PROFESSOR: So this is our adiabatic change. So now we can say several things. OK, if omega is changing slowly, the energy is changing slowly, but do we have something that changes even more slowly, something that really almost doesn't change?

What you need here is basically-- this was a very important discovery in classical mechanics. You need like two things that change. Everything is going to change slowly, but then there's going to be one thing that changes slowly and another thing that changes slowly, and they change kind of in the same way in such a way that the ratio or some combination of them doesn't change almost at all. That's what we're trying to get.

Anybody knows in classical mechanics what quantity here doesn't change much? Nobody. No clue? It's not obvious what doesn't change much, but here is the claim. Claim is that the quantity that doesn't change much is, in fact, the energy divided by omega.

The energy will change slowly. Omega will change slowly. But the ratio is almost not going to change at all. So here is the claim. There is an I of t called adiabatic invariant, which is basically H of t divided by omega of t, and it's almost constant. And this quantity has the units of energy times time.

I don't want to give away the whole story. But I think it's good if you, at this moment, think a second, well, what could it mean, or do I even have a clue why this could happen? And you think oh, quantum mechanics. The harmonic oscillator, what happened? The energy was equal to h omega times the level.

So kind of energy divided by omega is kind of a nice quantity. It's a quantum number. Quantum numbers are quantized, and they don't like to change, because how could an integer change slowly? As soon as it changes, it changes big. So a little bit of what we're getting at is the resistance of a system to change quantum level. When something is quantized, it cannot change slowly, and the adiabatic invariant is exploiting in classical physics that quantum property, if you wish.

So let's look at that. So the claim is that the name i is for adiabatic invariant, and we can verify it, and get some intuition as to why those very slowly. Now, I cannot prove that thing doesn't change. That would be too much, but it's going to change very slowly. You will appreciate that.

Let's see. Let's compute the derivative, di dt. So it's a ratio. So I have omega squared. Omega. I'm going to use dots, and I'm going to stop writing the factor, the key dependents. Omega H dot minus H omega dot. So what do we have here? Omega, H dot was calculated up there, m omega, omega dot x squared minus H p squared over 2m minus-- no, minus. Plus 1/2 m omega squared x squared times omega dot over omega squared.

And well, I still remember when I first saw that. I probably wanted the numerator to cancel and to do something very nice and simplify a lot. But it doesn't happen. So let's see what really happens. Well, you have this term, omega squared, omega dot, x squared m, omega squared, omega dot, x squared m. But the factors of 2 don't make it cancel. So it's there.

So let me write what we get when we simplify this. di dt is equal to omega dot over omega squared times 1/2 m omega squared x squared minus p squared over 2m. That term is clear. The p squared is that, and here, we cancel the 1, partially with a 1/2. So we've got this.

OK, so it doesn't look like it wants to be 0. But it's still very good. Let's see why that result is nice. Well, one thing you realize here is that it actually gave you kind of back the Hamiltonian with a different sine there. This is negative, and this will remain positive. So let's write this, this omega dot omega squared. And this is the kinetic energy minus the potential energy, well, the potential energy v of t minus the kinetic energy k of t in the harmonic oscillator.

Moreover, this quantity is already very small. So this thing is very small, but the fact that the adiabatic invariant is adiabatic that it's really good, should go beyond this. There should be something suppressing about this factor, because you know, this came from just the fact that things vary with omega dot. So what is happening? This is small and slowly varying.

This is neither small, nor slowly varying, in fact. Why? Potential minus kinetic energy. The potential energy in an oscillator goes up when the kinetic energy is 0. I see the oscillator goes to the end, stretches [INAUDIBLE] potential energy is large, the kinetic energy is 0. As it goes through the center, the equilibrium point, the kinetic energy is larger.

So this is oscillating. And it's very large, but now, you probably remember this fact about the harmonic oscillators. While the potential and kinetic energies oscillate, their averages are the same. So that's how this term is going to help you. The average of this quantity is roughly 0 over any period. And a period over a period, this quantity changes little.

So this is going to help us. Let me remind you here, suppose you have an oscillation, an x equals sine omega t, then the momentum would be m x dot, so m omega cosine omega t, and the kinetic energy minus the potential energy, if you do this little calculation, will go like omega squared cosine of 2 omega t, the v minus k. I leave for you that little calculation.

But it will go like cosine of 2 omega t, twice the period. And that thing tends to have a 0 average. So let's see what happens now. The idt to see what happens to it. Let's calculate I at t plus the period minus I at some t. So let's see how much I changes in a period.

So from here, we have the derivative. So we must do the integral from t to t plus t of the dI dt prime dt prime. So this will be the integral from t to t plus capital T of this whole thing, omega dot over omega squared of t times v of t-- it's all t prime, actually-- vt prime minus kt prime dt prime.

Let's see that. You have the derivative of I, so you can calculate the change in I by integrating with the derivative of I over mep. We've done that, and we've asked how much does this thing change over a period. Then we have that I of t plus t minus I of t, we have an integral over a period. We set this quantity very slowly and very little over a period, so roughly speaking, this is equal to omega dot over omega squared at t. It didn't change much over the integral. And then we have the integral over a period of the potential energy minus the kinetic energy.

And for a normal oscillator that is time independent, this quantity is strictly 0. If omega was not changing, this would be identically 0. So if omega is changing slowly, this quantity must be very close to 0. It's identically 0 when it doesn't change. Therefore, you see you got an extra suppression factor. The change in the adiabatic, so-called adiabatic invariant over time was already small, because everything goes slow. But there is an extra suppression due to the fact that these two energies have the same average over a period.

So you gain something. If the energy changes slowly, this energy over omega changes even much more slowly than that. So this is really exactly 0 for time independent omega, approximately 0 for slow omega, slowly changing omega. So that's the extra suppression factor, and that's what makes this an adiabatic invariant, something that really changes dramatically slower in a system in which everything is already changing slowly.