2 Part 2: Follow Through

There are a few different options for part 2, depending on what you want to do. Pick one of the following:

1. **Experimental**

   Build a small trebuchet and demonstrate that it works. For this part, you can work in a small group (up to 4 students), and the demonstration can take the form of a video showing your group and a functional trebuchet.

   BE SAFE. Keep the scale of your demonstration small ($\sim$ 1m max), and do NOT fire anything dangerous. Think squash ball or racquetball ball, and at most tennis ball (because golf balls are too hard).

   One of these might be a place to start:
   http://www.wikihow.com/Build-a-Trebuchet-(1-Meter-Scale)
   http://www.instructables.com/id/Trebuchet-Project/
   http://www.stormthecastle.com/trebuchet/how-to-make-a-trebuchet-out-of-popsicle-sticks.htm
   https://www.youtube.com/watch?v=Cm-AZSkzdxo

   Measure the distance the projectile travels, and compute the "distance efficiency" (i.e., $\epsilon_d = d_{\text{measured}}/d_{\text{max}}$). Compile this information into a brief report which includes the parameters of your trebuchet (lengths, masses, approximate starting and launch angles, etc.). **Your score for this part will be $200 \times \epsilon_d$ or 100, whichever is smaller**, such that you will need to achieve $\epsilon_d = 0.5$ to get full credit.

2. **Numerical**

   Designing a trebuchet that works well is not easy. Your job is to optimize the trebuchet length parameters with the goal of maximizing the energy transfer to the projectile.

   Use Mathematic, MATLAB®, or your favorite program to compute the velocity of the projectile at the point of release. For the sake of this calculation, assume that the release happens when $v_{2x} = v_{2y}$, that is, when it is moving up at 45°.

   You should assume that the beam is uniform so that its mass is

   $$m_b = \lambda \sqrt{m_1}\,(l_1 + l_2) \quad , \quad \lambda = \frac{1}{10} \frac{\sqrt{\text{kg}}}{\text{meter}}$$

   Note that the thickness of the beam scales with the square-root of the mass of the counter-weight. (The value of λ given above is roughly what you need with wood to keep it from bending too much.) From this, and treating the beam as a thin rod (i.e., of negligible cross-section), you can compute I_b for any choice of l_1 and l_2.

   To set the scale, assume a 30kg counter-weight and a 1kg projectile.

   Your objective is to find a collection of parameters (l_1, l_2, l_s), so that the energy transfer to the projectile is maximized. You can quantify this by computing the expected "energy efficiency" $\epsilon_E = E_{\text{projectile}}/U_0$. You should be able to find parameters which give $\epsilon_E > 0.5$ and a projectile velocity $v_2 > 20$m/s.

   Include with your optimized parameters plots of the motion of the trebuchet, similar to figures 4, 5 and 6, and the code you used to do the work.

   Note: if you want to modify the starting angles ϕ_{b0} and ϕ_{s0}, that is fine too, but you can also just take the values given in the table to reduce the number of parameters you need to think about in the optimization. In any case, assume that both masses are released from rest at $t = 0$.

3. **Analytical**

Medieval engineers did not have computers. Can you find an optimal design with pencil and paper?

Let me say up front that this is by far the hardest option for part 2. I eventually found an answer by careful choice of coordinates and separation of the motion into multiple phases in which I could make reasonable approximations for various trig functions. I also cross-checked my approximations against numerical solutions, so weed out the bad ones.

Should you choose to accept this mission, your objective is to find values of the sling and beam lengths (l_s, l_2) which maximize the kinetic energy of the projectile for a given m_1, m_2, and l_1. Assume that the starting angles are $\phi_{b0} = 3\pi/4$ and $\phi_{s0} = \pi/2$ and that the system starts at rest. As in the numerical section, you should assume that the beam is uniform with mass

$$m_b = \lambda \sqrt{m_1}\,(l_1 + l_2)$$

and use this to compute I_b for any choice of l_1 and l_2 given m_1 and λ.

4. **Optimal**

The best trebuchets have a hinged counter-weight and wheels. The wheels aren't just there so that you can move it, they are there because otherwise the hinged counter weight puts huge stresses on the frame (and wastes energy bouncing around its maximum achievable radius).

Here are the additional/modified parameters you will need (see figure 3):

m_f	mass of frame
l_1	distance from pivot to counter-weight attachment point
l_w	distance from the beam to counter-weight
ϕ_{w0}	starting counter-weight angle (~ 0)
x_{f0}	initial horizontal position of the frame ($= 0$)

Assume that the wheels work perfectly so that the frame and pivot are free to move horizontally without friction.

(a) Along the lines of problem 1 in part 1, compute the maximum energy efficiency of this design

$$\epsilon_E = E_{\text{projectile}}/U_0$$

where $E_{\text{projectile}}$ is the final projectile kinetic energy and U_0 in the initial stored potential energy relative to the minimum potential state (hint: consider conservation of momentum as well as energy). Continue to assume that the projectile launches at 45 degrees, and that you can neglect friction, etc as in part 1.

(b) Compute the Lagrangian of wheeled trebuchet and the resulting equations of motion (similar to problems 2 - 4 in part 1).

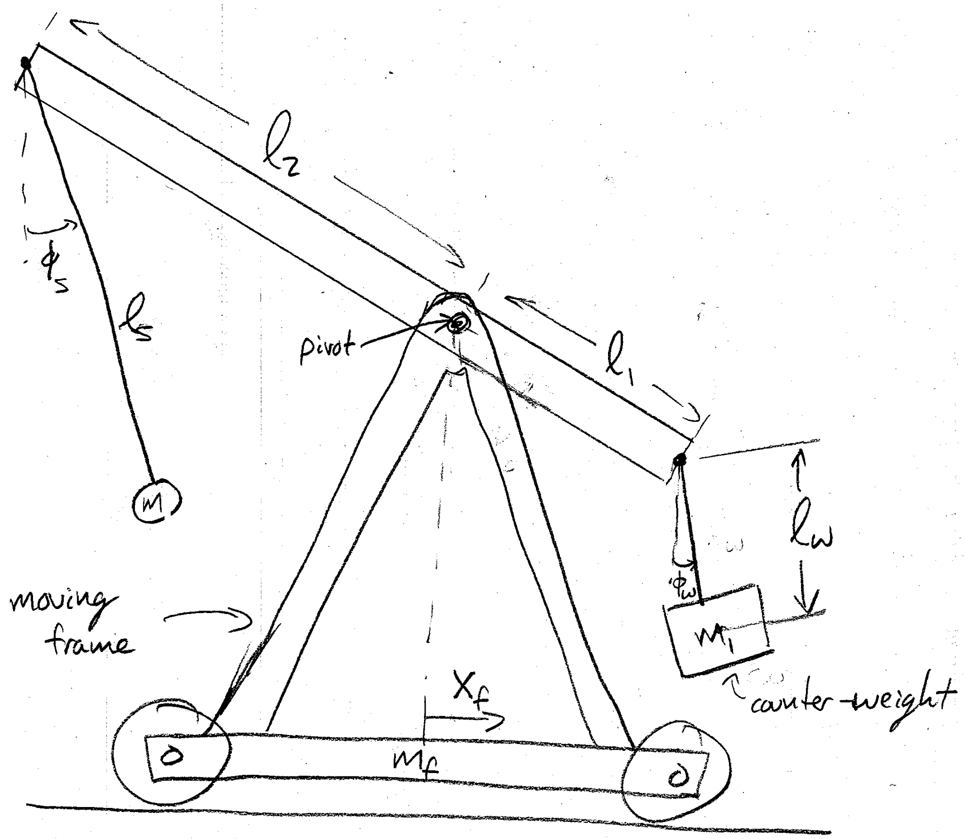

Figure 3: Wheeled trebuchet parameter description. Note: the projectile is still m_2.

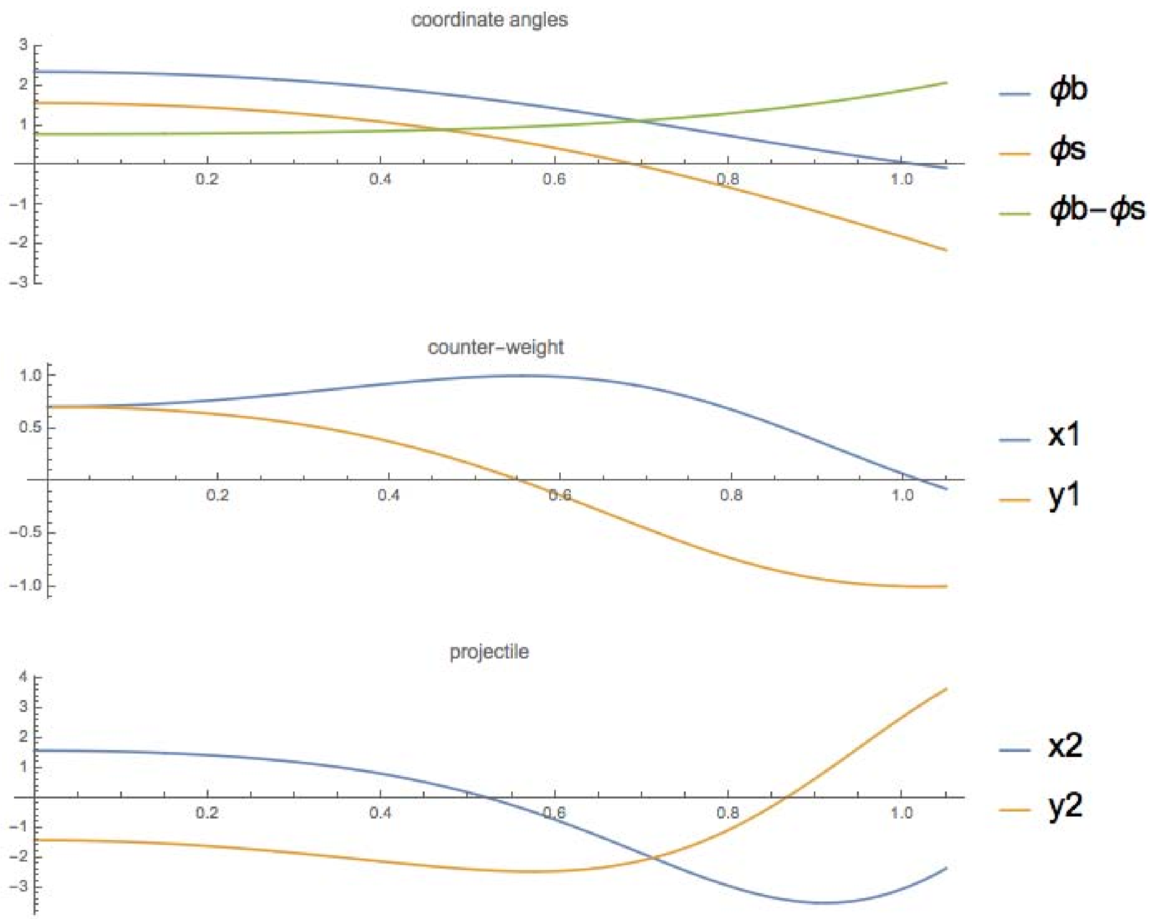

Figure 4: Motion of an example trebuchet. The angles used in the Lagrangian are shown, along with the Cartesian coordinates of the counter-weight and the projectile.

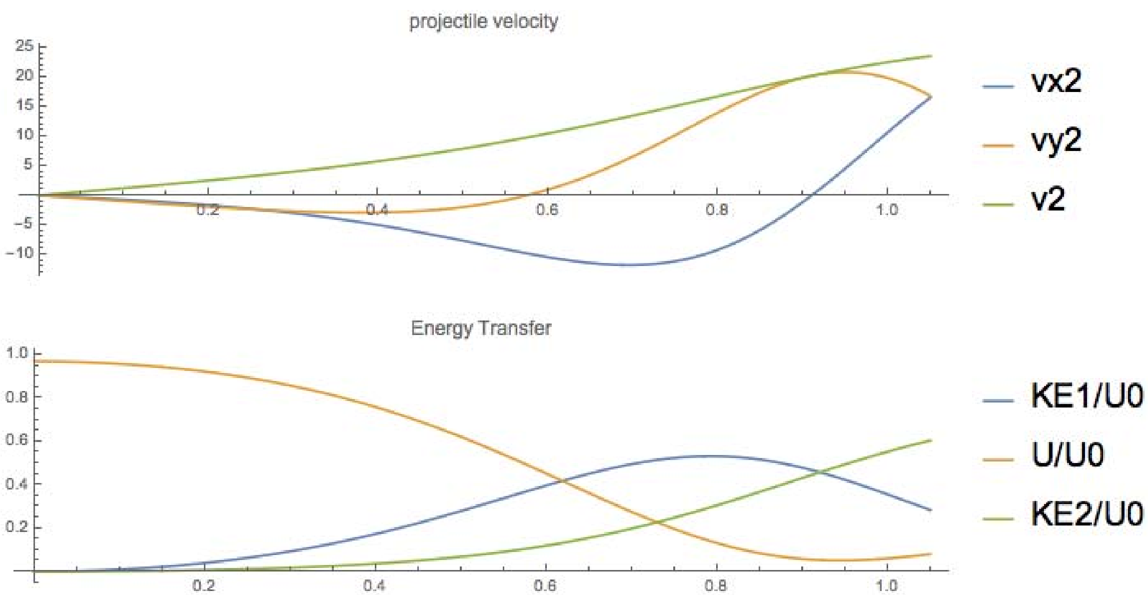

Figure 5: The velocity of the projectile is shown, along with the kinetic energy of the counter-weight and projectile relative to the stored potential energy.

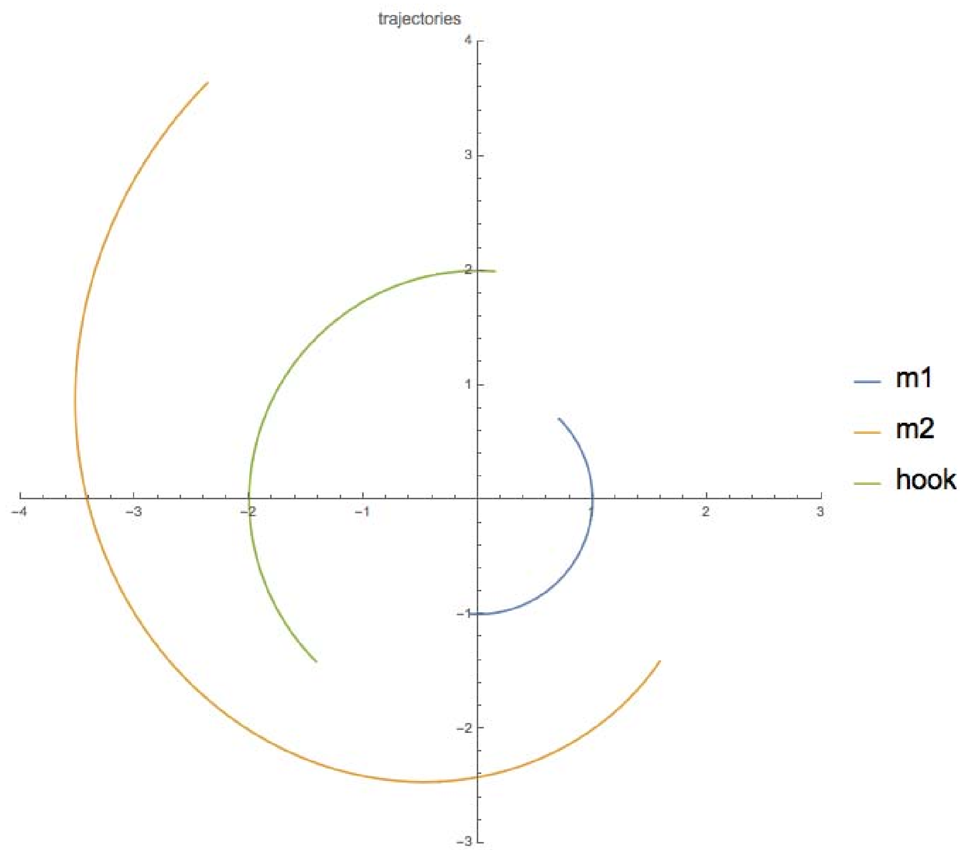

Figure 6: A parametric depiction of the trajectories shown in figure 4.