# The Global Positioning System and Relativity: A <10% test of general relativity everyday

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http://www-gpsg.mit.edu/~tah

# Overview

- Original design of the Global Positioning System (GPS)
- Use by “non-authorized” users
- Selected applications
- Relativistic effects

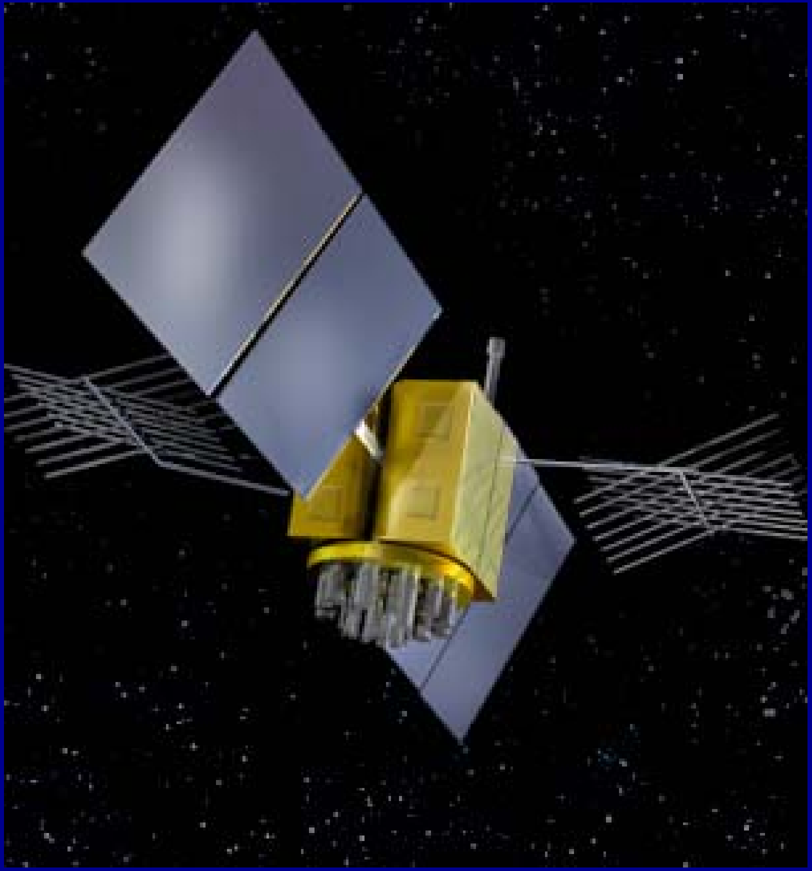
# GPS Original Design

- Started development in the late 1960s as NAVY/USAF project to replace Doppler positioning system
- Aim: Real-time positioning to < 10 meters, capable of being used on fast moving vehicles.
- Limit civilian (“non-authorized”) users to 100 meter positioning.

# GPS Design

- Innovations:
  - Use multiple satellites (originally 21, now ~28)
  - All satellites transmit at same frequency
  - Signals encoded with unique “bi-phase, quadrature code” generated by pseudo-random sequence (designated by PRN, PR number): Spread-spectrum transmission.
  - Dual frequency band transmission:
    - L1 ~1.575 GHz, L2 ~1.227 GHz
    - Corresponding wavelengths are 190 mm and 244 mm

# Latest Block IIR satellite (1,100 kg)



# Measurements

- Measurements:
  - Time difference between signal transmission from satellite and its arrival at ground station (called “pseudo-range”, precise to 0.1–10 m)
  - Carrier phase difference between transmitter and receiver (precise to a few millimeters)
  - Doppler shift of received signal
- All measurements relative to “clocks” in ground receiver and satellites (potentially poses problems).

# Measurement usage

- "Spread-spectrum" transmission: Multiple satellites can be measured at same time.
- Since measurements can be made at same time, ground receiver clock error can be determined (along with position).
- Signal

$$V(t,\vec{x}) \quad V_o \sin[2 \quad (ft \quad \vec{k}.\vec{x}) \quad C(t)]$$

$C(t)$ is code of zeros and ones (binary).

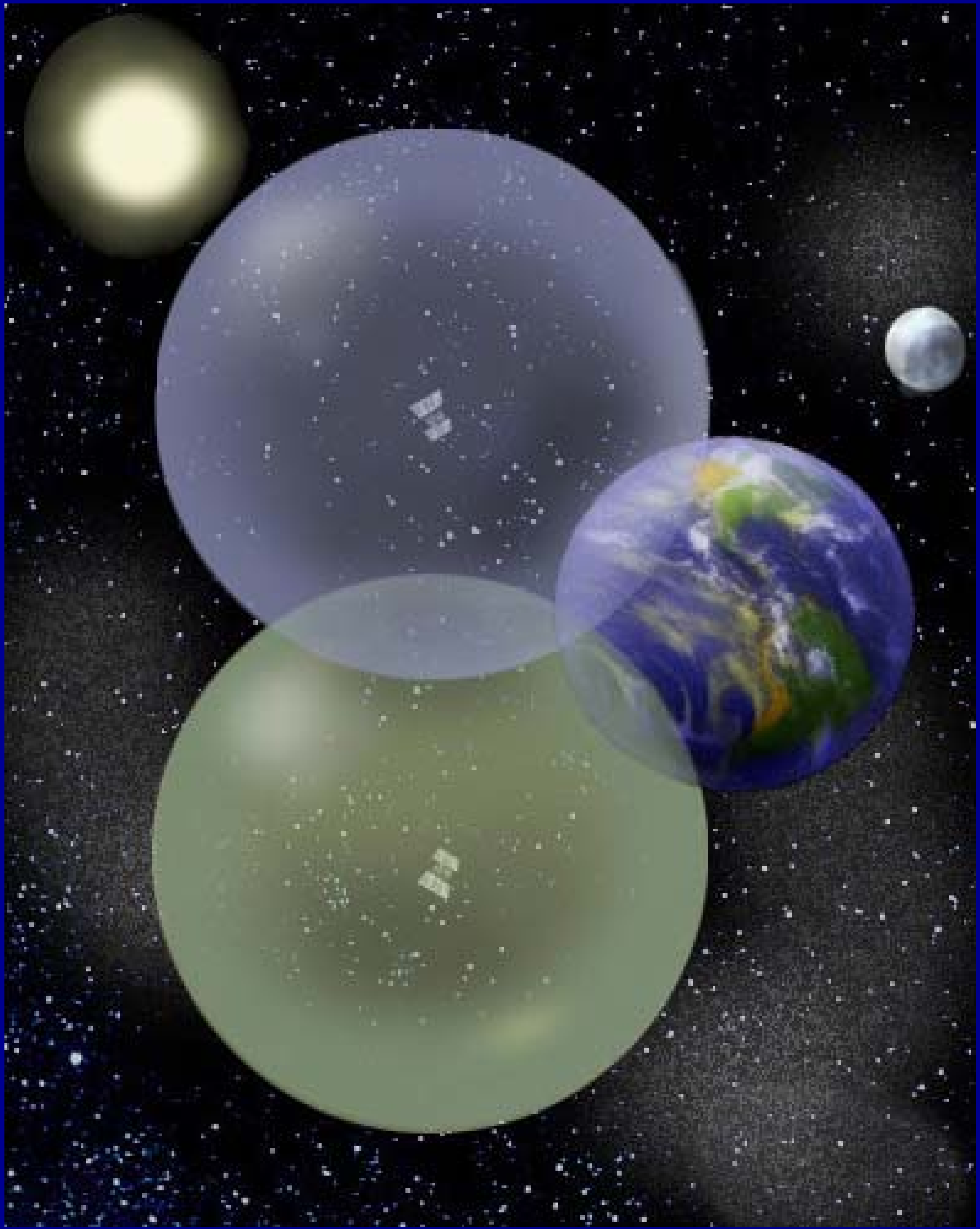
Varies discretely at 1.023 or 10.23 MHz

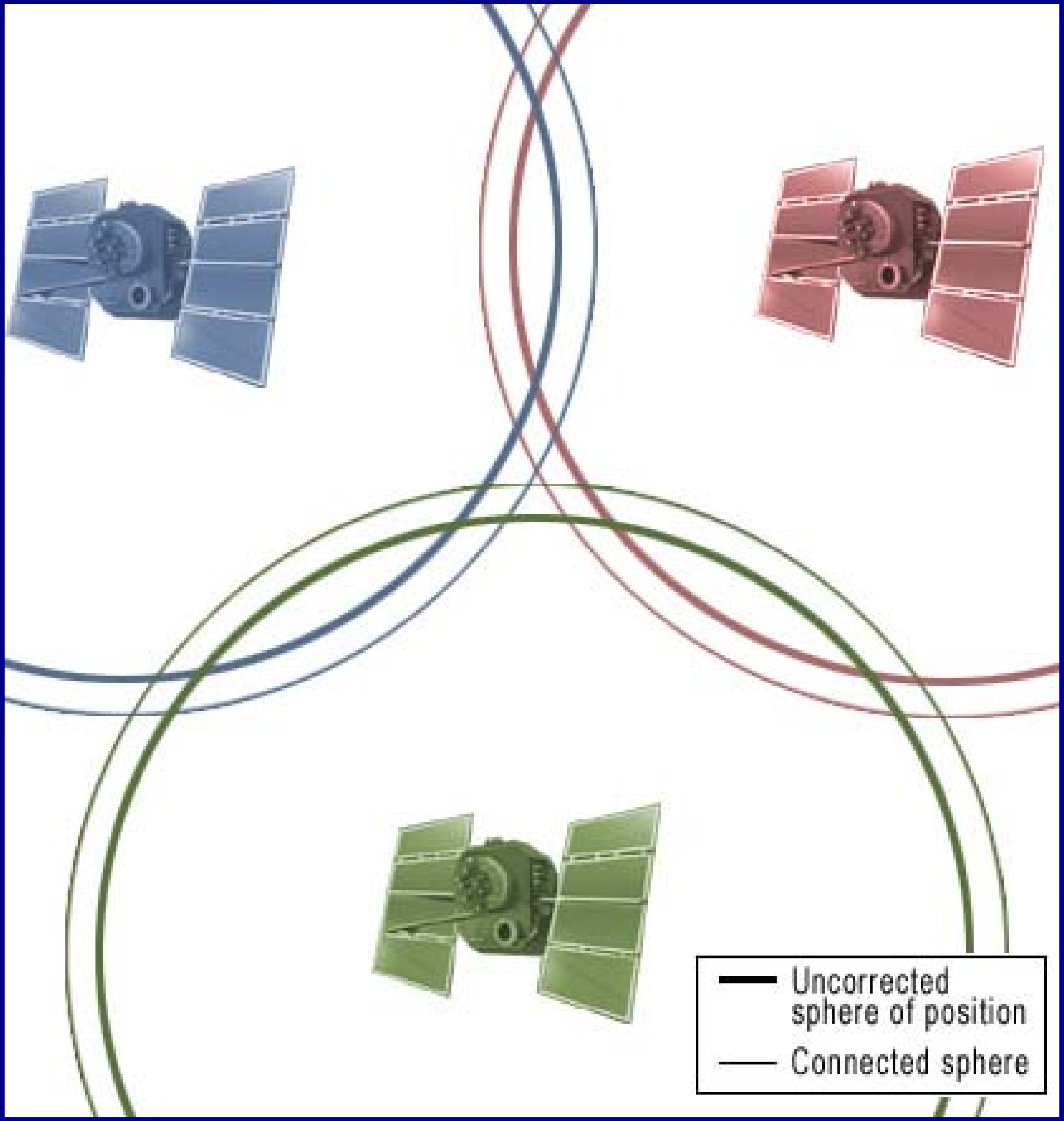
# Measurements

- Since the C(t) code changes the sign of the signal, satellite can be only be detected if the code is known (PRN code)
- Multiple satellites can be separated by "correlating" with different codes (only the correct code will produce a signal)
- The time delay of the code is the pseudo-range measurement.

# Position Determination (perfect clocks).

- Three satellites are needed for 3-D position with perfect clocks.
- Two satellites are OK if height is known)






# Position determination: with clock errors: 2-D case

- Receiver clock is fast in this case, so all pseudo-ranges are short

# Positioning

- For pseudo-range to be used for positioning need:
  - Knowledge of errors in satellite clocks
  - Knowledge of positions of satellites
- This information is transmitted by satellite in “broadcast ephemeris”
- “Differential” positioning (DGPS) eliminates need for accurate satellite clock knowledge.

# GPS security: SA

- To stop non-authorized users from getting the full accuracy of GPS, the military until May 2000 “corrupted” the GPS signals.
- Selective Availability (SA) “dithered” the clocks by time equivalent of ±100 meters.
- Turned off because ineffective

(Example shown later)

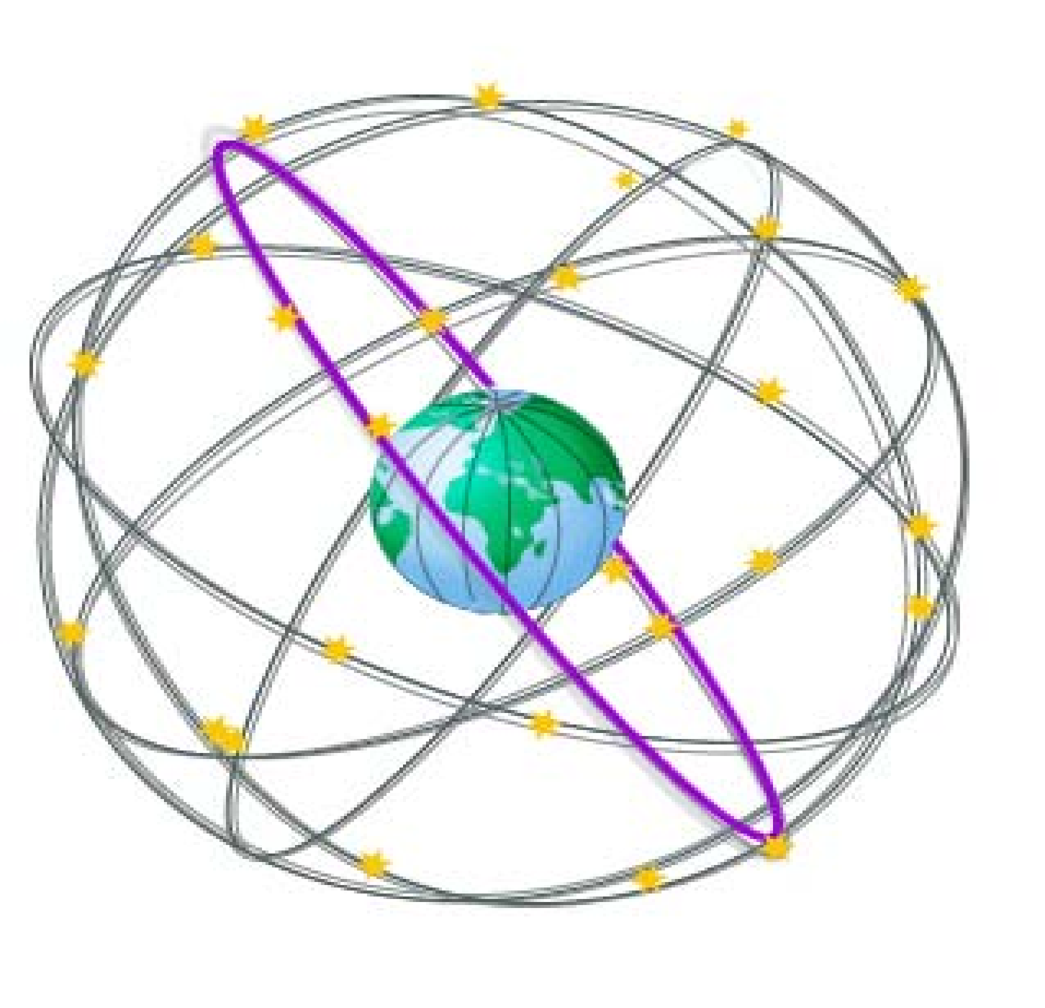
# GPS security: AS

- To ensure that military systems were not corrupted by false GPS transmission, Anti-spoofing (AS) in enabled on all satellites
- At L1 frequency: GPS satellites use two C(t) sequences: Course Acquisition C/A code and Precise Positioning code (P code)
- P-code in modified under AS to the Y-code which only authorized user know

# Satellite constellation

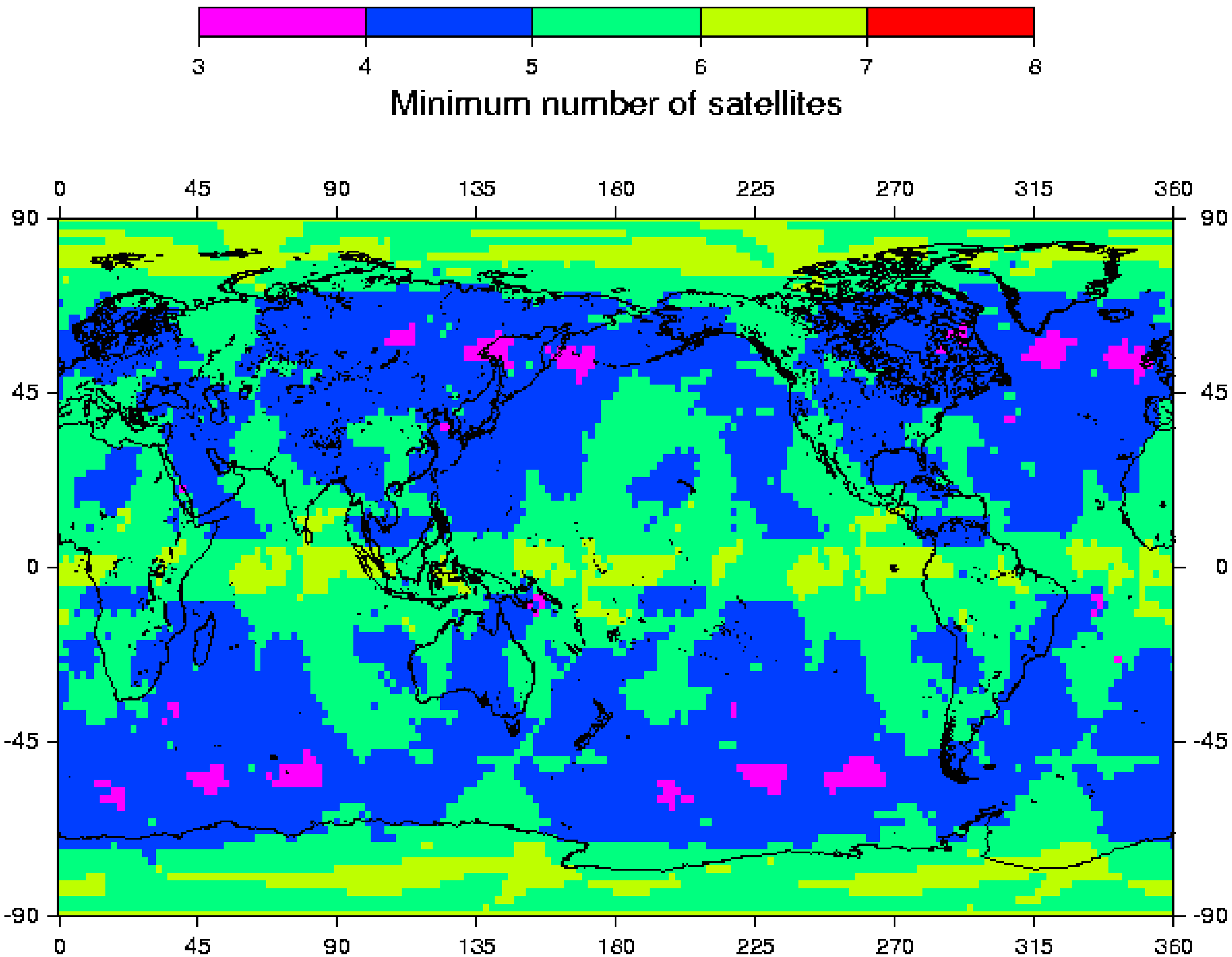
- Since multiple satellites need to be seen at same time (four or more):
  - Many satellites (original 21 but now 28)
  - High altitude so that large portion of Earth can be seen  (20,000 km altitude —MEO)

# Current constellation



- Relative sizes correct (inertial space view)
- “Fuzzy” lines not due to orbit perturbations, but due to satellites being in 6-planes at 55$^{o}$ inclination.

# Satellite Availability (smallest number above 15º minimum elevation)



# Positioning accuracy

- Best position accuracy with pseudo-range is about 20 cm (differential) and about 5 meters point positioning.
- For many applications we want better accuracy
- For this we use “carrier phase” where “range” measurement noise is few millimeters

# Carrier phase positioning

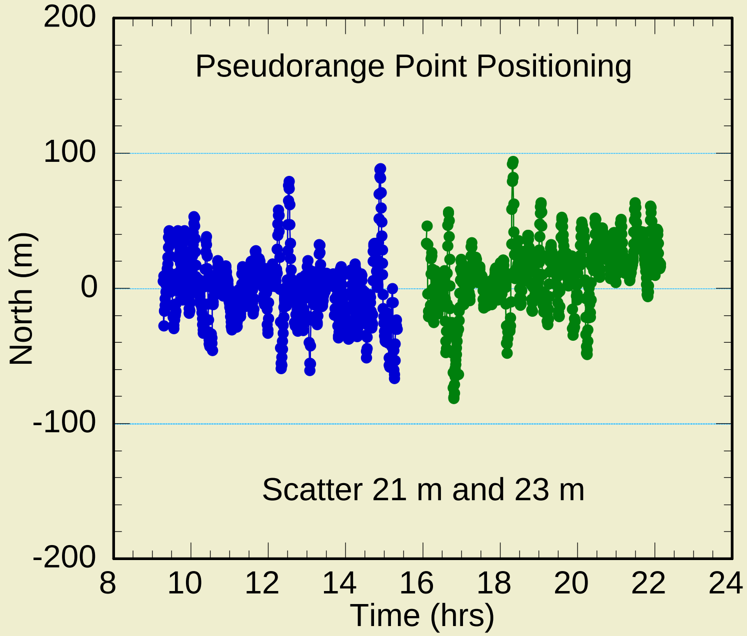
- To use carrier phase, need to make differential measurements between ground receivers.
- Simultaneous measurements allow phase errors in clocks to be removed i.e. the clock phase error is the same for two ground receivers observing a satellite at the same time (interferometric measurement).

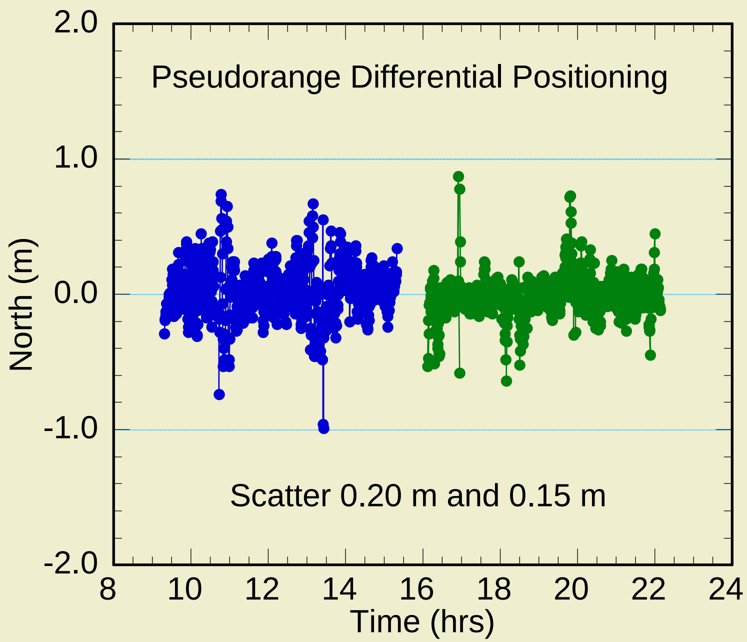
# Phase positioning

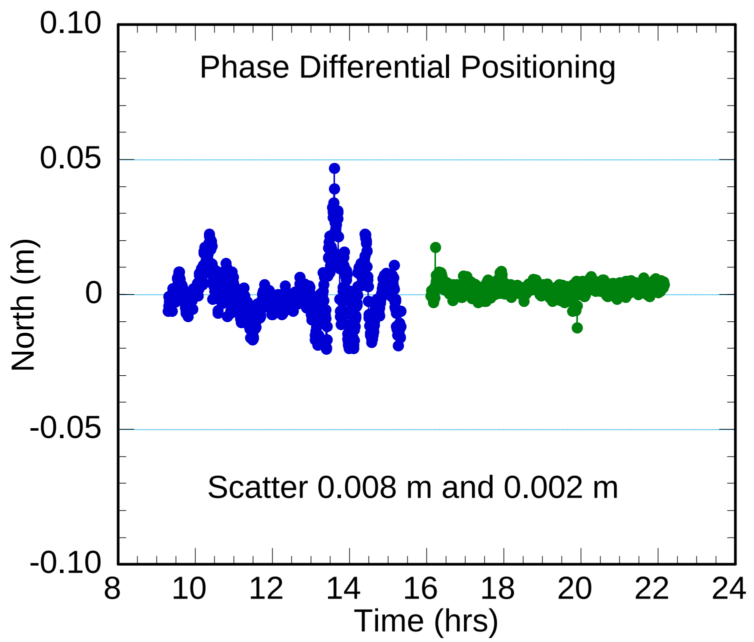
- Use of carrier phase measurements allows positioning with millimeter level accuracy and sub-millimeter if measurements are averaged for 24-hours.

# Examples of positioning results

SA on



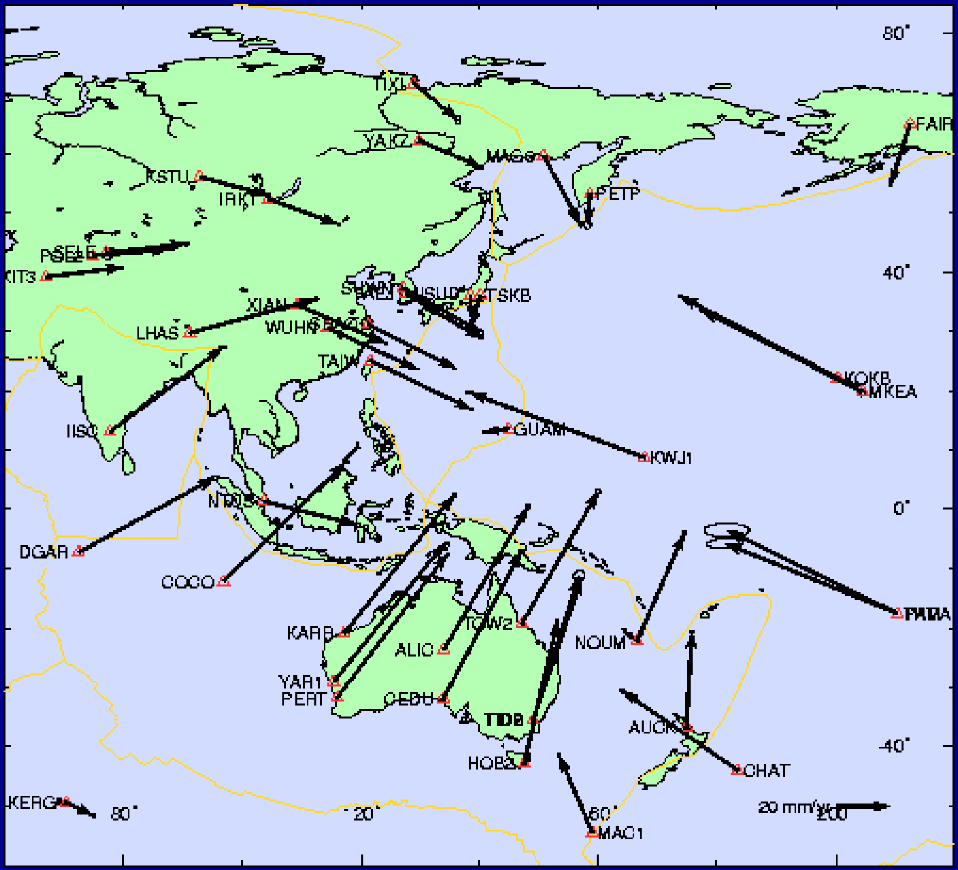
# Summary

- Use of differential measurements with carrier phase allows very precise position determination (independent largely of security features).
- We use these measurements in Earth science for deformation studies and atmospheric studies

# Tectonic Deformation Results

- “Fixed GPS” stations operate continuously and by determining their positions each day we can monitor their motions relative to a global coordinate system
- Temporary GPS sites can be deployed on well defined marks in the Earth and the motions of these sites can be monitored (campaign GPS)

# Example of motions measured in Pacific/Asia region

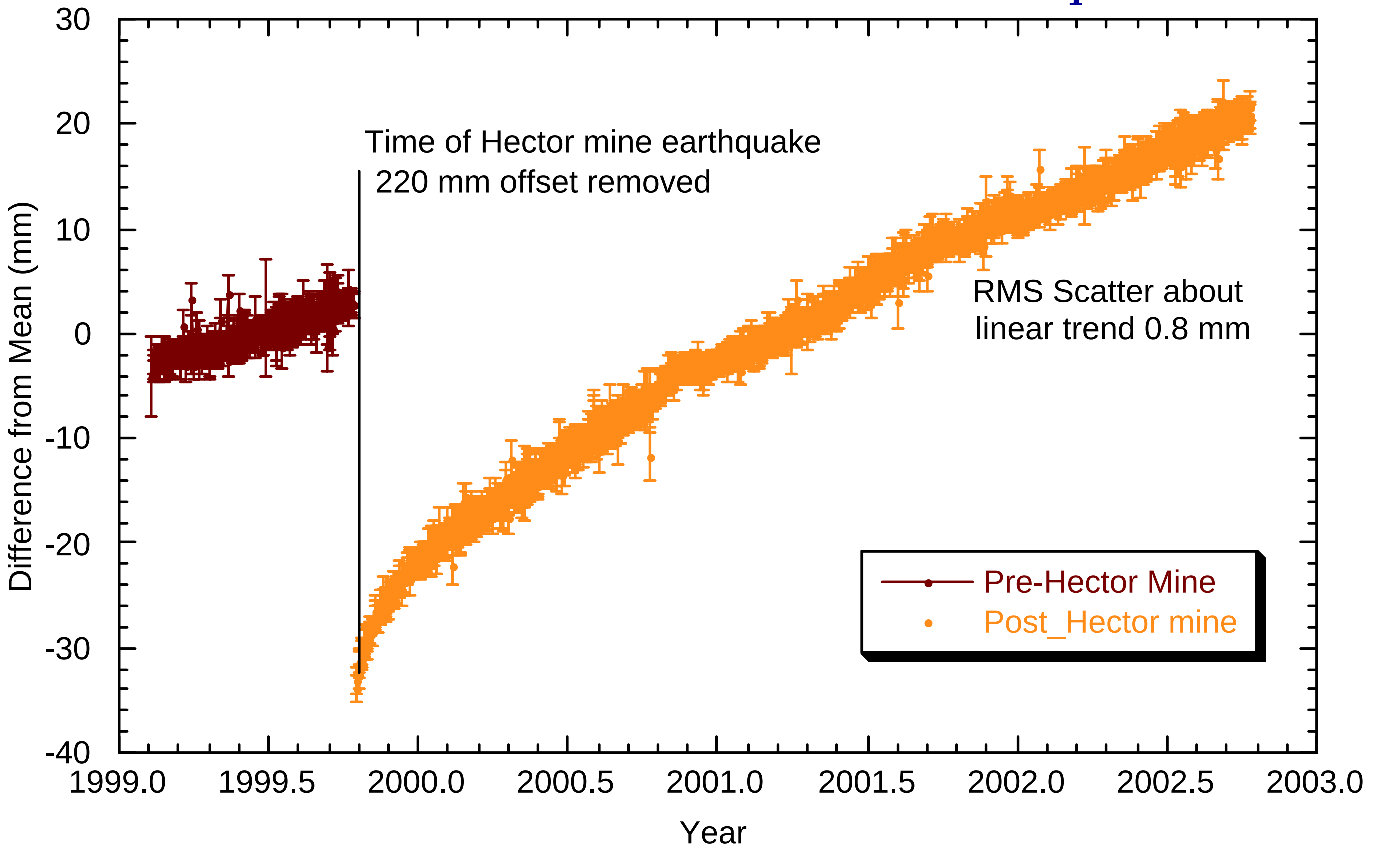



- Fastest motions are >100 mm/yr
- Note convergence near Japan

More at **http://www-gpsg.mit.edu/~fresh/MIT_IGS_AAC.html**

# Motion after Earthquakes. Example from Hector Mine, CA

Continued motion tells us about material characteristics and how stress is re-distributed after earthquake



# Relativistic effects

- General relativity affects GPS in three ways
  - Equations of motions of satellite
  - Rates at which clock run
  - Signal propagation
- In our GPS analysis we account for the second two items
- Orbits only integrated for 1-3 days and equation of motion term is considered small

# Clock effects

- GPS is controlled by 10.23 MHz oscillators
- On the Earth's surface these oscillators are set to $10.23x(1-4.4647x10^{-10})$ MHz (39,000 ns/day rate difference)
- This offset accounts for the change in potential and average velocity once the satellite is launched.
- The first GPS satellites had a switch to turn this effect on. They were launched with "Newtonian" clocks

# Propagation and clock effects

- Our theoretical delay calculations are made in an Earth centered, non-rotating frame using a “light-time” iteration i.e., the satellite position at transmit time is differenced from ground station position at receive time.
- Two corrections are then applied to this calculation

# Corrections terms

- Propagation path curvature due to Earth's potential (a few centimeters)

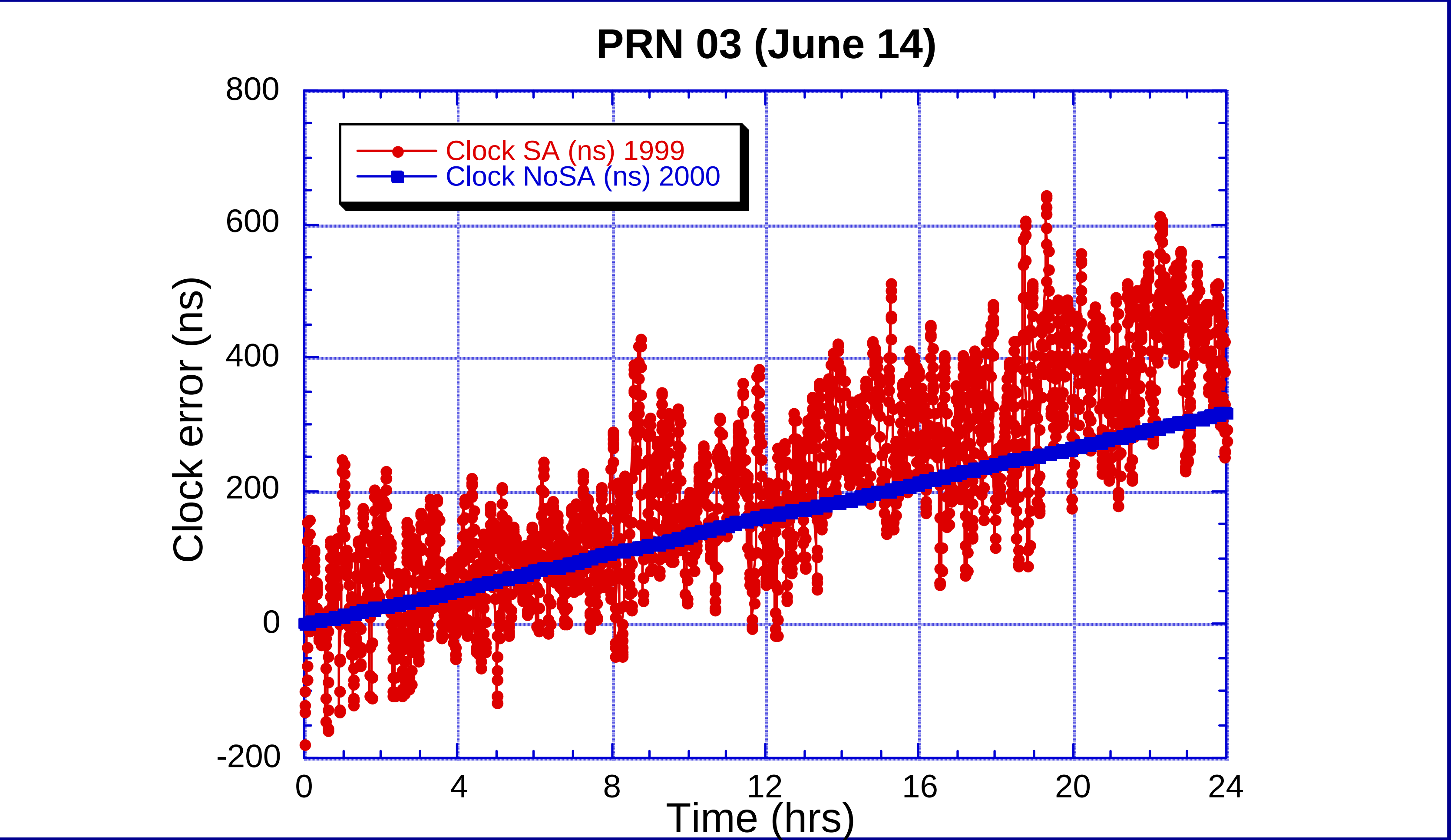
$$\frac{2GM}{c^3}\ln\frac{R_r \quad R_s}{R_r \quad R_s}$$

- Clock effects due to changing potential

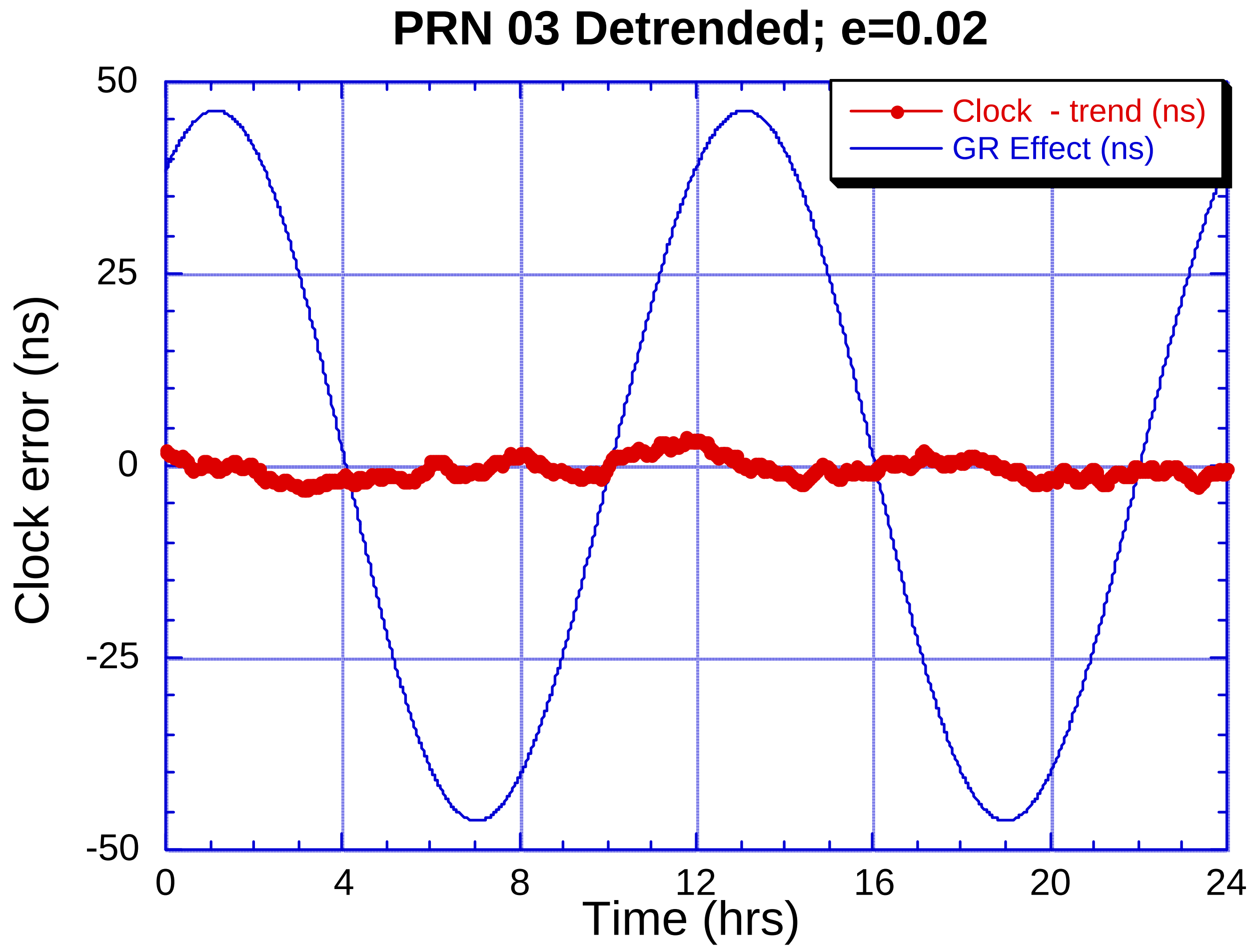
$$\frac{\sqrt{GM}}{c^2}e\sqrt{a}\sin E$$

- For e=0.02 effect is 47 ns (14 m)

# Effects of Selective Availability



# Relativistic Effects



# Tests of General Relativity

- In the parameterized post-Newtonian formulation, the time delay expression becomes:

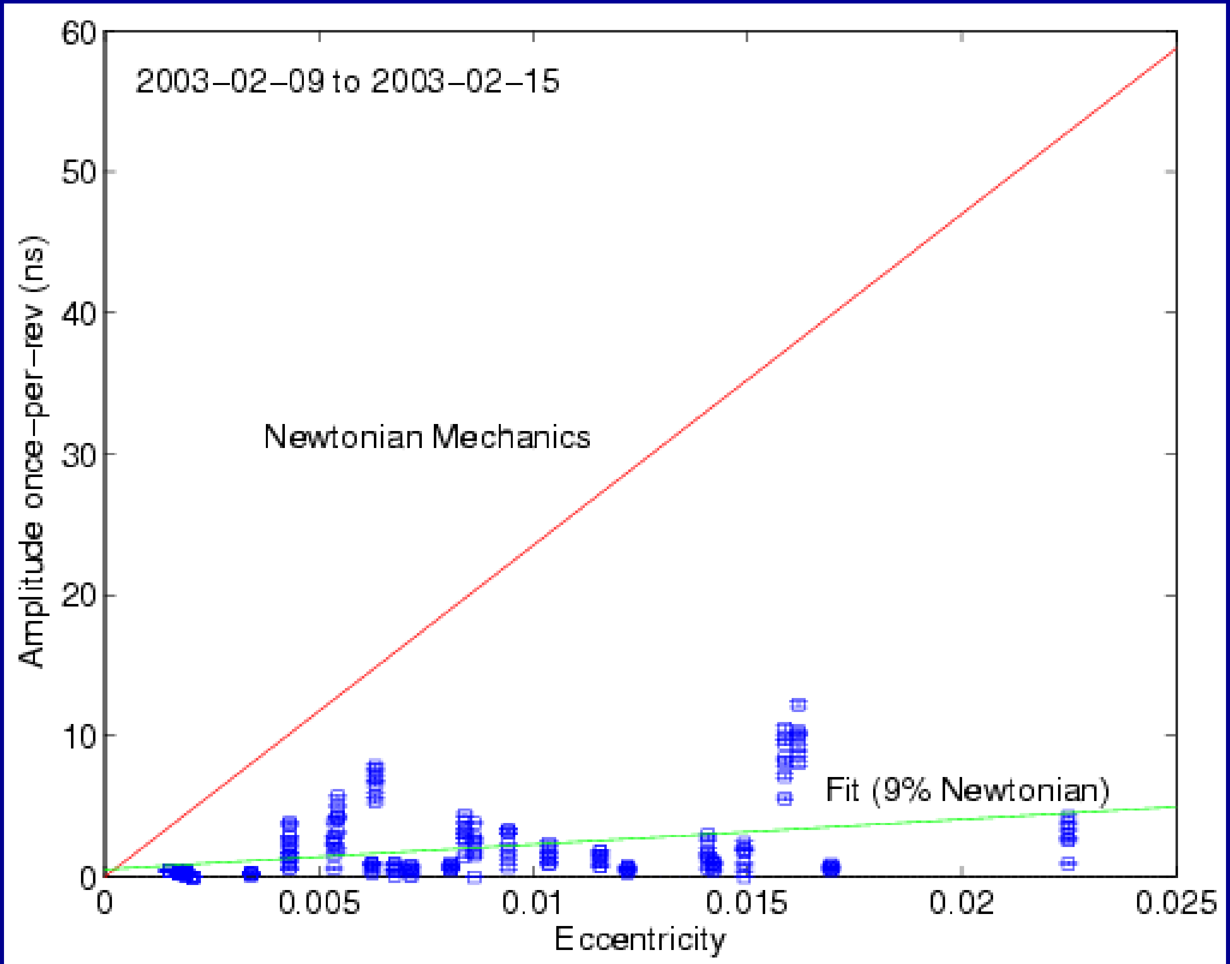
$$\frac{\sqrt{GM}}{c^2}\frac{(1 \quad )}{2} e\sqrt{a}\sin E$$

- In PPN,   is the gravitational term. In general relativity   = 1
- The clock estimates from each GPS satellite allow daily estimates of

# Using GPS to determine

- Each day we can fit a linear trend and once-per-revolution sin and cos terms to the each of the 27-28 GPS satellites.
- Comparison between the amplitude and phase (relative to sin(E)) allows and estimate of gamma to be obtained
- Quadrature estimates allows error bound to be assessed (cos(E) term)
- Problem: Once-per-orbit perturbations are common. However should not be proportional to eccentricity.

# Initial “quick” results




Amplitude comparison only

Consistent with GR to <10%

Only 1 week of data: Data after May 2000 could be used.

# Conclusions

- GPS dual-use technology: Applications in civilian world widespread
  - Geophysical studies (mm accuracy)
  - Engineering positioning (<cm in real-time)
  - Commercial positioning: cars, aircraft, boats (cm to m level in real-time)
- Relativistic effects are large but largely constant
- However due to varying potentials and velocities effects can be seen
- Some effects are incorporated by convention
- Need to keep in mind “negligible effects” as accuracy improves