MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.07: Electromagnetism II — December 2, 2012
Prof. Alan Guth

PROBLEM SET 10

DUE DATE: Friday, December 7, 2012. Either hand it in at the lecture, or by 6:00 pm in the 8.07 homework boxes.

READING ASSIGNMENT: Chapter 8 of Griffiths: *Conservation Laws*, and also Secs. 9.1–9.3, *Electromagnetic Waves: Waves in One Dimension, Electromagnetic Waves in Vacuum and in Matter*.

CREDIT: This problem set has 95 points of credit, plus the option of earning 15 points extra credit.

PROBLEM 1: CALCULATING THE FORCE BETWEEN TWO POINT CHARGES USING THE MAXWELL STRESS TENSOR *(15 points)*

Griffiths Problem 8.4 (p. 355).

PROBLEM 2: MOMENTUM STORAGE IN A PARALLEL-PLATE CAPACITOR IN A UNIFORM MAGNETIC FIELD *(15 points)*

Griffiths Problem 8.6 (p. 358).

PROBLEM 3: ANGULAR MOMENTUM AND A ROTATING SHELL OF CHARGE *(25 points)*

A total charge Q is uniformly distributed over the surface of a sphere of radius R. The sphere rotates about the z axis with angular velocity ω.

(a) Write down, using your book, previous notes, or homework, the magnetic field inside and outside the sphere. Write down also the electrostatic field inside and outside the sphere.

(b) Now consider the case where $\dot{\omega} \neq 0$. Calculate the Faraday induced electric field at the surface of the sphere as a function of θ. Calculate also the torque this field produces on the sphere.

(c) Suppose now the sphere has a mechanical moment of inertia I about the z axis. Show that if an external torque $\vec{\tau}$ is applied, the sphere undergoes an angular acceleration as if it had an additional moment of inertia I_{mag} due to the magnetic contributions. Calculate I_{mag}.

Now assume again that ω is constant:

(d) Calculate the energy stored in the magnetic field. Show that two-thirds is inside the sphere and one-third is outside the sphere. Verify that the total magnetic energy coincides with $\frac{1}{2} I_{mag}\omega^2$.

(e) Calculate the angular momentum stored in the fields. Explain why it should point in the z-direction. Verify that the magnitude of the angular momentum coincides with $I_{mag}\omega$.

PROBLEM 4: THE PRESSURE OF SUNLIGHT *(10 points)*

Griffiths Problem 9.10 (p. 382). Although you will find a tiny pressure, note that the energy flux from the sun is actually very large.

PROBLEM 5: THE MAXWELL STRESS TENSOR FOR A MONOCHROMATIC LINEARLY POLARIZED PLANE WAVE *(15 points)*

Griffiths Problem 9.12 (p. 382). Do all your calculations with time-averaged quantities.

PROBLEM 6: REFLECTION AND TRANSMISSION OF A PLANE WAVE AT NORMAL INCIDENCE *(15 points)*

Griffiths Problem 9.13 (p. 386).

PROBLEM 7: ENERGY CONSERVATION IN THE PRESENCE OF MAGNETIC MONOPOLES *(15 points extra credit)*

In Eq. (7.43) (p. 327), Griffiths gives the form of Maxwell's equations as extended to include the possibility of magnetic monopoles:

$$\begin{aligned} \nabla \cdot \vec{E} &= \frac{\rho_e}{\epsilon_0} \qquad & \nabla \times \vec{E} &= -\mu_0 \vec{J}_m - \frac{\partial \vec{B}}{\partial t} \ , \\ \nabla \cdot \vec{B} &= \mu_0 \rho_m \qquad & \nabla \times \vec{B} &= \mu_0 \vec{J}_e + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \ . \end{aligned}$$

In Eq. (7.69) (p. 342), he gives the Lorentz force law for monopoles:

$$\vec{F} = q_m \left(\vec{B} - \frac{1}{c^2} \vec{v} \times \vec{E} \right) \ .$$

Note that the factor of $1/c^2$ in the second term (or something with the same units) is needed for the term to have the correct dimensionality.

Generalize Poynting's theorem to this case. You will find that there is no need to modify the formulas for the Poynting vector or for the energy density of an electromagnetic field. You should be able to show that with the extra terms in Maxwell's equations, and with the inclusion of a term for the work done on magnetic monopoles, energy is still conserved. Would energy conservation still work if the force on a magnetic monopole were given by

$$\vec{F} = q_m \left(\vec{B} + \frac{1}{c^2} \vec{v} \times \vec{E} \right) \qquad \text{(wrong!) ?}$$

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