PROFESSOR: OK. That is the first step. But to make things really clear, we have to do the second step that it involves the Bi solution. So let me say something about Bi as well. So Bi has similar asymptotic expansions. A Bi solution, if psi, you have an exact Bi solution. When psi on the left would behave as minus 1 over pi, this is for u negative. 1 over u to the 1/4 sine of 2/3 u to the 3/2 minus pi over 4. And psi on the right of x would behave as 1 over square root of pi, 1 over u to the 1/4, e to the 2/3, u over 3/2.

OK. This is the asymptotic behavior of B. So what's noteworthy about it? First, that it oscillates for u negative. That's to the left. It oscillates. Instead of a cosine, a sine, the same phase shift. And then that on the other side for u positive, it blows up. So it uses the other exponential.

And this all is reasonable. And in fact, it allows you to connect as well. So in this case, we would have this term looks like this one. And the left one looks like-- so indeed, if you had the Bi solution, the matching requires that the D term over there is matched to the B term over here because that's how the Bi solution is connected.

So we now need to relate the B to the D. So I will write down the relation here. So in this case, there's no factor of 2, but there is a minus sign. So we have B must be equal to minus D. So you can take B equals to minus 1 and D equals to 1. And we get 1 over square root of kfx sine x to A. Kfx prime dx prime minus pi over 4 connects in principle to minus 1 over square root of kappa of x e to the integral from A to x, kappa of x prime dx prime.

I think I have it all right. B is there. B is minus 1. It's over there. D is equal to 1. That's the D term. That's the other connection condition.

OK. So the technicalities are gone. But the concept now requires a serious discussion. I'll give you a little time to take it in.

We've connected every functions. That was the first work we did at the beginning of this lecture. We did, in the top blackboard justify, this expansions. We discussed how the Bi function would originate from a counter integral as well. And we didn't derive the Bi asymptotic expansions, but they're similarly derived. And then we used those to connect things.

And now the question is whether these equations can be used. If I know the solution on the left is like that, can I say the solution on the right is like that? Or if I know the solution on the right is like that, can I say the equation is on the left? And the same thing here. If I know the solution on the left, do I know the solution on the right? And if I know the solution on the right, do I know the solution on the left?

Now it looks like yes, that's what you derive. But remember, it's not quite simple because our whole discussion assume that OK, the potential is strictly linear. But in reality, the potential starts to deviate. So these things that were right here are not exact asymptotic expansions. That is the exact dominant term of the asymptotic expansions. But here there may be extra terms because the solution does not correspond to a potential is exactly linear. And anyway, this is not an exact solution of the differential equation.

So there is a possibility of ambiguities here that actually indicate the following. I claim this connection can only be done in this way. You can only take this equality from the right to the left. That is, if you know that you just have a decaying exponential, you know it connects to this function. Why? Because if you know you have a decaying exponential, there's 0 chance there is a growing exponential on the right. You may have a barrier that extends forever, and there's definitely only a decay in exponential, however the barrier looks.

If there's only a decaying exponential, you know that this growing exponential is just not there. The coefficient here is 0. So you don't have the ambiguity that maybe you have a little bit of a growing exponential that connects to this. So you can go from here to there.

On the other hand, you cannot quite go from the left to the right. Why is that? The reason is the following. If you just know this is on the left, you would say, OK, now I predict a decaying exponential there. But your calculation is not totally exact, so there could be an infinitesimal wave of the other type on the left, an infinitesimal wave of this kind. But that infinitesimal wave of this kind goes into a growing exponential, and a growing exponential eventually overtakes this one and ruins your solution.

So you cannot really use this equation from right to left. The small possibility of error on the left side, because you don't have exact solutions, translates into an ambiguity concerning a positively growing exponential, and it would overtake this. So if you say oh, I have this, therefore I have this decaying exponential, this may be a very inaccurate thing because maybe you have a little bit of the sine that you didn't see because it was much smaller than this term. But that would give you a growing exponential. So you cannot use that equation in that direction.

Similarly here, you have another direction. The direction is this one. It's crucial. You can go that way, but you cannot go the other direction as well. And the reason is kind of similar. Suppose you have here, you say, OK, I have this growing exponential. And say somebody comes in, no, there's a little bit of cosine here. And you say well, but a little bit of cosine gives me a decaying exponential that is irrelevant compared to this one. So you are safe in going from this side to that side. You're going into growing exponential. Any error on the left will produce a small error on the right.

Of the other hand, you cannot go, honestly, from right to left because if you had a decaying exponential-- this is a growing exponential. If you had the decaying exponential, you wouldn't see it here. This one takes over. But the decaying exponential produces a nice cosine that is quite comparable to this one. So a decaying exponential that you don't see here produces on this side a term that is comparable to this one. So you also cannot go from this direction to that direction.

So these are the connection formulas and these are the directions. We've done this for this configuration in which we have a turning point of this form at the point A, in which you're allowed to the left and forbidden to the right. For convenience, it's useful to have a formula-- let's go here-- where you have the opposite direction in which you have a B here, and you have allowed to the right and forbidden to the left.

I will write those formulas because they are many times used and you will need them in general. And Griffiths and other books don't have the patience to give you all the formulas.

This is kappa of x prime, vx prime. This is growing or decaying exponential? This should be called the growing exponential on the left. This is the forbidden region. As x becomes more and more left, the interval is bigger. So this is a growing exponential on the left. 1 over k of x sine b to x, k of x prime, vx prime minus pi over 4. And this one, again, just like that one, you go into the growing exponential. So that's that relation.

And I kind of write these things following this picture. So you're allowed to the right and forbidden on the left side. The allowed way function the right, the forbidden way function to the left. 1 over kappa of x, exponential of minus x to B. Kappa of x prime vx prime goes into-- a decay in exponential you can follow into the other region. And that's what we do here, as before. The decaying exponential is safe because if you know there's just a decaying exponential, there's no possibility of having the growing exponential.

So these are your conditions for that turning point, and that's how you use them. So one example that we will not get to discuss now, but will be in the notes, is tunneling across a barrier. And you will use the connection conditions in this case here, and for this case, the other ones that we've done. So you need for tunneling both connection conditions. It's a very nice exercise. I recommend that you play with it, and we'll put it in the notes as well.

So let's stop here.