8.08 Problem Set # 2

Feb. 9, 2005
Due Feb. 16, 2005

Problems:

1. **Curie's Law from canonical ensemble:**
Consider a spin-1/2 in a magnetic field B. The $S_z = 1/2$ state has an energy $g\mu_B B/2$ and the $S_z = -1/2$ state has an energy $-g\mu_B B/2$. Assume the spin is in contact with a heat bath of temperature T.
(a) Find the probability $P(1/2)$ for the spin to be in the $S_z = 1/2$ state and the probability $P(-1/2)$ for the spin to be in the $S_z = -1/2$ state.
(b) Find the average spin $\langle S_z \rangle$.
(c) Find the spin susceptibility $\chi = \frac{\langle S_z \rangle}{B}\Big|_{B\to 0}$.

2. Problem 12.9 in K. Huang's book
(c) Show the equipartition of the energy. That is the averages of the potential energy and the kinetic energy of a particle are given by $\langle \frac{p_i^2}{2m} \rangle = \langle \frac{1}{2} m\omega^2 q_i^2 \rangle = \frac{1}{2} k_B T$.

3. **Cooling by adiabatic demagnetization:**
(a) Consider N spin-1/2 spins in a magnetic field B. Initially, the system has a temperature T. If we slowly reduce the magnetic field to zero, what becomes the temperature of the system? (Hint: the entropy remains unchanged in the above adiabatic process.)
(b) Consider N spin-1/2 spins in a magnetic field B. The spin system is in thermal contact with an ideal gas of N particles in a volume V. Initially, the two systems have a temperature T. Assume $g\mu_B B \gg k_B T$. If we slowly reduce the magnetic field to zero, what becomes the temperature of the gas?

4. Problem 6.3 in K. Huang's book

5. Problem 6.4 in K. Huang's book
Assume the air is an ideal gas. You may want to do (b) first. $\frac{\gamma-1}{\gamma}$ is just a constant. Find the value of the constant. (Hint: the entropy per particle does not depend on height z.)