

PROFESSOR: All right, spin orbit.

We have it still there, delta H spin orbit, orbit.

Well, we know what V is. So that derivative, dv/dr can be taken care of, the $1/r$. That gives you $e^2/2m c^2 1/r^3 S \cdot L$. And you remember that $S \cdot L$, from 805, $J^2 - S^2 - L^2$. The reason that's why you tend to do addition of angular momentum, because S and L , calculating the matrix elements of this thing is very easy, in that base is because every state there has a fixed value of S^2 , a fixed value of L^2 . And J^2 plus a couple of possibilities.

So we must work with the couple basis. Basis. And therefore, we can attempt to find E_1 of a N, L, J, MJ , spin orbit, equal $e^2/2m c^2$ and $L \cdot J, MJ, S, L, R^3$. The R^3 has to stay inside the expectation value, because the expectation value includes integration over space. So this is a very important.

N, L, J, MJ . And again the useful question, the couple basis will have we have degeneracies. All the states are degenerate there. So this time, we fixed n , because the degeneracies happen only when you fix n . So do we have the right to do this, to use the formula form perturbation, the non-degenerate perturbation theory to do this calculation?

And the answer is yes, because the perturbation $S \cdot L / R^3$ commutes with L^2 , with J^2 , and with J_z . you need all that because you can have degeneracies by having different L values. And that would be taken care by this operator that has different eigenvalues when L is different.

You can have the degeneracies involving different J values. This would be taken by this operator. And you can have degeneracies when m has different value so that involves the J_z operator. So you really need a perturbation that commutes with all of them.

And why does it commute with all of them? You can see it in several ways. Let's do L^2 . L^2 is Casimir, it commutes with any L_i . It doesn't even think about S because it doesn't know anything about S . So it commutes with S . So L^2 with any L_i and commutes with S . And L^2 is an invariant, it commutes with r^2 because r^2 is rotational invariant. So everything commutes with that.

In order to do the other ones, you can also think in terms of this matrix J^2 over here and S^2 and do all of them. You should do it and convince yourself that they all commute. So we can do this. If we can do it, it's good because then we can evaluate these quantities.

So let's do a little of the of the evaluation. So this $E_{1n l m j}$ is equal to. Let's evaluate the $S \cdot L$ part by using $\frac{1}{2} j^2 - s^2 - L^2$. So that gives you a factor of $\frac{\hbar^2}{4m^2 c^2}$, with this 2 over there. So you get $e^2 \hbar^2$ over $4m^2 c^2$, J^2 times $J^2 + 1 - L^2 + 1 - \frac{3}{4}$, times $n l m j$, $\frac{1}{r^3}$, $m l m j$.

OK. That should be clear from the fact that you have a $j^2 - s^2$. That's a $\frac{3}{4}$ spin is always $\frac{1}{2}$, and L^2 . This is known. It's equal, in fact, to $n l m l + 1$ over cubed $n l m l$. Which is equal, let me discuss that again. It's a little-- $a_0 l, l + 1, l + \frac{1}{2}$.

OK. So this is a known result. It's one of those expectation values that you can get from Feynman, Hellman, or for other recursion relations. And this is always computed in the original uncoupled basis. But we seem to need it in the coupled basis. So again, are we in trouble? No. This is actually the same.

And it is the same only because this answer doesn't depend on m_l . Because this states involve various combinations of m_l and m_s . But doesn't depend on m_l , so the answer is really the same. So these things are really the same.

Happily, that simplifies our work. And now we have $E_{1 n l j m}$ is equal to $E_{n 0}$ -- this is yet another notation, this is the ground state energy here-- $m c^2 n, j, j + 1 - l, l + 1 - \frac{3}{4}$, over $l, l + \frac{1}{2}, l + 1$. OK this is spin orbit.