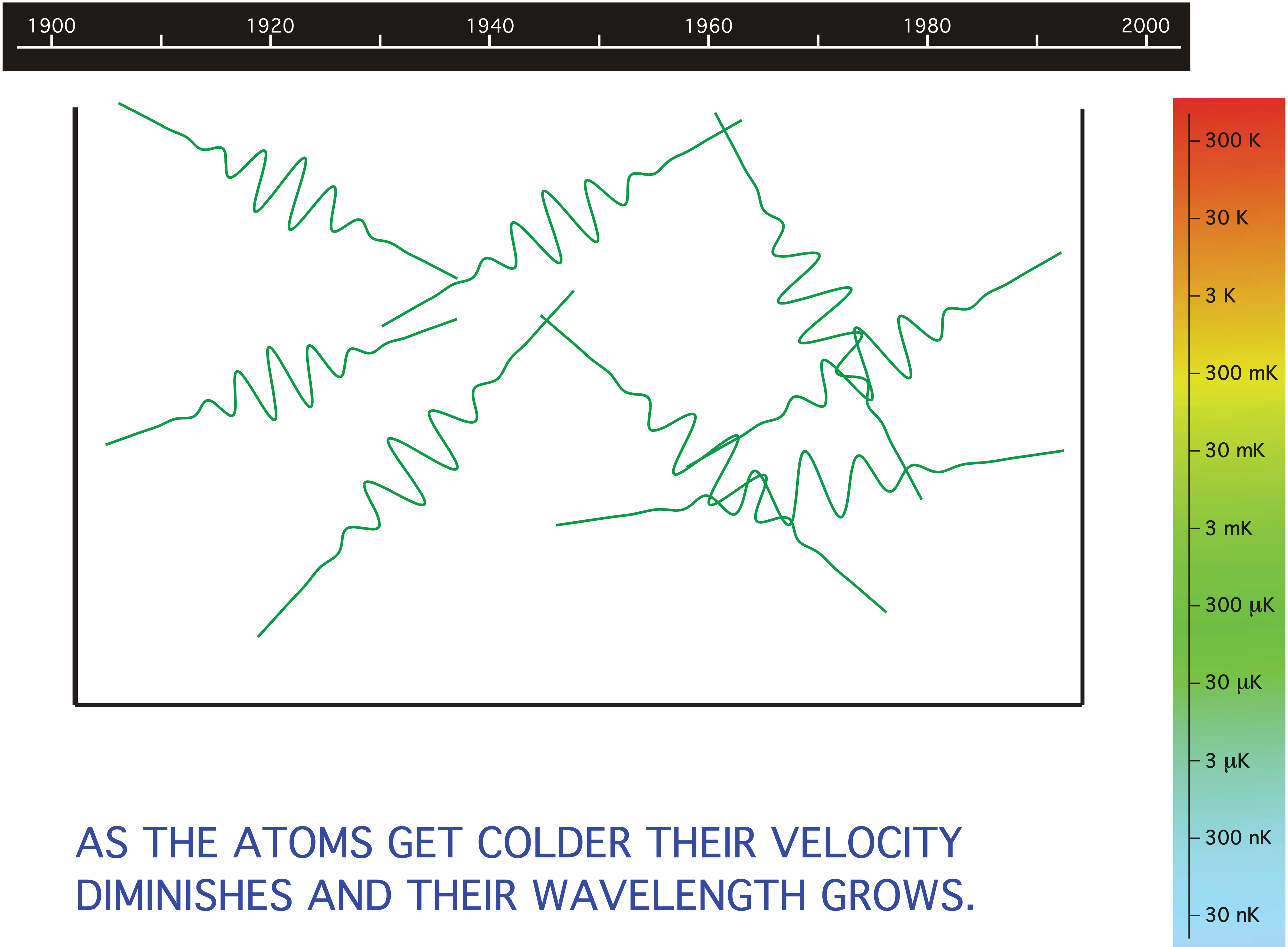
1921
"for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect"



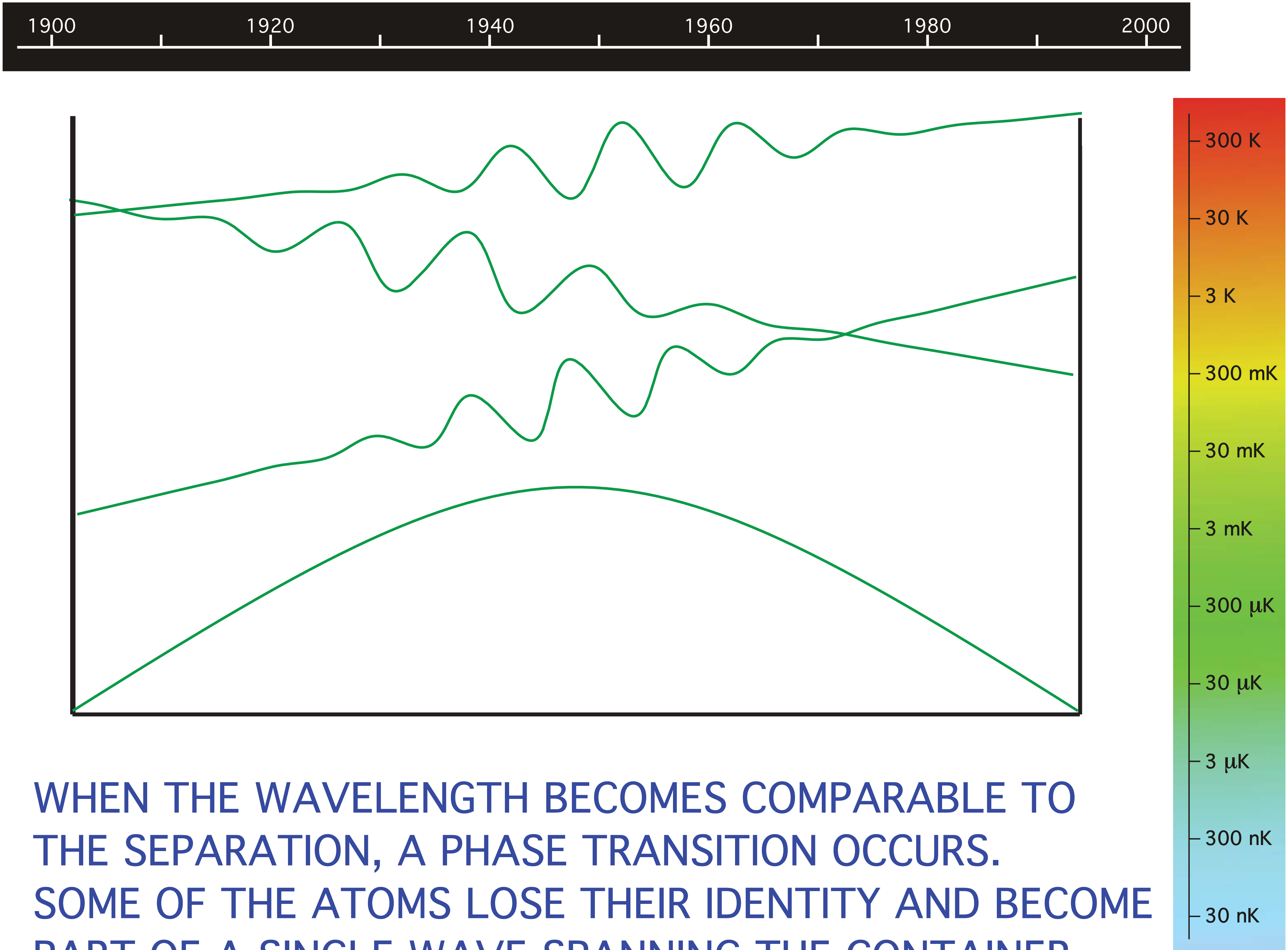




THEY APPLIED THEIR THEORY TO THE SIMPLEST POSSIBLE CASE: A GAS OF <u>NON-INTERACTING</u> ATOMS

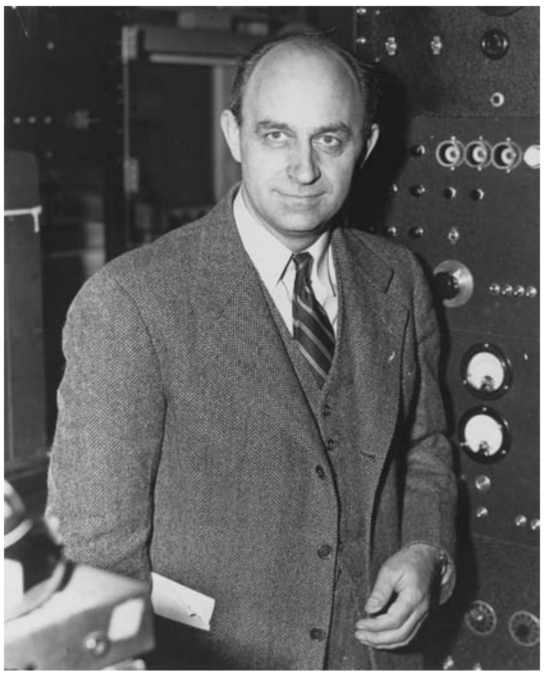


AS THE ATOMS GET COLDER THEIR VELOCITY DIMINISHES AND THEIR WAVELENGTH GROWS.



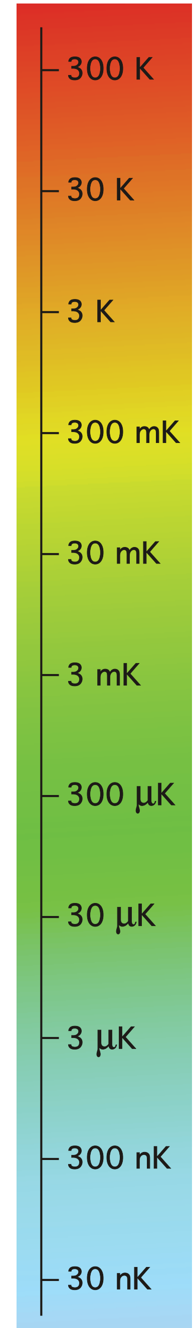
WHEN THE WAVELENGTH BECOMES COMPARABLE TO THE SEPARATION, A PHASE TRANSITION OCCURS. SOME OF THE ATOMS LOSE THEIR IDENTITY AND BECOME PART OF A SINGLE WAVE SPANNING THE CONTAINER.

HALF THE ATOMS IN THE WORLD FOLLOW THE RULES OF BOSE AND EINSTEIN AND ARE CALLED "BOSONS". THE OTHER HALF FOLLOW RULES SET OUT BY ENRICO FERMI AND PAUL DIRAC AND ARE CALLED "FERMIONS".



Enrico Fermi 1938
"for his demonstrations of the existence of new radioactive elements produced by neutron irradiation, and for his related discovery of nuclear reactions brought about by slow neutrons"

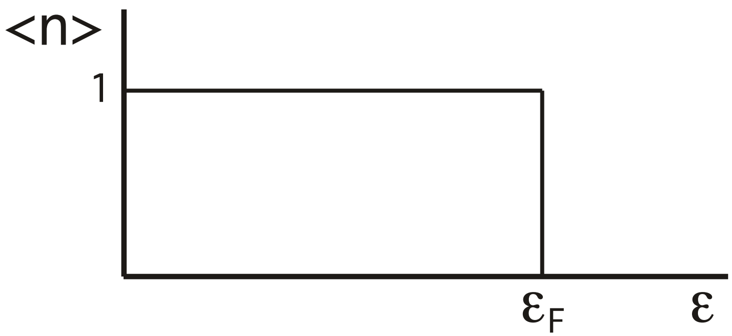
P.A.M. Dirac 1933 (with Erwin Schrodinger)
"for the discovery of new productive forms of atomic theory"

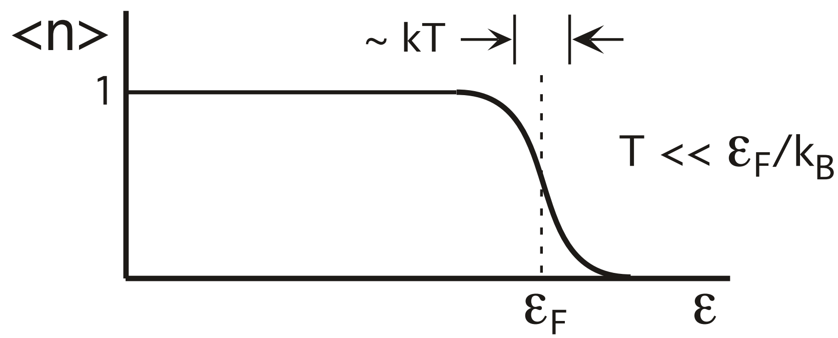


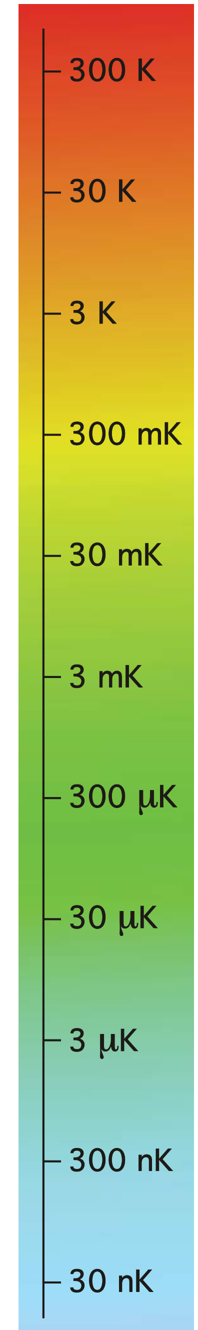
T = 0

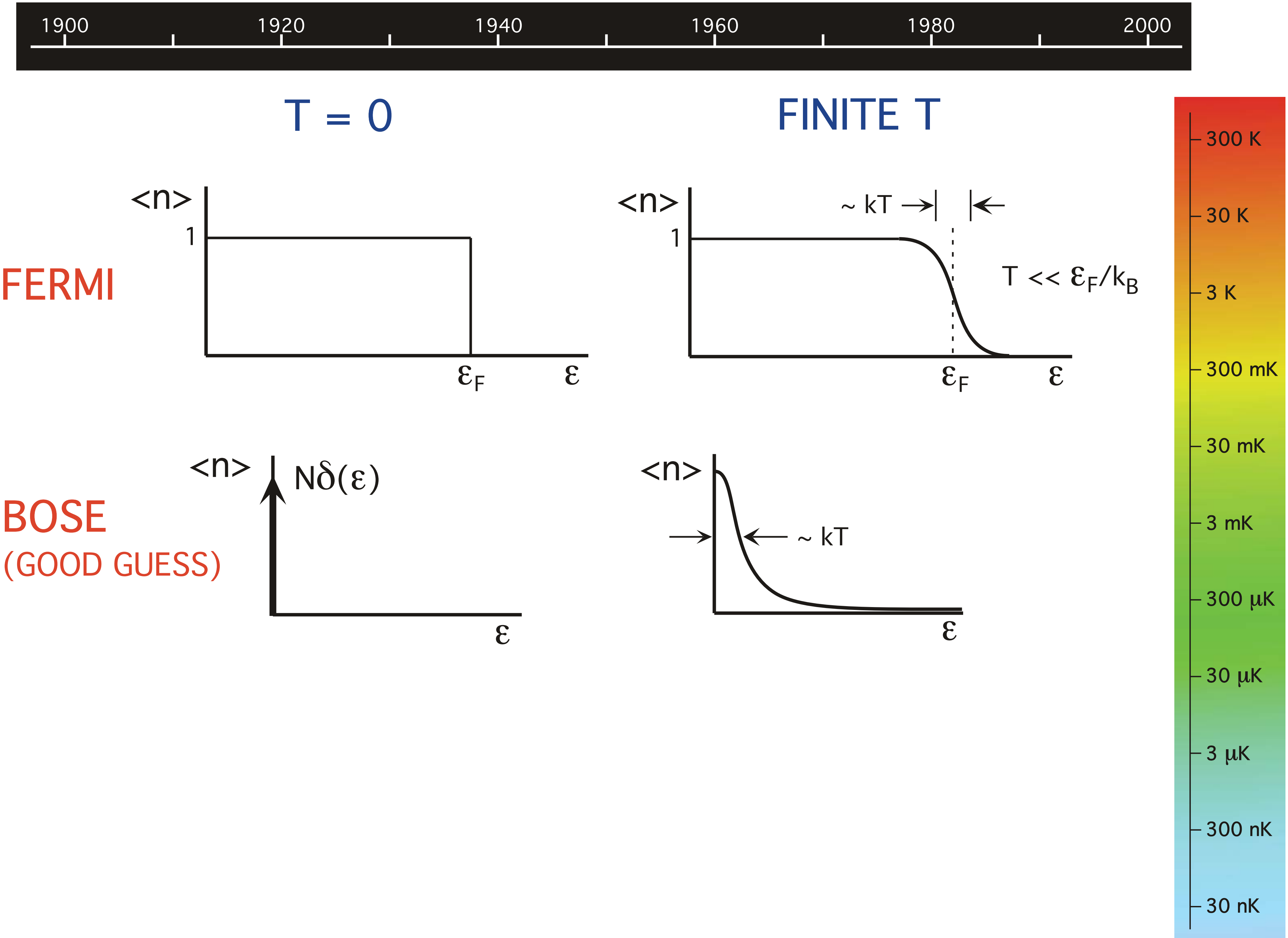
FINITE T

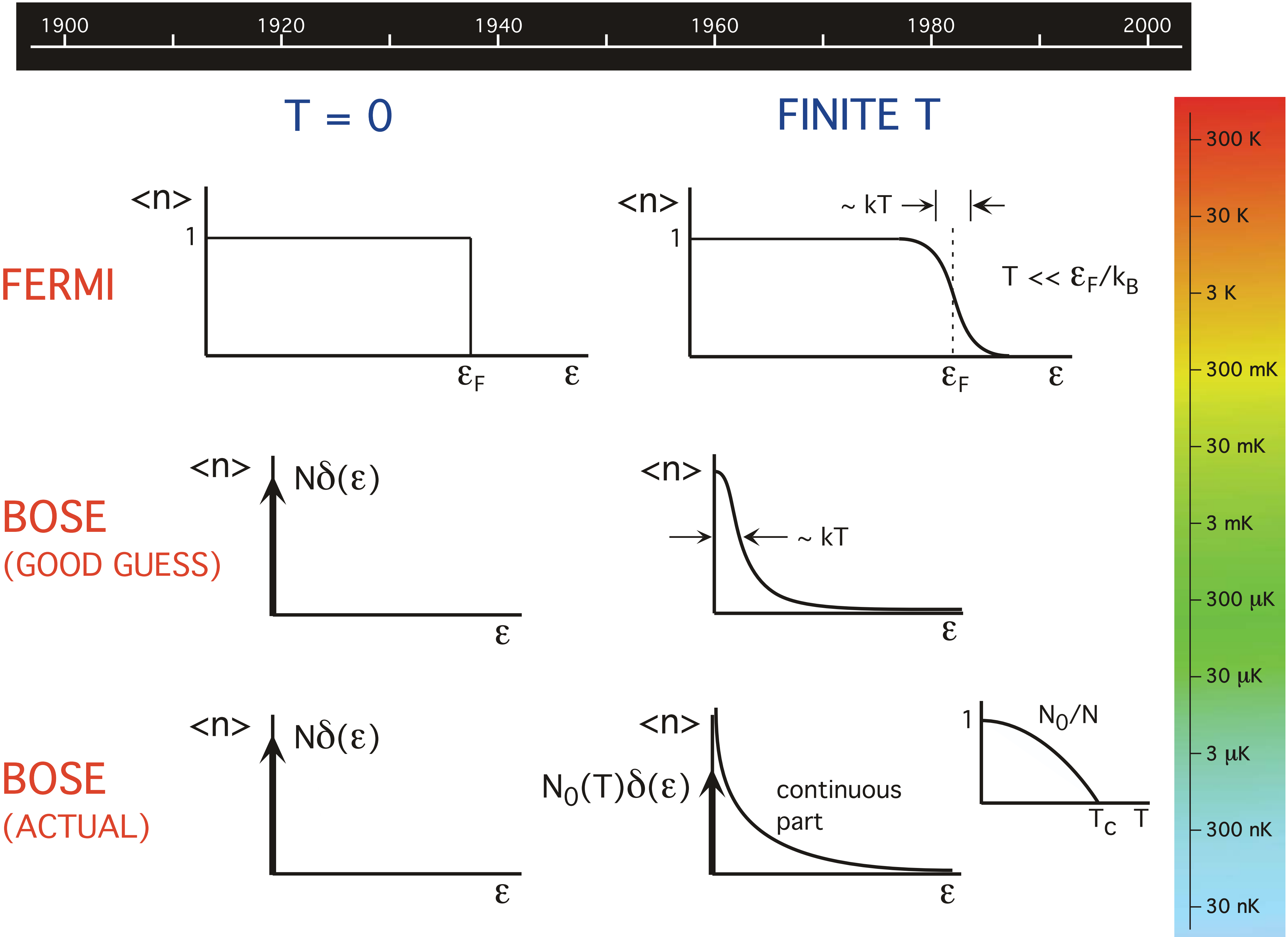
FERMI








1900
1920
1940
1960
1980
2000
T = 0
FINITE T
FERMI
<n>
1
ε_F
ε
<n>
1
~ kT
T << ε_F/k_B
ε_F
ε
BOSE
(GOOD GUESS)
<n>
Nδ(ε)
ε
<n>
~ kT
ε
300 K
30 K
3 K
300 mK
30 mK
3 mK
300 μK
30 μK
3 μK
300 nK
30 nK



1900
1920
1940
1960
1980
2000
T = 0
FINITE T
FERMI
<n>
1
ε_F
ε
<n>
1
~ kT
$T \ll \varepsilon_F/k_B$
ε_F
ε
BOSE
(GOOD GUESS)
<n>
$N\delta(\varepsilon)$
ε
<n>
~ kT
ε
BOSE
(ACTUAL)
<n>
$N\delta(\varepsilon)$
ε
<n>
$N_0(T)\delta(\varepsilon)$
continuous
part
ε
1
N_0/N
T_c
T
300 K
30 K
3 K
300 mK
30 mK
3 mK
300 μK
30 μK
3 μK
300 nK
30 nK

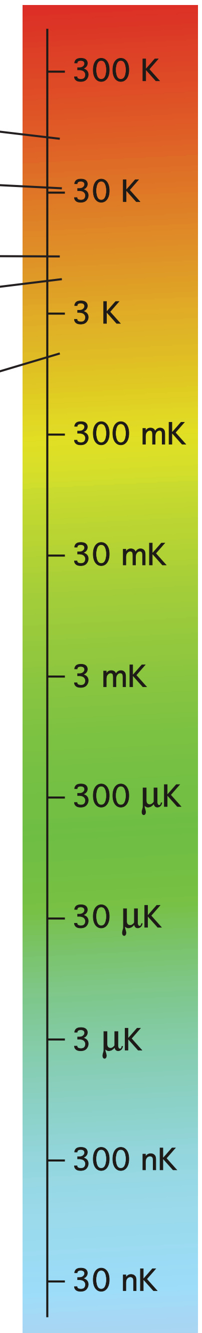
O_2 LIQUEFIES AT 90K

O_2 FREEZES AT 50K

H_2 LIQUEFIES AT 20K

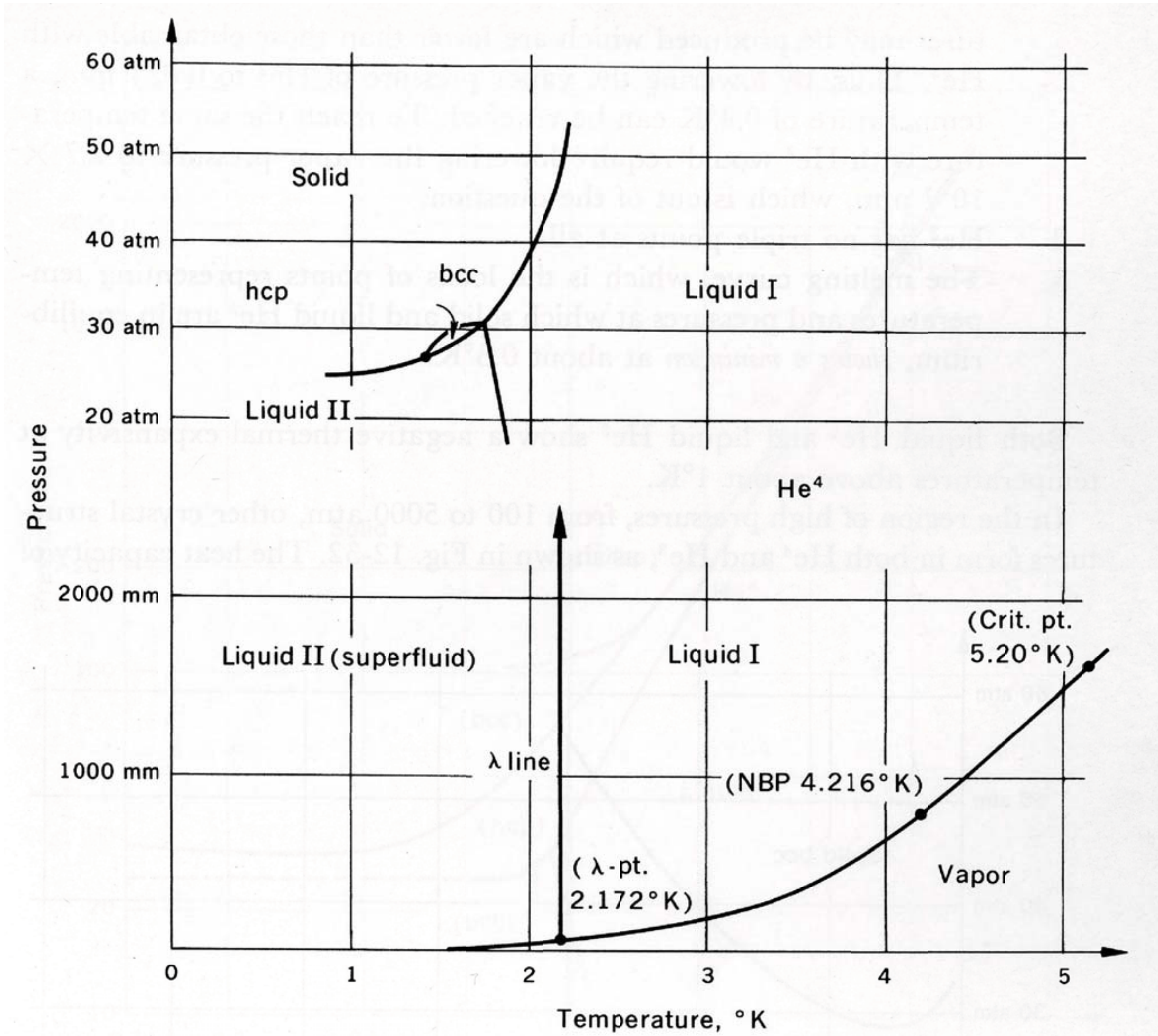
H_2 FREEZES AT 14K

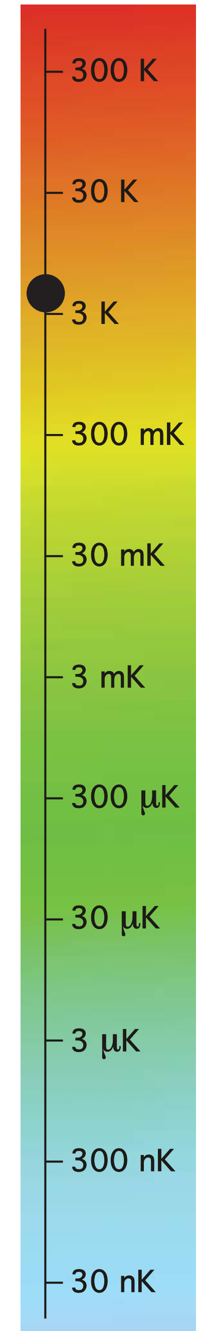
He LIQUEFIES AT 4K



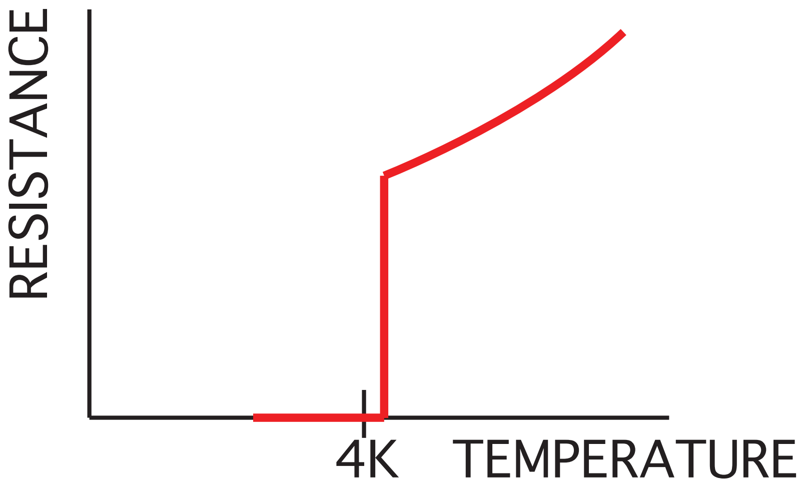
REAL ATOMS OR MOLECULES DO INTERACT WITH EACH OTHER AND UNDERGO LIQUEFICATION AND FREEZING DUE TO THESE INTERACTIONS.

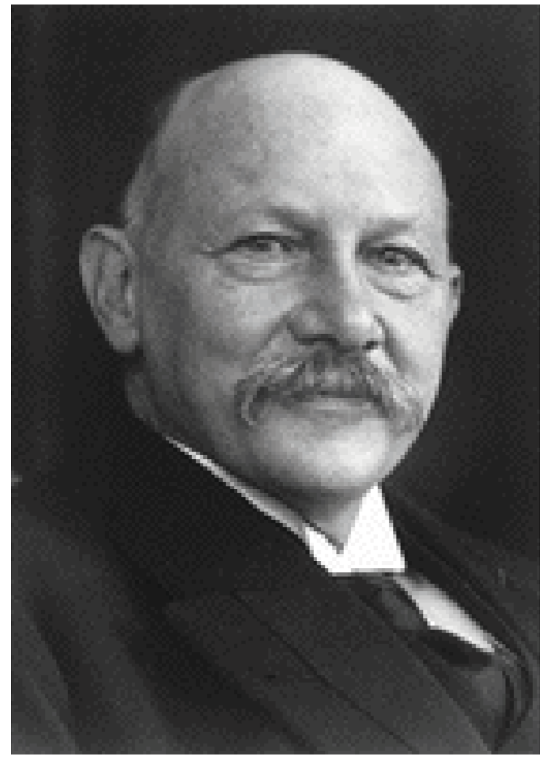
TEMPERATURES BELOW 4.2 K CAN BE ACHIEVED IN ^{4}He BY PUMPING ON THE VAPOR ABOVE THE LIQUID



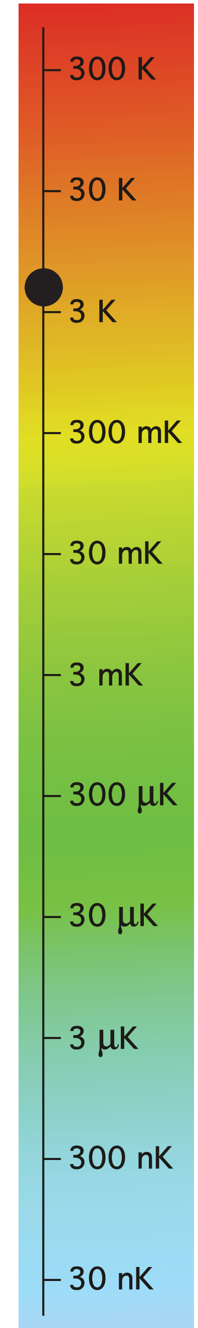


SUPERCONDUCTIVITY WAS DISCOVERED BY KAMERLINGH ONNES IN 1911.



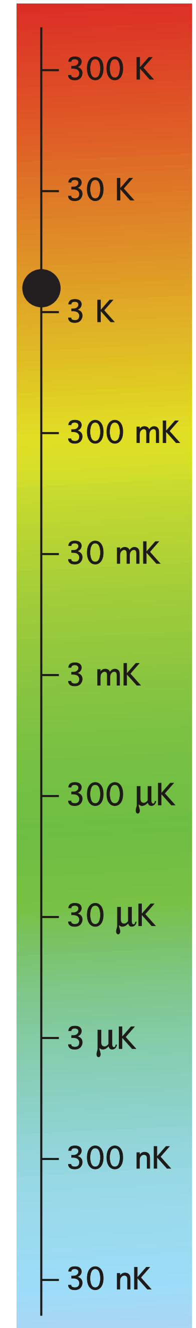


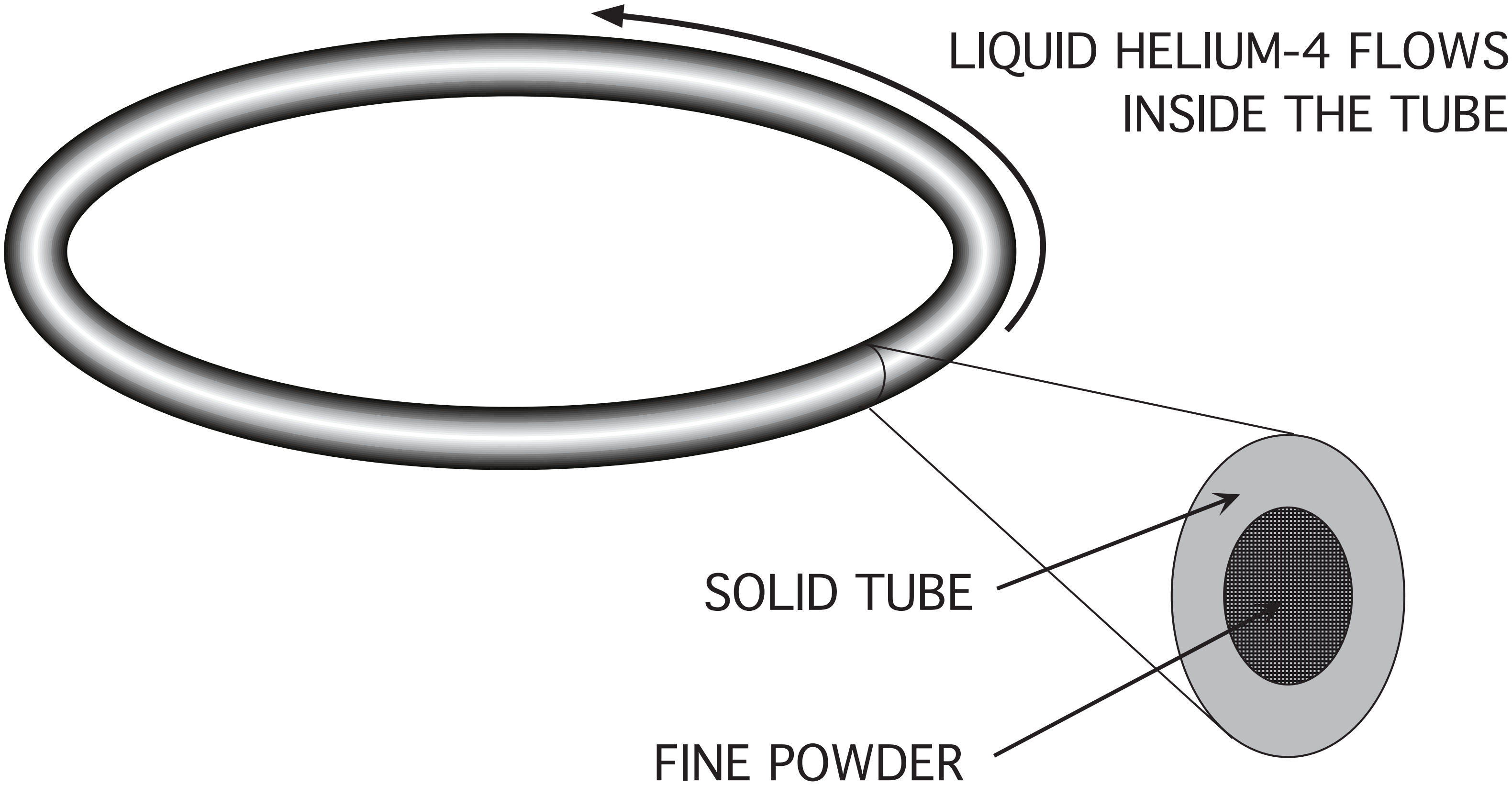
1913
"for his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium"

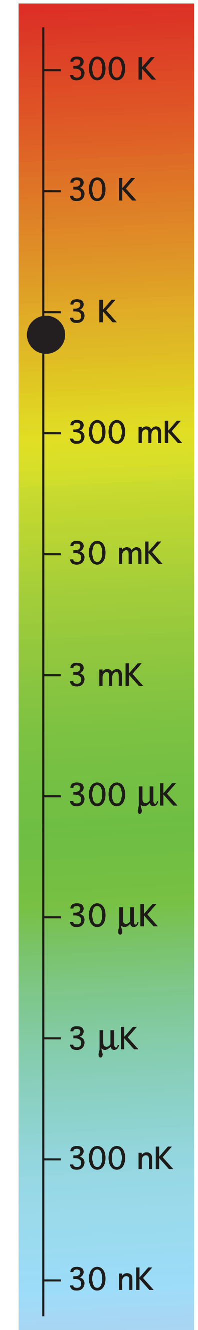


The Nobel Prize in Physics 1972

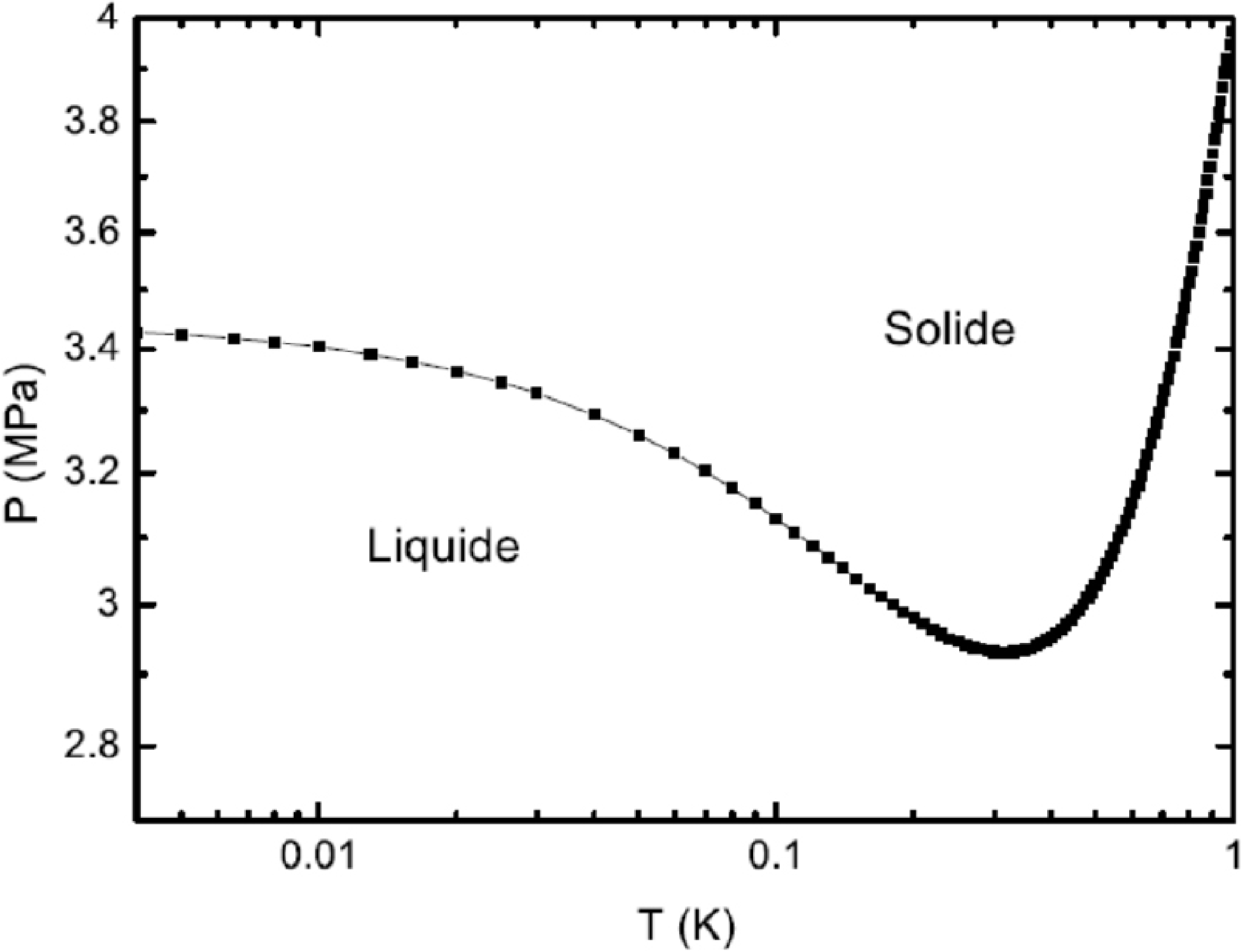
The Nobel Prize in Physics 1972 was awarded jointly to John Bardeen, Leon Neil Cooper and John Robert Schrieffer "*for their jointly developed theory of superconductivity, usually called the BCS-theory*".





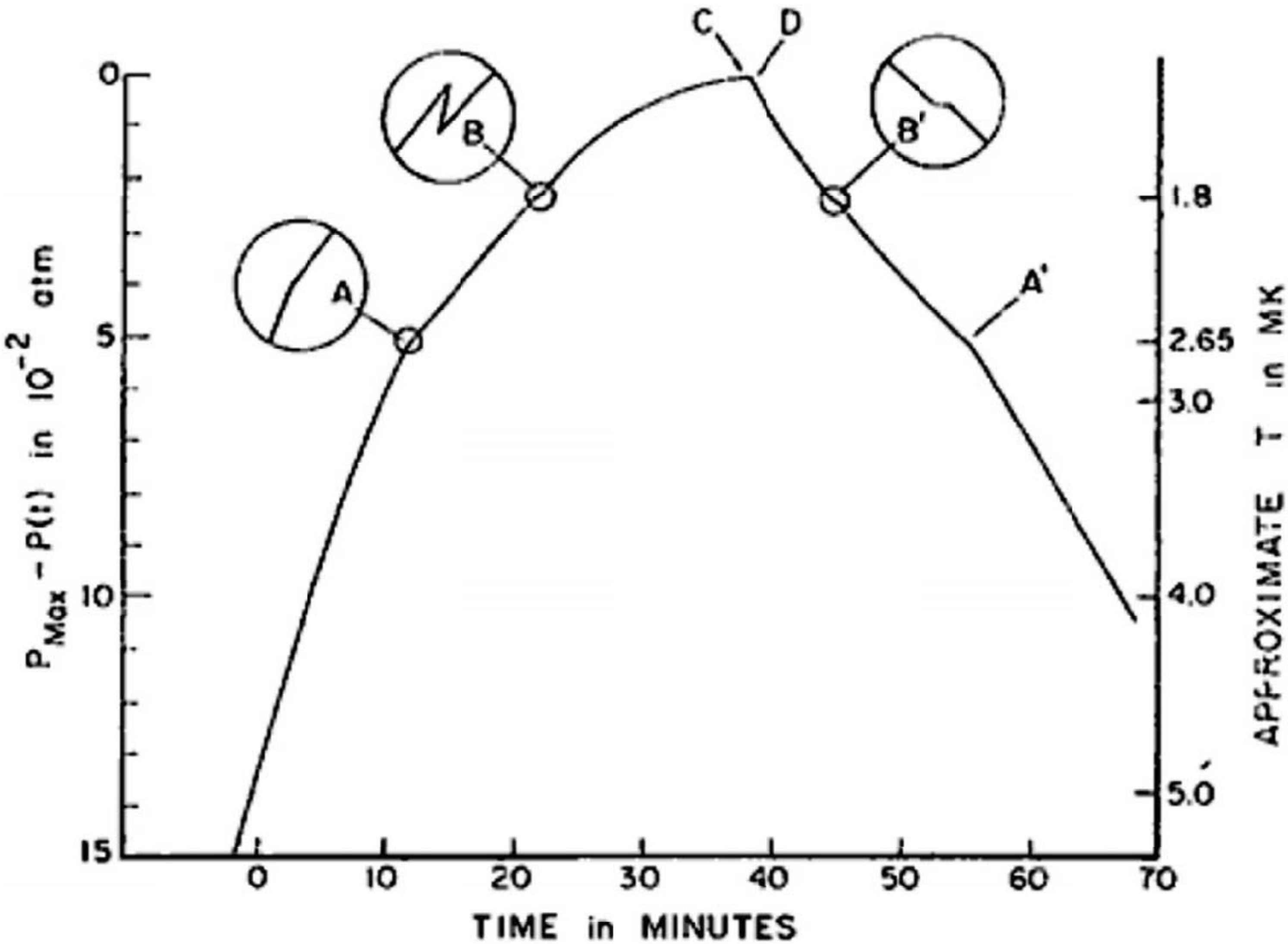


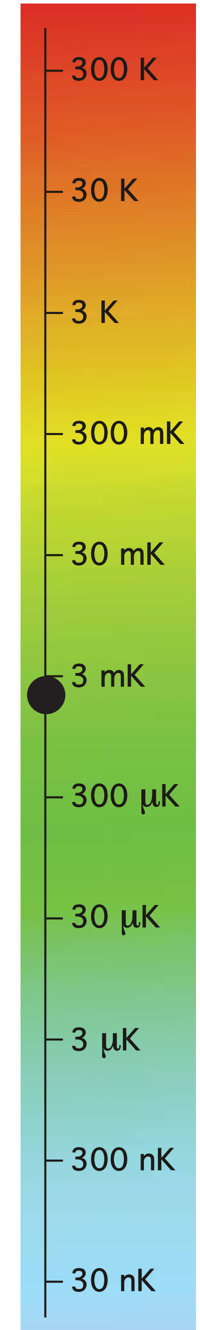
SUPERFLUIDITY WAS DISCOVERED IN HELIUM-4 IN THE 1930s AT A TEMPERATURE OF 2 KELVIN.



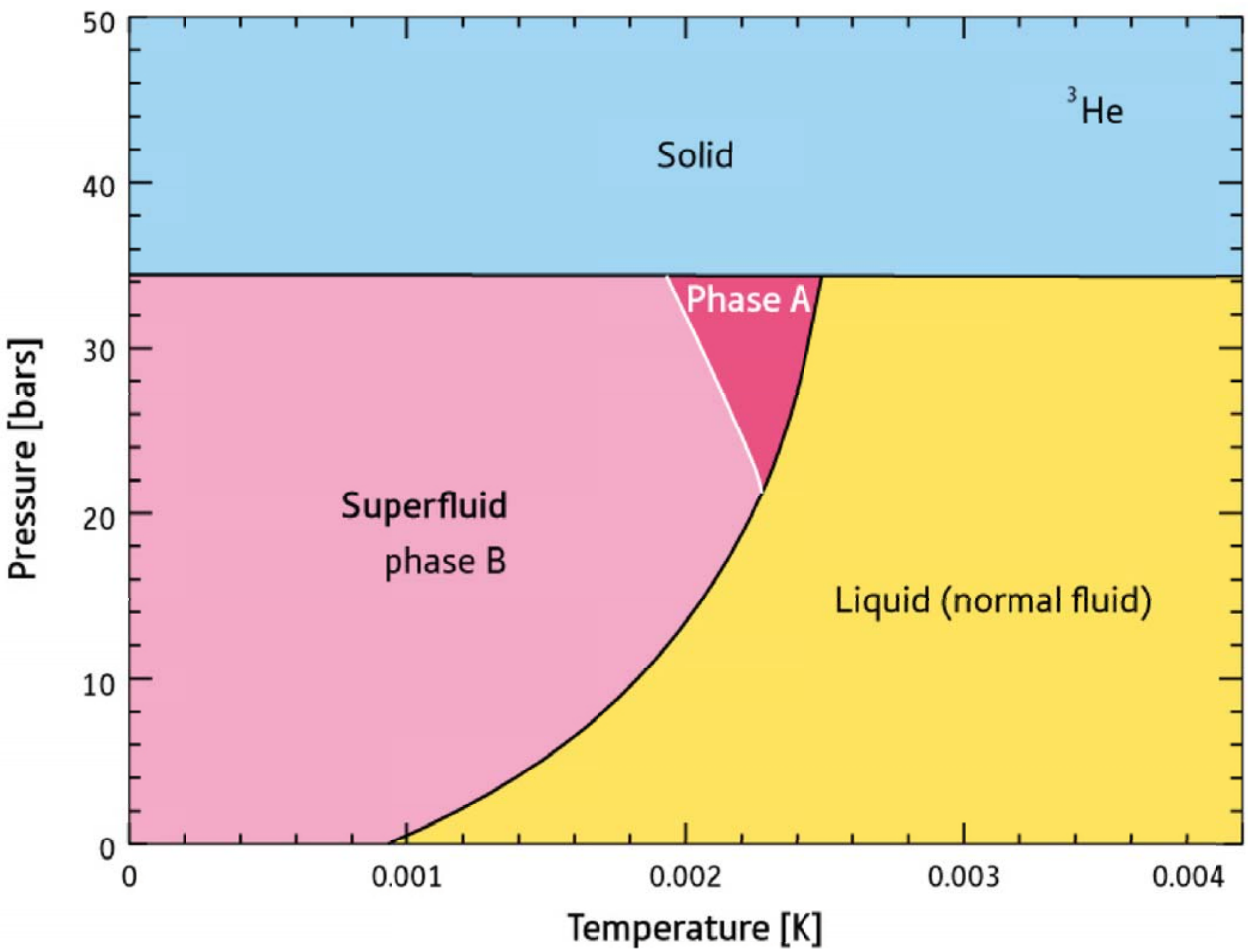
4
3.8
3.6
3.4
3.2
3
2.8
P (MPa)
Solide
Liquide
0.01
0.1
1
T (K)

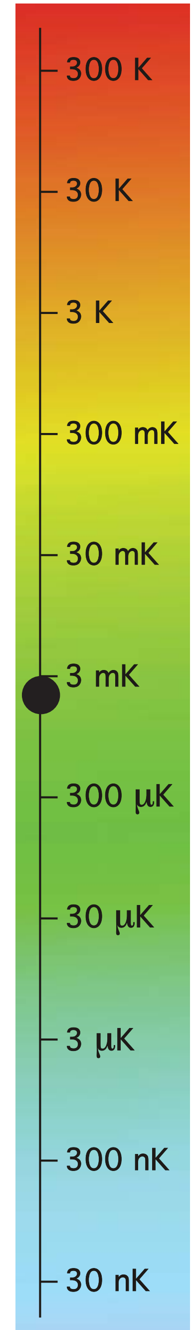
DOUG OSHEROFF SEES STRANGE GLITCHES ON THE MELTING CURVE OF ^3He





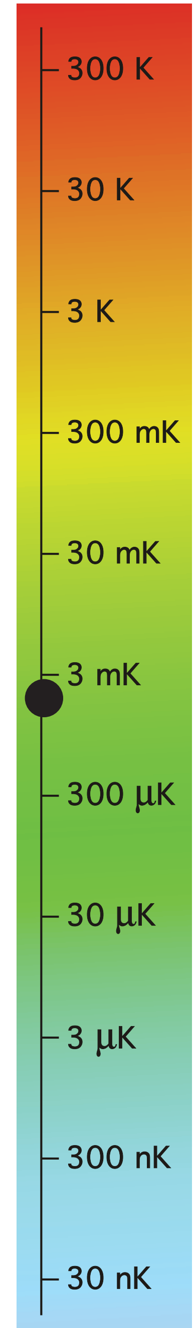
WHICH TURNED OUT TO BE TWO DIFFERENT SUPERFLUID PHASES OF ^3He



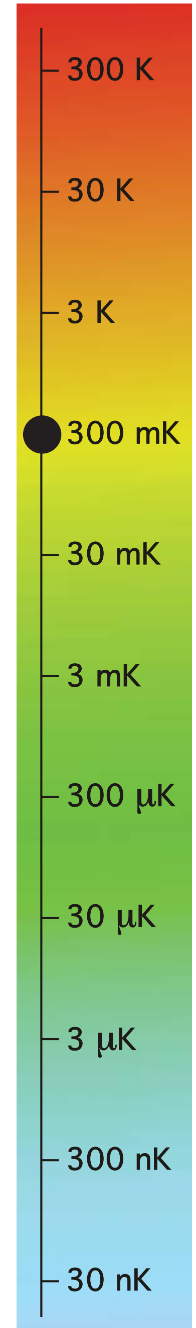


The Nobel Prize in Physics 1996

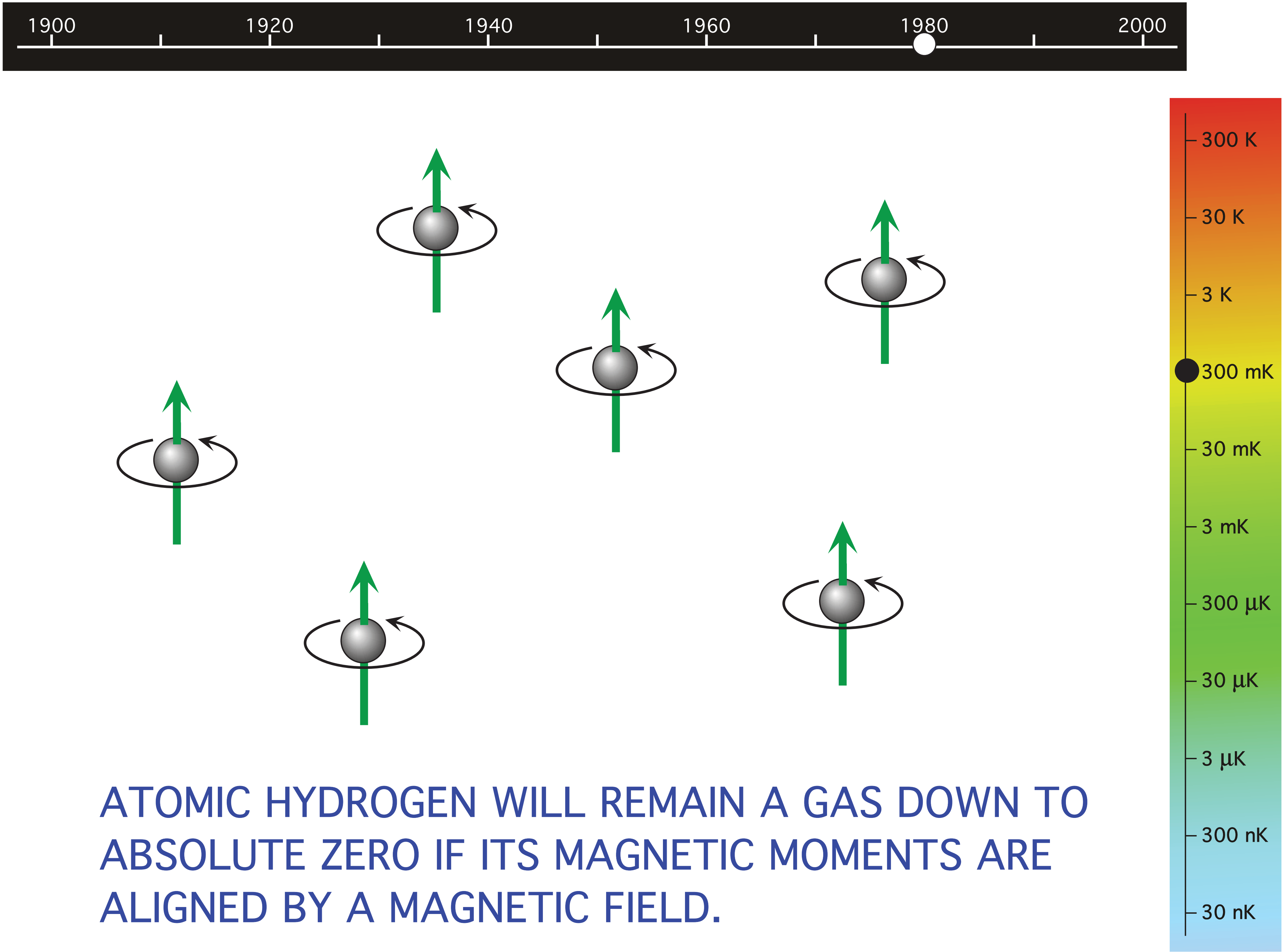
The Nobel Prize in Physics 1996 was awarded jointly to David M. Lee, Douglas D. Osheroff and Robert C. Richardson "*for their discovery of superfluidity in helium-3*".



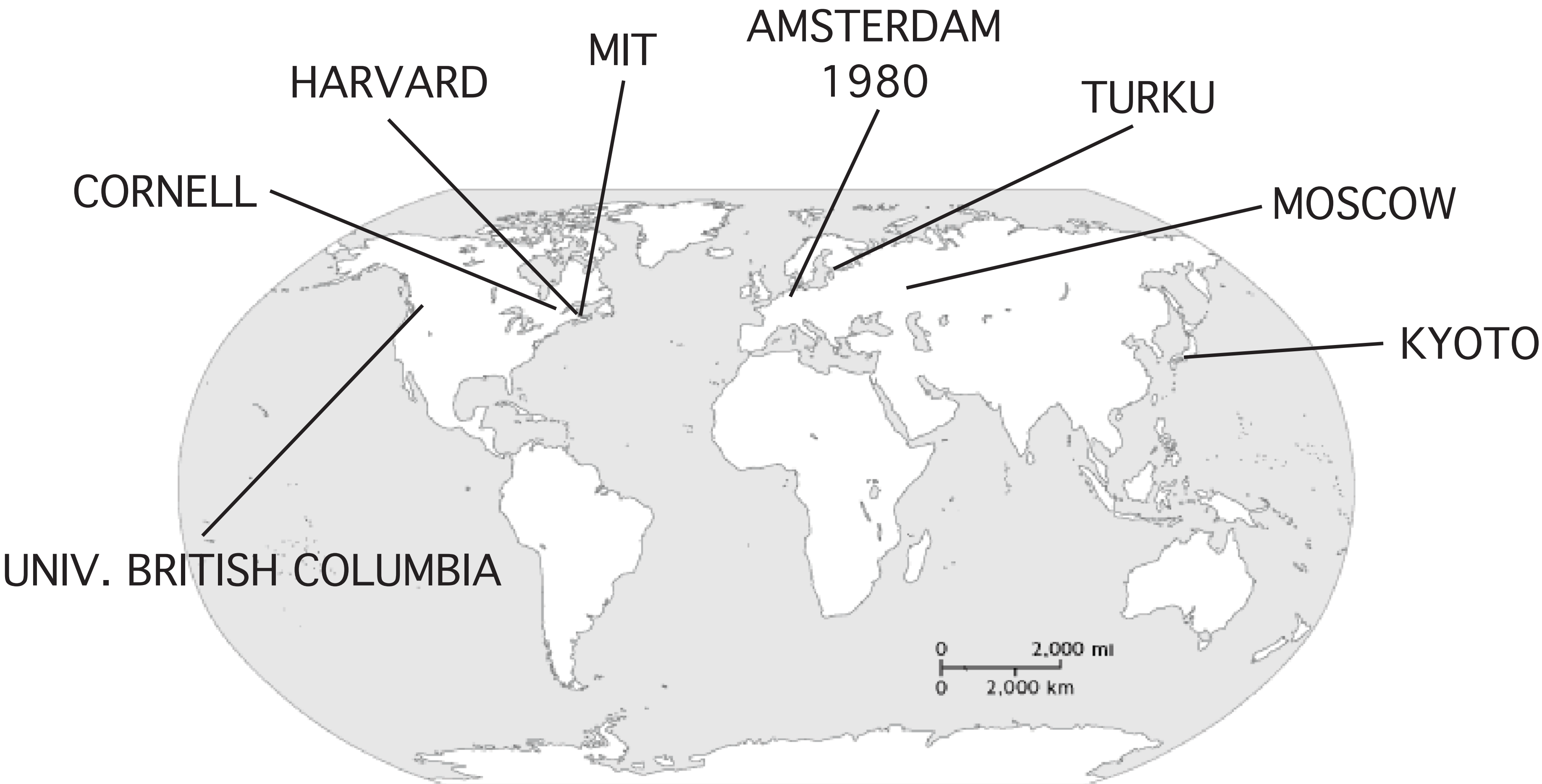
THE SEARCH BEGINS FOR BOSE-EINSTEIN CONDENSATION IN A REAL GAS

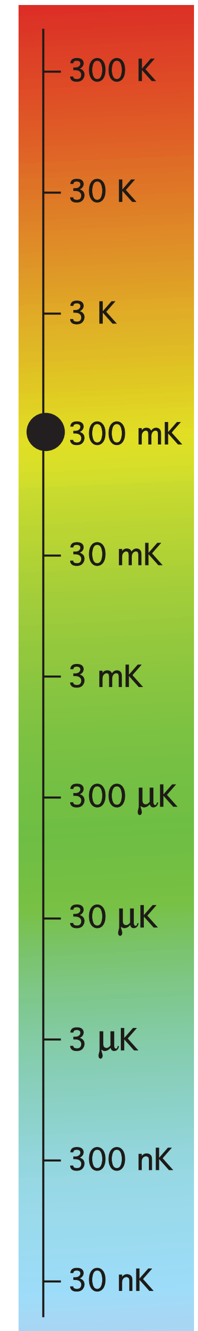


See the article *Possible "New" Quantum Systems*
In Physics Review Letters, Volume 36, Number 15 pages 910–913 (1976)

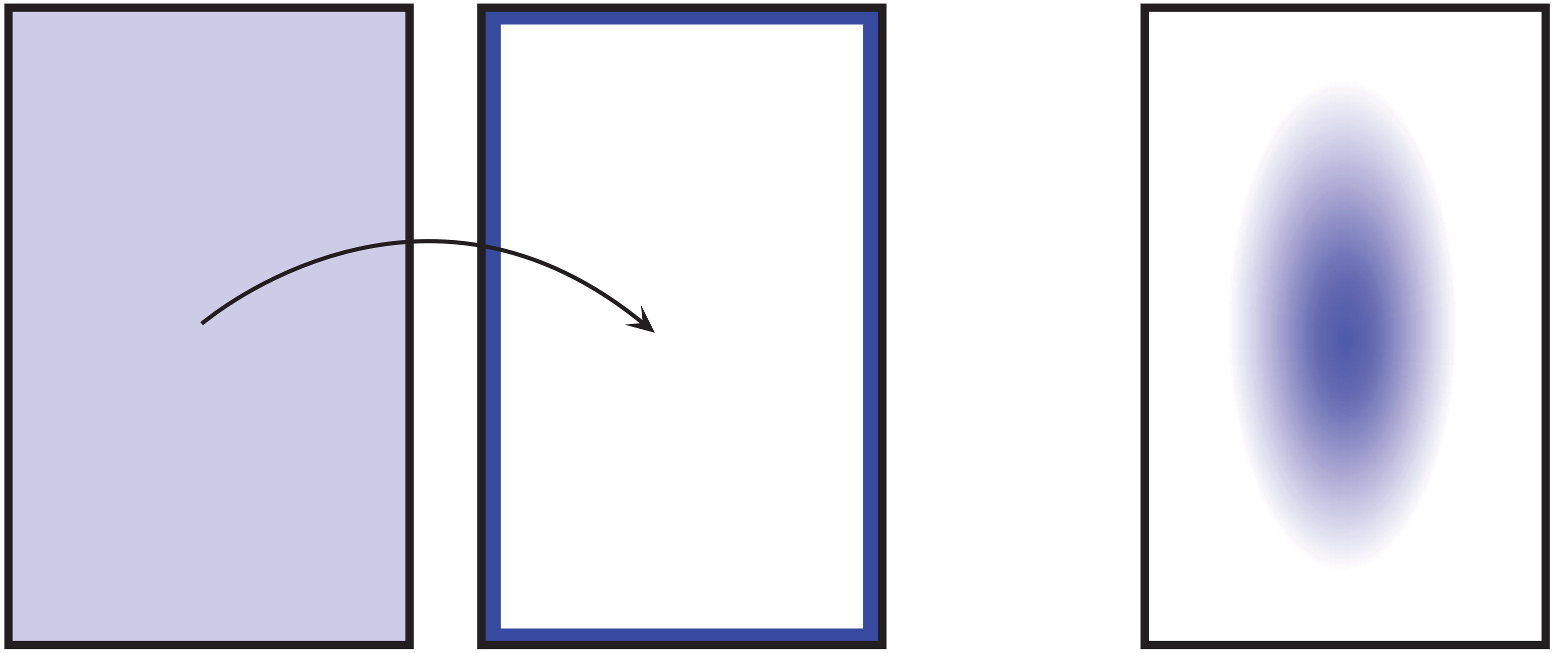


ATOMIC HYDROGEN WILL REMAIN A GAS DOWN TO ABSOLUTE ZERO IF ITS MAGNETIC MOMENTS ARE ALIGNED BY A MAGNETIC FIELD.



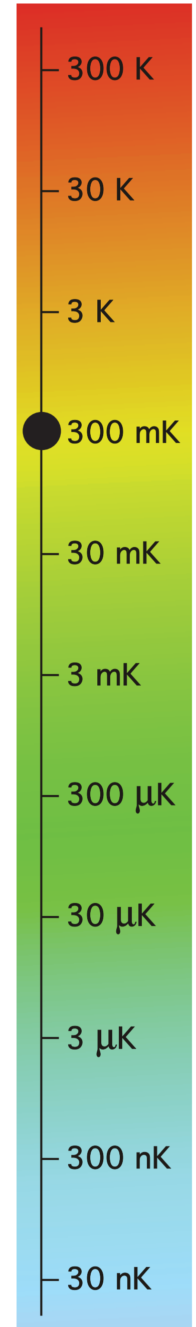


GROUPS WORKING ON SPIN-POLARIZED ATOMIC HYDROGEN



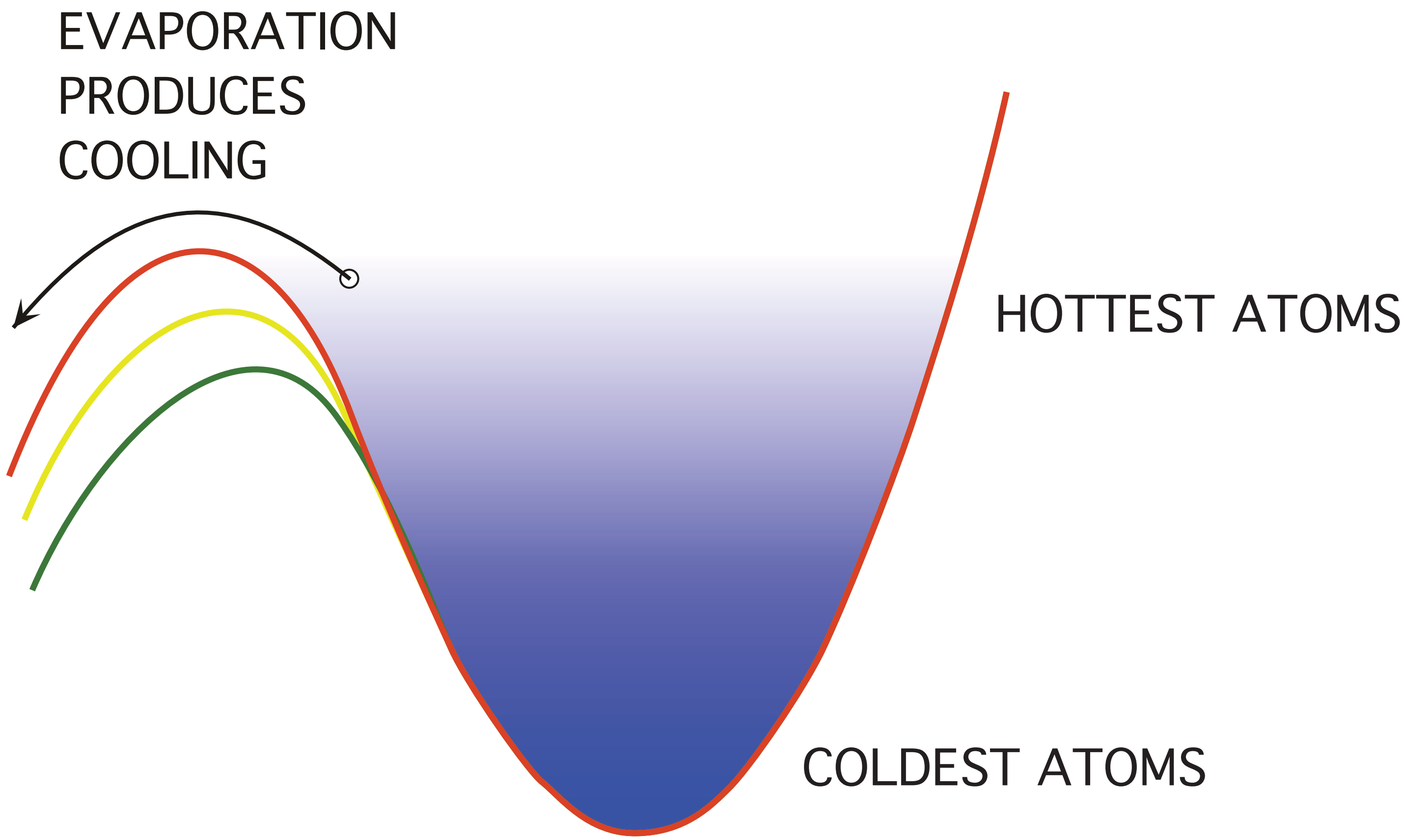
CONFINEMENT BY WALLS

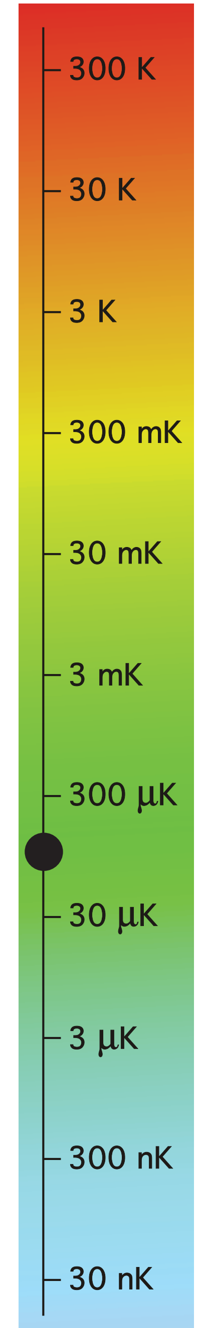
TRAPPING BY A MAGNETIC FIELD



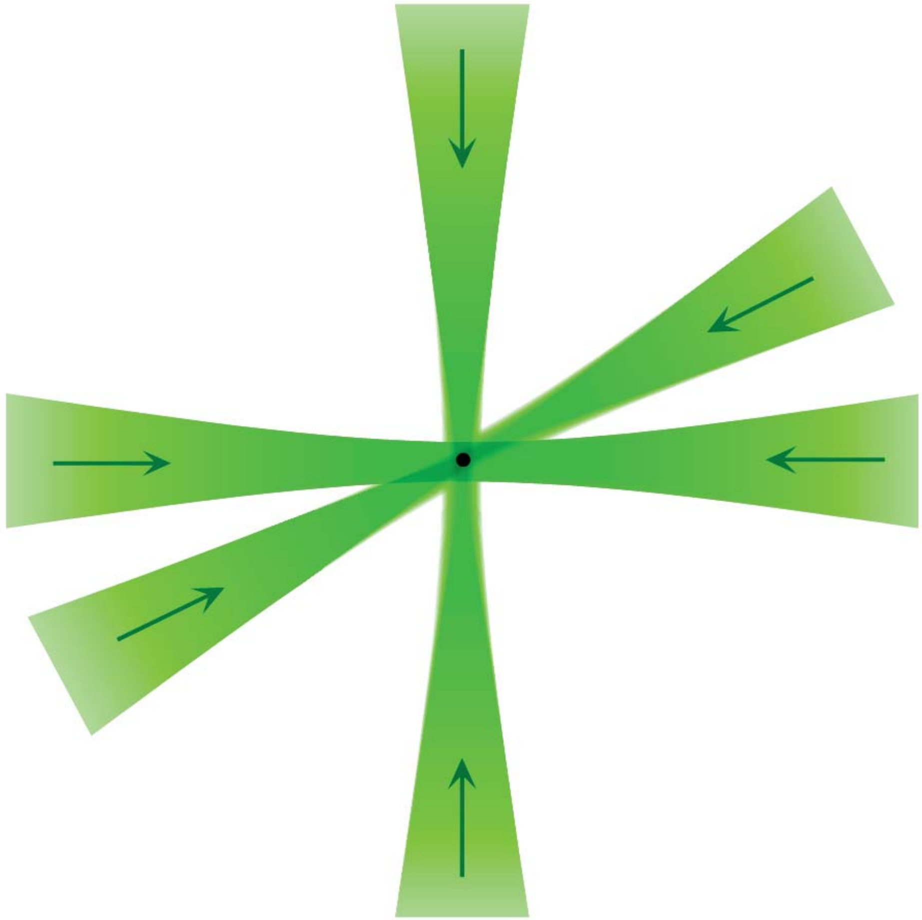
WALLS CAUSE THE MOMENTS TO FLIP; THEN THE ATOMS RECOMBINE INTO MOLECULES AND FREEZE OUT.

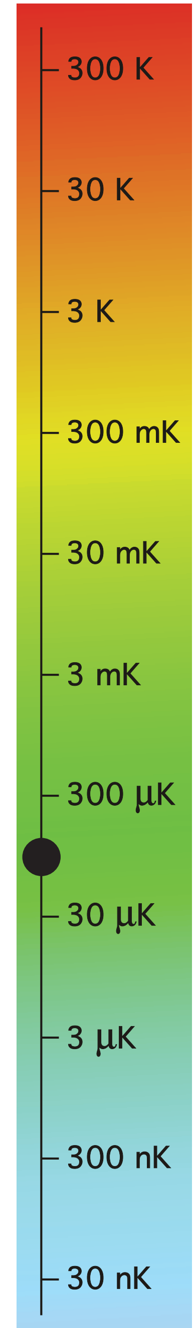
A MAGNETIC TRAP KEEPS THE ATOMS OFF THE WALLS.





IN 1986 HARALD HESS, A POSTDOCTORAL FELLOW IN MIT'S HYDROGEN GROUP, PROPOSES MAGNETIC TRAPPING AND EVAPORATIVE COOLING.

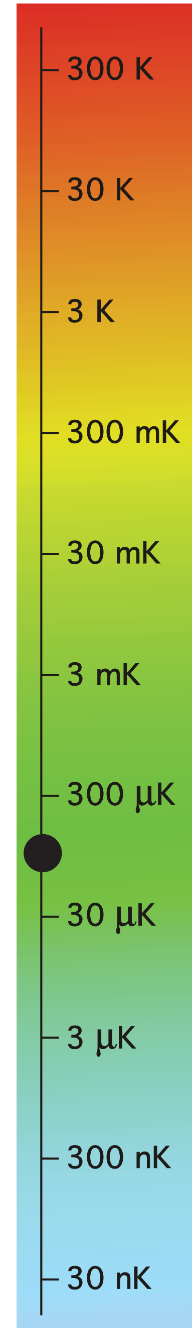




BEGINNING IN THE 1980s, METHODS WERE DEVELOPED TO COOL ATOMS INTO THE MICROKELVIN REGION OF TEMPERATURES USING LASERS.

The Nobel Prize in Physics 1997

The Nobel Prize in Physics 1997 was awarded jointly to Steven Chu, Claude Cohen-Tannoudji and William D. Phillips "for development of methods to cool and trap atoms with laser light".

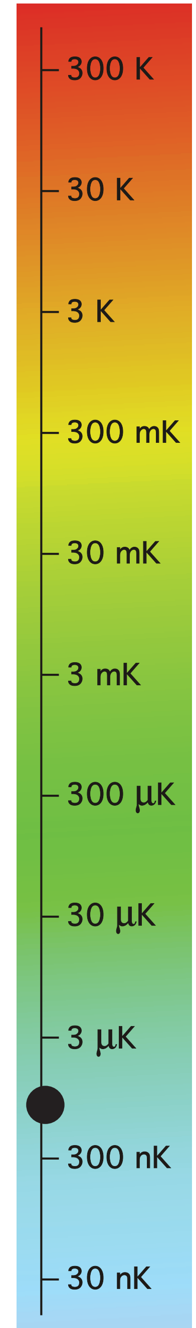


LASER COOLING WORKS BEST WITH CERTAIN ATOMS SUCH AS LITHIUM (Li), SODIUM (Na), and RUBIDIUM (Rb).

BUT LASER COOLING ALONE CAN NOT GET THESE ATOMS COLD ENOUGH TO ACHIEVE BEC.

FOR THE FINAL STAGE OF COOLING ONE MUST TURN TO EVAPORATIVE COOLING.

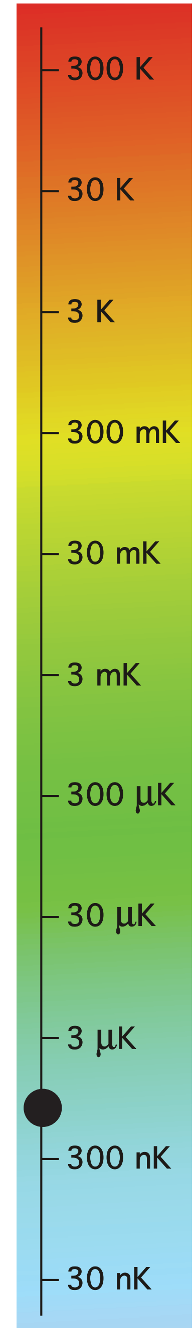
THEN THE RACE BEGAN: LOWER THE TEMPERATURE WHILE INCREASING THE DENSITY.



- ERIC CORNELL & CARL WIEMAN
  JILA (NIST AND UNIV. OF COLORADO)
  Rb

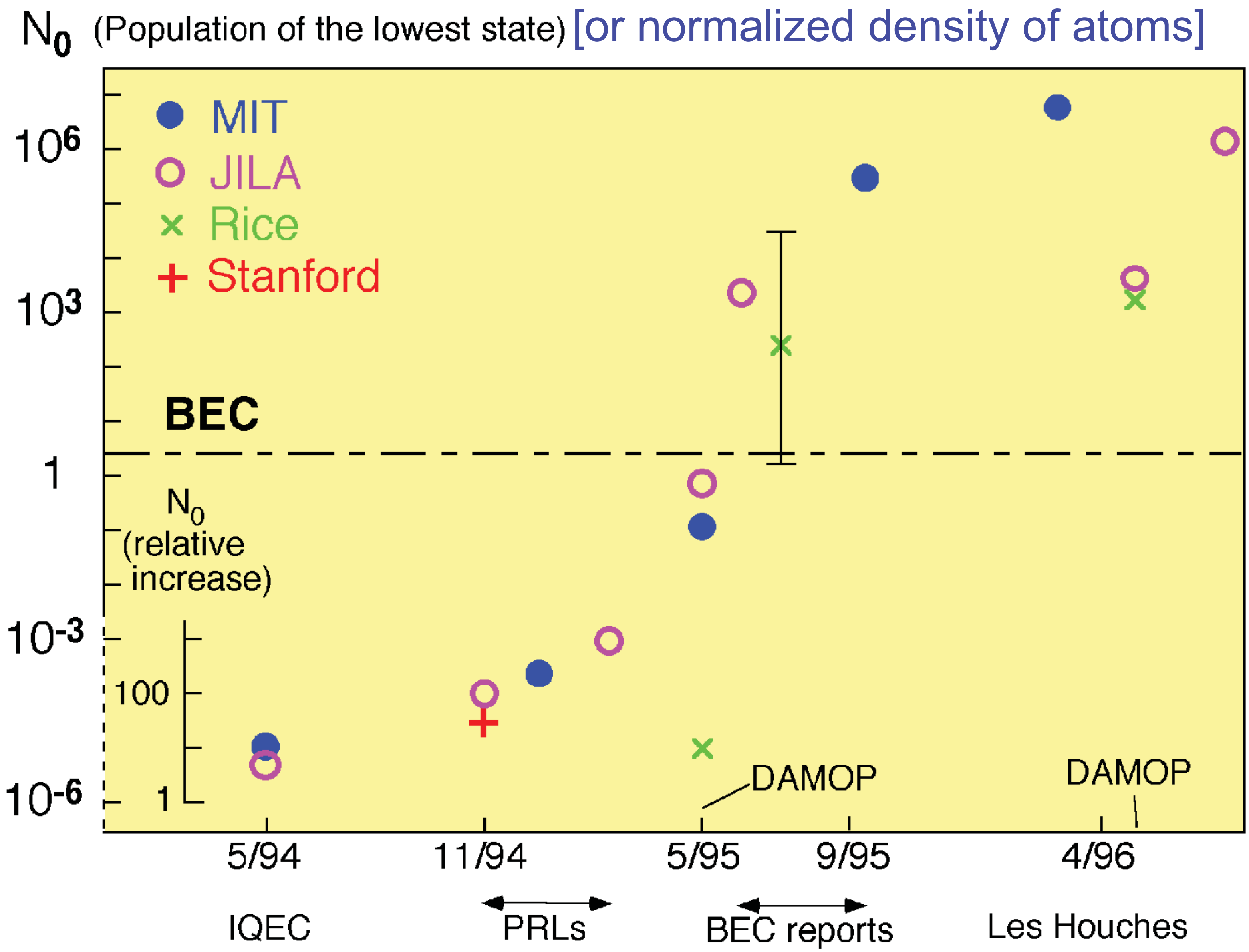
- RANDALL HULET
  RICE UNIVERSITY
  Li

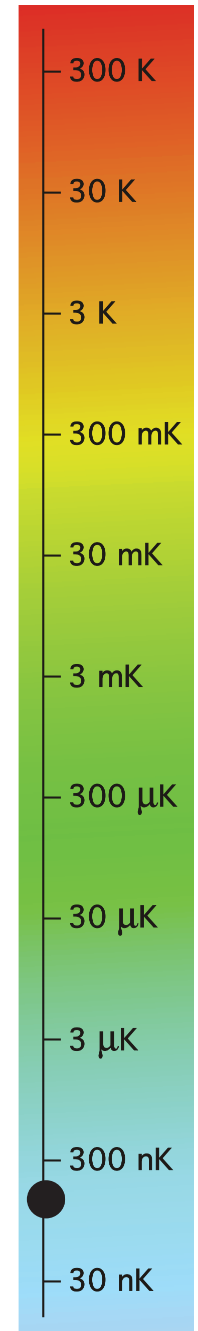
- WOLFGANG KETTERLE
  MIT
  Na

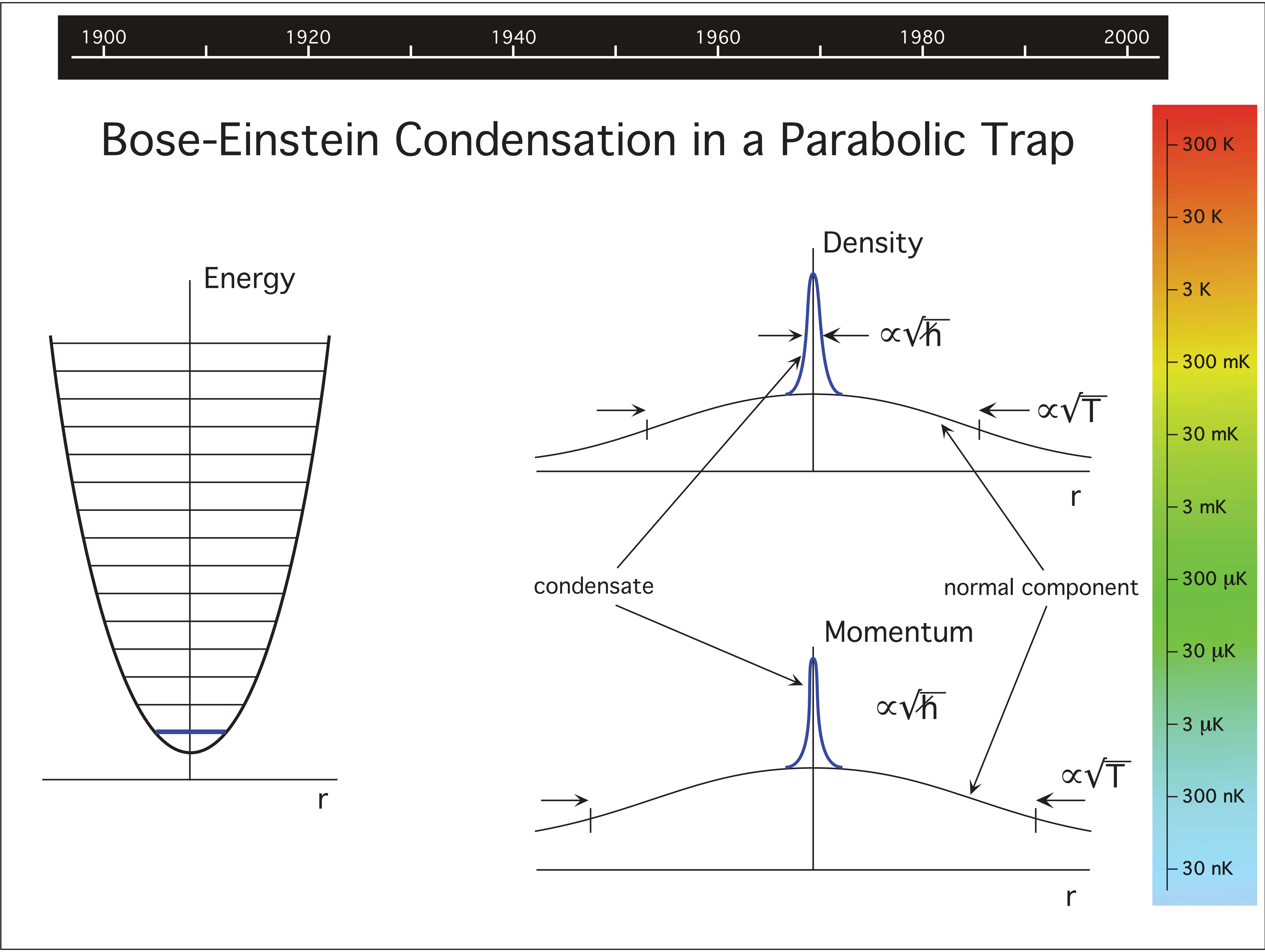


Race to BEC

Evaporative cooling of alkali atoms

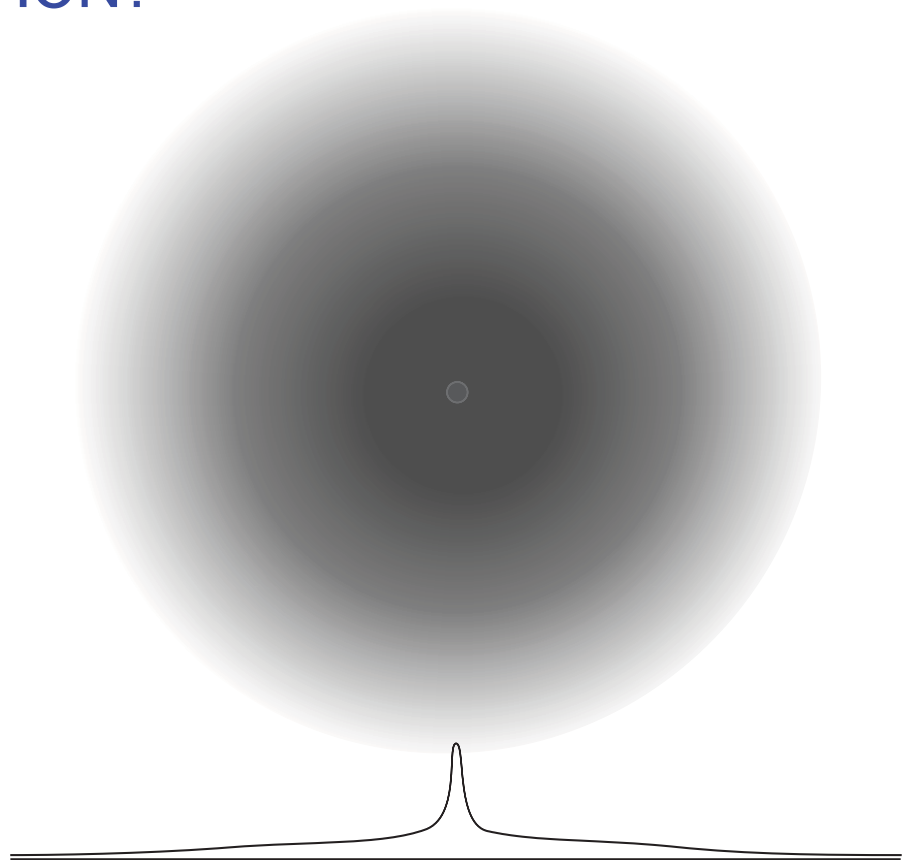






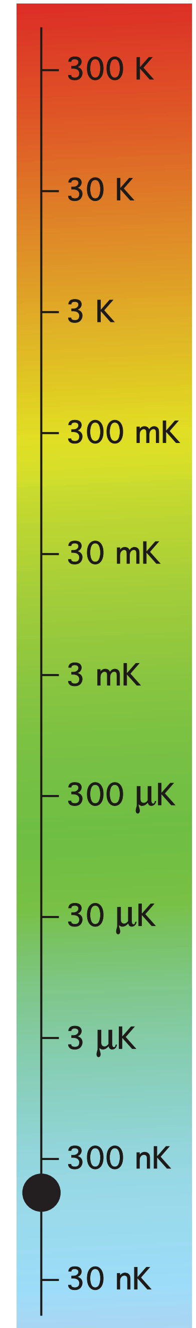
1900
1920
1940
1960
1980
2000
Bose-Einstein Condensation in a Parabolic Trap
Energy
r
Density
∝√ħ
∝√T
r
condensate
normal component
Momentum
∝√ħ
∝√T
r
300 K
30 K
3 K
300 mK
30 mK
3 mK
300 μK
30 μK
3 μK
300 nK
30 nK

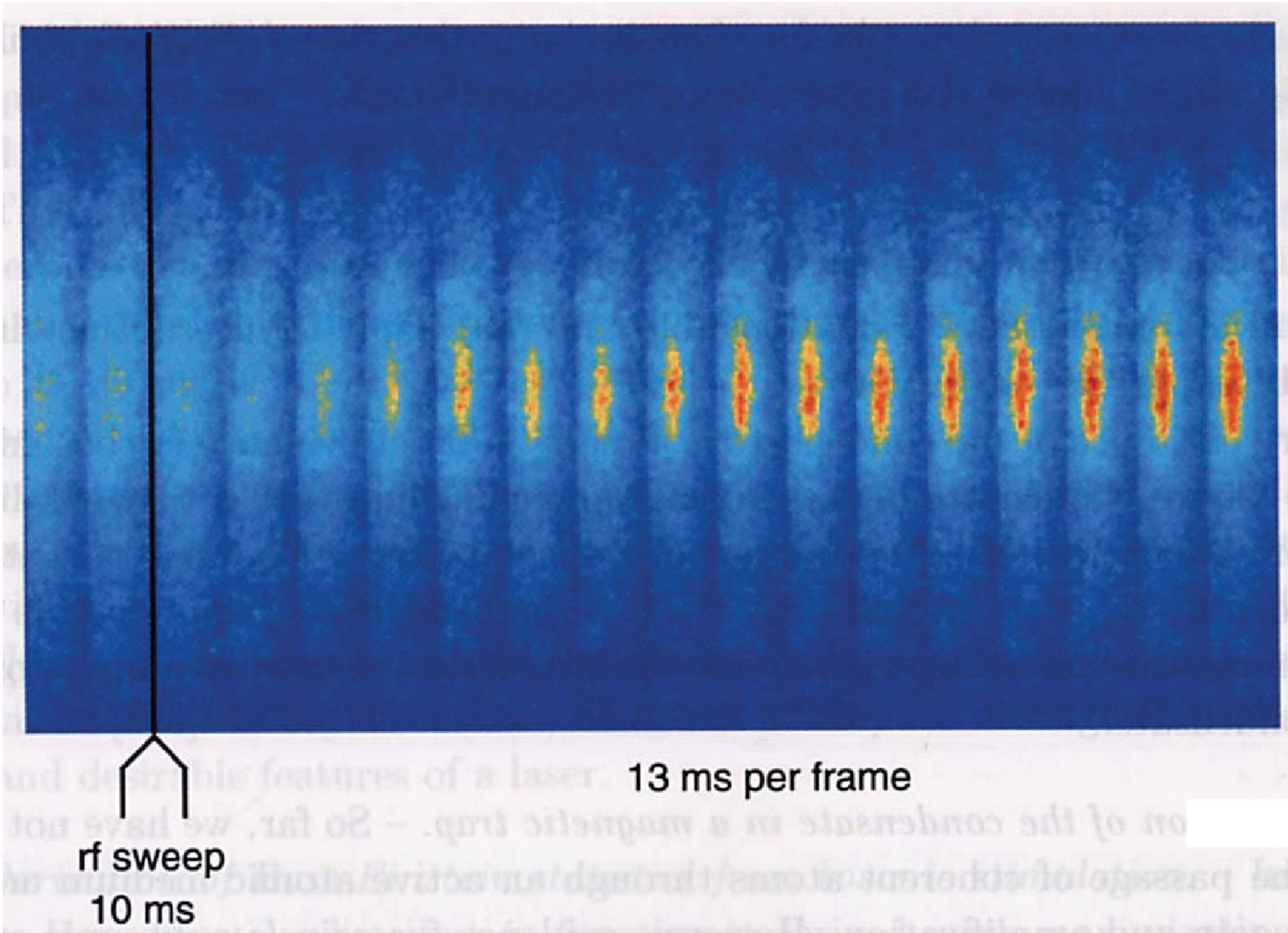
HOW DID THEY KNOW THEY HAD BOSE-EINSTEIN CONDENSATION?



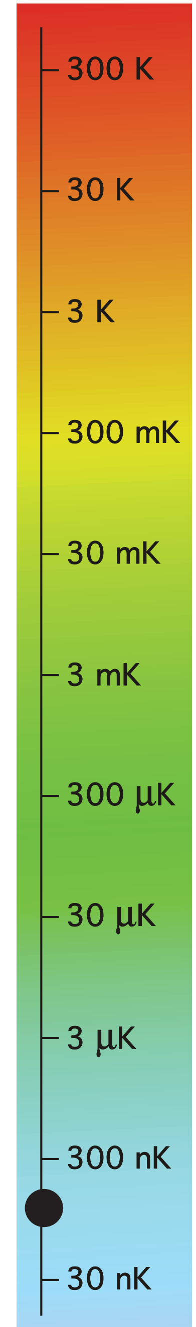
IN THE TRAP, ATOMS IN THE CONDENSATE ARE ALMOST AT REST, THE REMAINDER HAVE THERMAL SPEEDS.

WHEN THE TRAP IS TURNED OFF THE THERMAL ATOMS SPEED AWAY, BUT THE CONDENSATE ATOMS REMAIN NEAR THE ORIGIN.



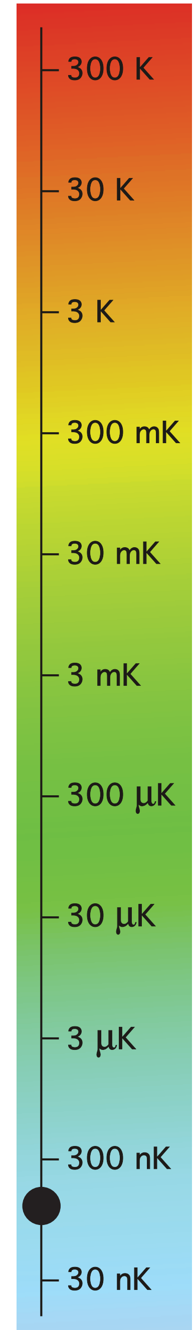


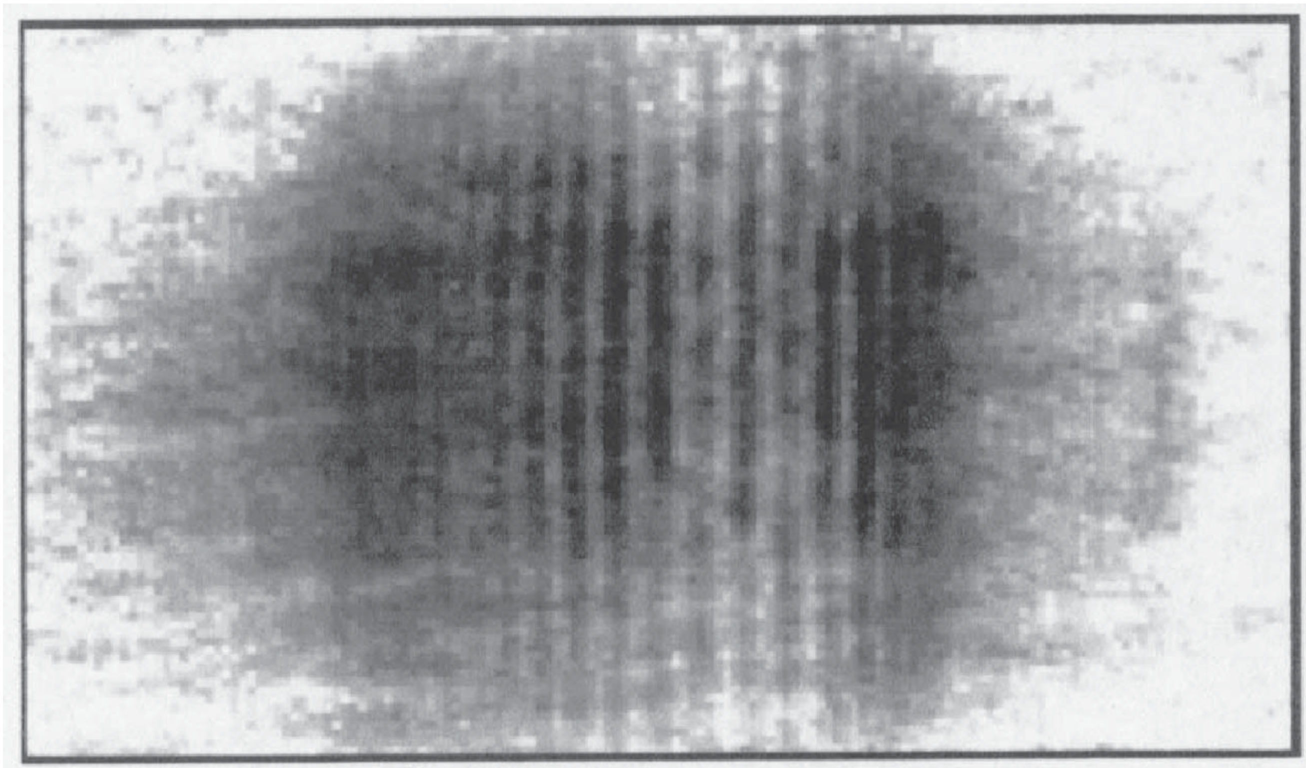
SUCCESSIVE REAL SPACE IMAGES OF A SODIUM CONDENSATE FORMING IN A KETTERLE TRAP

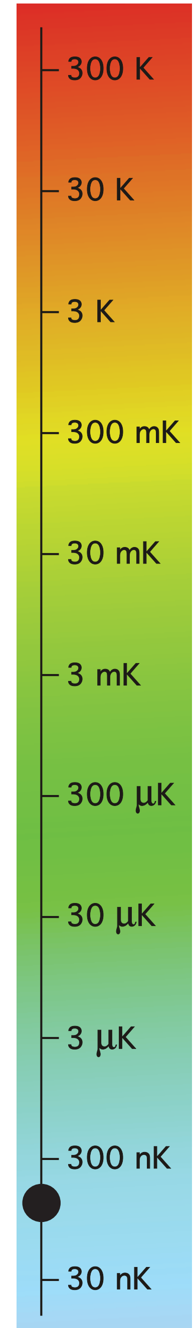


The Nobel Prize in Physics 2001

The Nobel Prize in Physics 2001 was awarded jointly to Eric A. Cornell, Wolfgang Ketterle and Carl E. Wieman "*for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates*".







INTERFERENCE OF MATTER WAVES