8.251 – Homework 1

B. Zwiebach Spring 2007

Due Tuesday, February 13.

1. (10 points)

   Quick calculation 2.2 (p. 20).

   Quick calculation 2.3 (p. 24).

2. (10 points) A matrix L that satisfies (2.44) defines a Lorentz transformation. Show that

   (a) If L_1 and L_2 are Lorentz transformations so is the product $L_1 L_2$.

   (b) If L is a Lorentz transformation so is the inverse matrix L^{-1}.

   (c) If L is a Lorentz transformation so is the transpose matrix L^T.

3. (10 points) Problem 2.2, part (a) only.

4. (10 points) Problem 2.3.

5. (10 points) Consider the (x, y) plane described with a complex coordinate $z = x + iy$. We have seen that the identification $z \sim e^{\frac{2\pi i}{N}} z$ with $N \geq 2$ a positive integer, can be used to construct a cone. Consider two relatively prime integers M and N, with $M < N$ and the identification

$$z \sim e^{2\pi i \frac{M}{N}} z\,, \quad M,\, N \geq 2\,. \tag{1}$$

   One may naively believe that a fundamental domain is provided by the points that satisfy the constraint $0 \leq \arg(z) < 2\pi \frac{M}{N}$. Experiment with low values of M and N to convince yourself that this is *not* a fundamental domain. Determine a fundamental domain for the identification in (1).

   Hint: There is a lovely theorem that follows from Euclid's algorithm for the greatest common divisor: Given two integers a and b, relatively prime, there exist integers m and n such that $ma + nb = 1$ (m and n are not unique). This result should be useful once you have thought a bit about the problem. Finding m and n is not easy unless you use Euclid's algorithm: try, for example, solving $187m + 35n = 1$, for some integers m and n.

6. (10 points) Problem 2.4.

7. (20 points) Problem 2.7