PROFESSOR: OK, so our discussion at the beginning was based on just taking the state, those instantaneous energy island states, and calculating what phases would make it satisfy the Schrodinger equation. And we found those are the phases that came close to satisfying the Schrodinger equation, but not quite. So in order to do this under a more controlled approximation, let's do a calculation where we put all the information in.

So if you have a state psi of t, we'll write it as a superposition of states of instantaneous eigenstates. So this is a general solution. Maybe general [? an ?] [? sets ?] for a solution. The wave function-- since those instantaneous energy eigenstates are complete orthonormal, this form a ON basis, orthonormal basis at all times-- at any time, it's an orthonormal set of states-- we should be able to write our state as that superposition.

So what we're going to do is now kind of re-do the analysis of the [INAUDIBLE] approximation more generally so that we see, in fact, equations that show up that you can solve in general. So the Schrodinger equation is i h bar dvt of psi is equal to H psi. So let's look at what it gives us here.

So we'll have i h bar sum over n. And I have to differentiate this state. So we get Cn dot-- dot for time derivatives-- psi n plus Cn psi n dot-- this is a time derivative of this state-- is equal to H of psi-- this is the sum over n Cn of t-- H of psi n, is equal to E n of t psi n of t. OK, so that's your equation. Now let's see in various components what it gives you.

So to see the various components, we form an overlap with a psi k of t. So we'll bring in a psi k of t. And what do we get?

Since these states are orthonormal, psi k, when it comes here, this is a function of time. It doesn't care. Psi k hits a psi n. That's a Kronecker delta. The sum disappears. And the only term that is left here is Ck dot. So we get i h bar Ck dot from this term.

And let's put the second term to the right hand side. So let's just write what we get from the right hand side and from this term. So from the right hand side, we have the psi k on that thing that is on the right. That, again, hits this state and produces a Kronecker delta. So we get Ck Ek of t from the term that was on the right hand side.

And here, however, we don't get rid of the sum because psi k is not orthonormal to psi n dot.

Psi n dot is more complicated. So what do we get here? Minus i h bar the sum over n psi k psi n dot inner product Cn.

OK, that's pretty close to what we want. But let's write it still in a slightly different way. I want to isolate the Ck's. So from that sum, I will separate the Ck part. So we'll have Ek of t.

And there's going to be a term here, when we have n equal k, so I'll bring it out there-- minus i h bar psi k psi k dot Ck. And the last term now becomes i h bar the sum over n different from k psi k psi n dot Cn. OK, so this is the form of the equation that is nice and gives you a little understanding of what's going on.

That's a general treatment of trying to make a solution from instantaneous energy eigenstates. Here were your instantaneous energy eigenstates. We tried to make a solution. That is the full equation. What did we do before? We used just one of them. We took one instantaneous energy eigenstate and we tried to make a solution by multiplying by one thing, and then we tried. But then it doesn't work because when you have just one coefficient, say k, with some fixed k, you have this equation. But then you couple to all other coefficients where n is different from k.

So what we did before was essentially, by claiming that this term is small, just focus on this thing, and this is an easily solvable equation that, in fact, gives the type of solution we have there. When you have C dot equal to this-- I'll write it in our previous approximation, so in the approximation where the last term is negligible. And we would see why it could be negligible. Then we get just i h bar Ck dot equals Ek of t minus i h bar psi k psi k dot Ck.

And this thing is solved by writing Ck of t is equal to e to the 1 over i h bar integral from 0 to t of this whole thing Ek of t prime minus i h bar psi k psi k dot of t prime dt prime times Ck of 0. This is a differential equation. So i h bar times the time derivative of this-- if you apply a i h bar time derivative, you differentiate with respect to time, then exponent, you get 1 over i h bar that cancels this i h bar. And the derivative of the exponent is this factor-- just this standard, first order, time dependent differential equation.

So last time we said we ignored possible couplings between the different modes represented by this term, and we just solved this equation, which gave us this, which is exactly what we've been writing here. e to the I theta of k comes from the first term on that integral. And e to the i gamma of k comes from the second term on that integral. These are the same things.

So what is new here is that there is a coupling. And you cannot assume that just Ck evolves in time some particular k, and the others don't. The others will get coupled. In particular, if you have that at time equals 0-- t equals 0-- some Ck of 0 is equal to 1, but all the other ones, Ck primes at 0, are equal to 0 for all k prime different from k, so your initial condition is you are in the state k at time equals 0. That's why you have Ck at time equals [? 0 to ?] [? a 1. ?] And the others are 0 at different times.

If you look at your differential equation and try to see what happens after a little time, well, we know Ck dot is going to change, is going to have a non-trivial value. This is going to happen. But these things are not going to remain 0. Those other states are going to get populated, in particular, i h bar Ck prime dot. Look at the top equation. Apply for k equal to k prime and look at time equals 0. What do you get at time equals 0?

Well, you would get all these factor-- Ek prime times Ck prime at time equals 0. Well, that's 0. That's nothing.

But from the last term, what would we get? Minus i h bar sum over n different from k prime of psi k prime psi n dot Cn at time equals 0. And here, well, the only one that is different from 0 at time equals 0 is Ck. And k, when n is equal to k, is allowed because we said k prime is different from k. So there is one term here. This is minus i h bar psi k prime psi k dot. Only when n equals to k you get something here because this is the only term that exists. And it's equal to 1.

So immediately, at time equals 0, the other coefficients start changing. You start populating the other instantaneous energy eigenstates. So there is real mixing in this top thing that says it's not rigorous to claim that you stay in that energy eigenstate. It starts to couple.

And if it starts to couple, then eventually you make transitions. The only hope, of course, is that that term is really small. And basically, we can argue how that term becomes a little small by doing a little calculation. And then we can be rigorous and take a long time to get to the conclusion, or we can just state it. And that's what we're going to do after analyzing that term a little more.