PROFESSOR: So, adiabatic results. So last time we just followed and tried to use an instantaneous eigenstate to construct the solution of the Schrodinger equation. Our result was that we couldn't quite construct the solution of the Schrodinger equation. What we wrote didn't exactly solve the Schrodinger equation. But we claimed that it was important and interesting. And therefore let me remind you of what we said.

So we showed that the wave function, psi of t, could be written as a constant here, e to the i theta of t, e to the i gamma of t times this wave function. And that wave function here was what we called an instantaneous eigenstate of the Hamiltonian. So H of t acting on psi of t was, at any instant of time, an eigenstate.

Nevertheless, we said that these eigenstates are not solutions of the Schrodinger equation in general. They solve this funny time-- inspired by the time-independent Schrodinger equation, this instantaneous eigenstate condition. But when you try to solve the Schrodinger equation, this psi of t would not solve it.

So we tried to add things, and we constructed what seemed to be close to a solution. And there it is. What was theta of t? It was a dynamical phase, it's called, and is defined by this integral. If E of t was a constant, it would be a minus E times t over H bar. And this would be the familiar phase with which you evolve energy eigenstates.

The second term, this gamma of t, was a little more intricate. We define an intermediate thing called nu of t, which was i times the inner product of the instantaneous eigenstate with the derivative with respect to time of the instantaneous eigenstate.

We argued that this factor over here was imaginary. Therefore we put an i so that nu is real. And the gamma of t-- so these were definitions. It's defined as 0 to t dt prime nu of t prime.

OK. So the claim was that if you happen to have a wave function that, at time equals 0, it's in one of those instantaneous eigenstates, it would remain in such instantaneous eigenstate up to phases that are calculable.

OK. So let's make this a little more explicit, in the sense of an adiabatic theorem. So I'll also put here that this is not an exact solution. So I would say the wave function at time t is roughly given by that quantity. It's not exactly, because it's not an exact solution of the Schrodinger

equation.

So if we want to state the adiabatic theorem a little more precisely, we consider a set of instantaneous eigenstates. Instantaneous eigenstates. And that is H of t psi n of t equal E n of t psi n of t.

And here n will go from 1 up to possibly infinity. It just doesn't have to stop. It can go on forever.

OK. Equipped with the instantaneous eigenstates, you might decide that you initially want to be in a state. So psi at t equals 0 is given by psi K at 0. That's your initial condition.

Now, you look at this and say, OK, when is the adiabatic result that you're going to stay roughly in that energy eigenstate going to be true? This is going to require that, as you have the energy E K as a function of time-- here is the energy E K as a function of time. 0. So I'll write it here.

You should have that this doesn't get mixed, or doesn't coincide, with some other energy, like something like that, the crossing, or that they touch with another one. You should keep them separate. You know that the general perturbation theory makes life complicated.

And if you find another state that gets very close, the adiabatic theorem or the adiabatic approximation might go wrong. So we want other energy eigenstates. Here is E K minus 1, and here is E K plus 1. So you have all these energy eigenvalues as a function of time. And you want that, for example, E K-- you order all your energies, and you want that E K of t is definitely less than E K plus 1 of t, which is less than or equal to the other ones. And this is definitely greater than E K minus 1 of t, which is greater or equal than the other one.

So the instantaneous energy eigenstates that you are focused in should be well separated from the other ones. Otherwise you could run into difficulties. So that's definitely necessary for this to hold.

Then the adiabatic theorem says that psi of t will be approximately equal to e to the i theta K of t, e to the i gamma K of t times psi K of t.

So I'm keeping a little difference of notation, trying to be careful. These wave functions, at all times, I put them with a line below the psi. And these instantaneous energy eigenstates, I don't put that. So a little difference that should help you determine this.

Now, what are these theta K or gamma K? Are just the same quantities. I might as well write them for completeness. Theta K of t is what you had before. Minus 1 over H bar 0 to t. But this time E K of the t prime. Then you have nu of t. Nu K is equal to i psi K of t psi K dot of t. And finally the gamma K of t is the integral from 0 to t of nu of t prime dt prime K. Everything with K, corresponding to the Kth eigenstate.

So this is a more precise version already of the statement of the adiabatic approximation. You want to make it even more precise? We'll say something about the error in this formula. In a sense, the hard work in the adiabatic approximation is telling you how much error there is.

So we will not go through proof of the error. We'll just motivate it. And perhaps this will be discussed in recitation. It's an interesting subject, and goes back to work of Born, Max Born, the one that gave the probabilistic interpretation of quantum mechanics, and Fock, of Fock space states of the harmonic oscillator. They did the first work doing that.

Then a Japanese fellow, Kato, in the middle '50s-- so Born and Fock was 1928, I believe. And in the '50s, Kato improved the analysis of the adiabatic theorem. And this whole thing has many applications to the theory of molecules and spin states and systems. We will begin to see some of this today. And had the big revival with the discovery of Berry that there's quite a lot of physics in this phase. This phase here-- this is a normal phase that you know for energy eigenstates, slightly generalized. But this phase is very different. And that's what Barry elaborated and explained and showed that it could be observable in some cases. And it was a pretty nice discovery.

So basically, the adiabatic statement here says that you don't jump to another instantaneous energy eigenstate, you remain in that instantaneous energy eigenstate.

OK. Another thing that is important is that sometimes you say, well, these are phases. When you have states, the phases are unobservable over all phases of a state. You calculate the state after a while. It has a phase. Well, that phase of your physical state doesn't make any difference.

Well, that's true. The overall phase of a state doesn't make any difference. But if you have a system that is in a superposition of two quantum states, one corresponding to one instantaneous energy eigenstate and one corresponding to another-- so there's psi equal to psi 1 and psi 2, one for each-- psi 1, by the linearity of quantum mechanics, is going to

develop its own phase. Psi 2 is going to develop its own phase. And now the relative phase is observable.

When you have a superposition of states, the relative phase is observable. And therefore, if the two states evolve with different phases, you can have observable consequences. So there is something that can happen here, and something that can be measured.