Lecture 4 - Topics

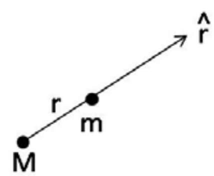
- Gravitational potentials
- Compactification
- Large extra dimensions

Reading: Zwiebach, Sections: 3.7 - 3.10

Gravitation

Newtonian Gravity

$$F_m = -(GMm/r^2) \cdot \hat{r}$$

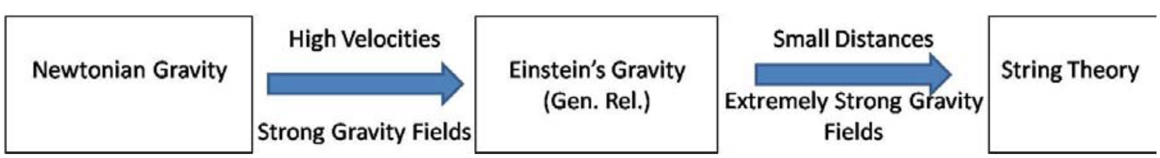


When working in regime with high velocities or strong gravity fields, instead work with:

Einstein's Gravity

When working in a regime with small distances or extremely strong gravity fields, instead work with:

String Theory



$$-ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu \overset{GR}{\Rightarrow} -ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$g_{\mu\nu}(x) = g_{\nu\mu}(x)$$

Still symmetric, still invertible. Signature (-,+,+,+) (1 negative eigenvalue, 3 positive eigenvalues)

If gravity weak, approximate $g_{\mu\nu}(x) = \eta\mu\nu + \underbrace{h_{\mu\nu}(x)}_{\text{small fluctuation}}$

In E & M, used $A_\mu(x)$: $A_\mu \to A_\mu + \partial_\mu$

Could define new coordinates:
$x'^\mu = x^\mu +^\mu (x)$
$h^{\mu\nu} = \eta^{\mu\alpha}\eta^{\nu\beta}h_{\alpha\beta}$
$h^{\mu\nu} \stackrel{GT}{\to} h'^{\mu\nu} = h^{\mu\nu} + \partial^\nu_\mu + \partial^\mu_\nu$

There's a dramatic change of language when moving from Newtonian gravity to Einstein's gravity, but we'll use the same language for string theory as GR. (May eventually need to change language for string theory as we understand it better)

Planckian Units

3 Fundamental Constants:

$$G = 6.67 \times 10^{-11}\frac{m^3}{kg \cdot s^2}$$
$$c = 3 \times 10^8 \frac{m}{s}$$
$$\hbar = 1.06 \times 10^{-34}\frac{kg \cdot m^2}{s}$$

Suppose want fundamental length, mass, and time: l_p, m_p, and t_p

$$G = 1 \cdot \frac{l_p^3}{m_p t_p^2}$$
$$c = 1 \cdot \frac{l_p}{t_p}$$
$$\hbar = 1 \cdot \frac{m_p l_p^2}{t_p}$$
$$l_p = \sqrt{\frac{G\hbar}{c^3}} \approx 1.6 \times 10^{-33}\text{cm}$$
$$m_p = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-5}\text{g}$$
$$t_p = \sqrt{\frac{l_p}{c}} \approx 5.4 \times 10^{-44}\text{s}$$

Note: l_p small, t_p extremely small, m_p fairly large.

m_p	$m_p c^2$	m_{electron}	$m_{\text{electron}} c^2$	m_{proton}	$m_{\text{electron}} c^2$
10^{-5}g	10^{19} GeV	10^{-27} g	0.5 MeV		1 GeV

New accelerator LHC will accelerate protons to maximum of 7000 GeV which is much smaller than 10^{19} GeV.

Cosmological Constant

Energy density (mass density) $\approx 0.7 \times 10^{-29} \frac{\text{g}}{\text{cm}^3}$

Note critical mass density of universe (to make it flat) is $\approx 10^{-29} g/cm^3$

In Planckian units, $m_p/l_p^3 = 5.3 \times 10^{93} \frac{g}{cm^3}$

$$\frac{\text{Planckiang vacuum energy}}{\text{True vacuum energy}} = \frac{5.3 \times 10^{93}}{0.7 \times 10^{-29}} = 7.6 * 10^{122}$$

That's quite an error! Odd because in an equation with order 1, expect solution to also be order 1 unless equation very peculiar.

E & M:
$F = q\vec{E}$
$\nabla \cdot \vec{E} = \rho$
$E = -\nabla\Phi$
$\nabla^2\Phi = -\rho$

Gravity:
$\vec{F} = m\vec{g}$
$\vec{g} = -\nabla V_g$

$\vec{g}$: gravitational field
V_g: gravitational potential (energy/mass) in all dimensions like $\vec{E}$ = (energy/charge)

In 4D:

$$\Phi = \frac{Q}{4\pi r}$$

$$V_g^{(4)} = -GM/r$$

$$\nabla^2 V_g^{(4)} = 4\pi G\rho_m$$

In D-dim: $\nabla^2 V_g^{(D)} = 4\pi G^{(D)}\rho_m$ (Note: D = number of space-time dimensions, d = number of space dimensions, $D = d+1$)

Note: not keeping G the same.
LHS has units the same in all dimensions.

RHS's ρ_m has units that changes with dimension so G must too.

$$\nabla \cdot \vec{g} = -4\pi G \rho_m$$

$$[G^{(5)}]\frac{M}{L^4} = [G]\frac{M}{L^3} \Rightarrow [G^{(5)}] = L \cdot [G]$$

$$l_p = \sqrt{\frac{G\hbar}{c^3}} \Rightarrow G = \frac{l_p^2 c^3}{\hbar} \Rightarrow [G] = L^2\frac{c^3}{\hbar} \Rightarrow [G^{(5)}] = L^3\frac{[c^3]}{[\hbar]}$$

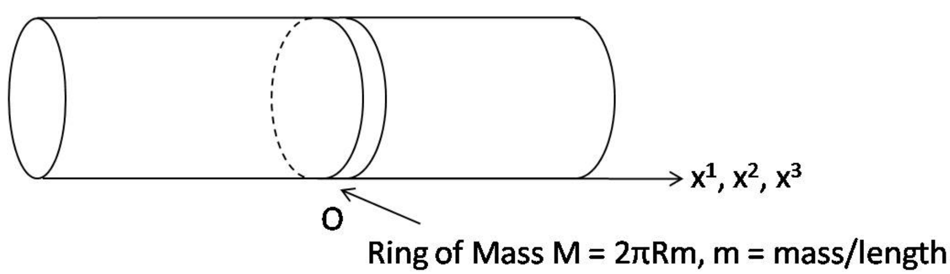
$$G^{(5)} = (l_p^{(5)})^3 c^3/\hbar$$

$$\frac{G^{(5)}}{G} = \frac{(l_p^{(5)})^3}{(l_p)^2}$$

In general:

$$\boxed{\frac{G^{(D)}}{G} = \frac{(l_p^{(D)})^{D-2}}{(l_p)^2}}$$

Imagine a 5D world with a $G^{(5)}$. Compactify 1 dimension subject to effectively 4D x^1, x^2, x^3, x^4



$$\nabla^2 V_g^{(5)} = 4\pi G^{(5)} \rho_m^{(5)}$$

$$\rho_m^{(5)} = m \cdot \delta(x^1)\delta(x^2)\delta(x^3)$$

$\rho_m^{(5)}$ above has correct units, but correct value? Check by integrating over volume:

$$\int dx_1 dx_x dx_3 \int_0^{2\pi R} dx_4 \rho_m = M$$

For a 4D observer:

$$\rho_m^{(4)} = M\delta(x^1)\delta(x^2)\delta(x^3) = 2\pi\rho_m^{(5)}$$

$$\nabla^2_{x_1,x_2,x_3} V_g(x_1,x_2,x_3) = 4\pi\left(\frac{G^{(5)}}{2\pi R}\right)\rho_m^{(4)}$$

$$G = G^{(5)}/2\pi R$$

So the extra length is $2\pi R =$length of compact dimension l_c

If had more compact dimestions, all circles, would get:

$$\frac{G^{(D)}}{G} = (l_c)^{D-4} = \text{ volume of compact space}$$

Experimental Implications

With current particle accelerators, have explored $l \approx 10^{-16}$ cm.
Since $E = \hbar c/l$, an l of $\approx 10^{-18}$ cm correlates to $E = 20$TeV.

Say $l_p^{(D)} = 10^{-18}$cm ($l_p = 10^{-33}$cm) for an acc. that could detect

Find l_c:

$$\frac{G^{(D)}}{G} = \frac{(l_p^{(D)})^{D-2}}{l_p^2} = (l_c)^{D-4}$$

Take $D = 5$:

$$l_c = (l_p^{(5)})^3/l_p^2 = (10^{-18})^3/(10^{-33})^2 = 10^{12}\text{cm} = 10^7\text{km}$$

If this were the case, we would have seen this dimension already!

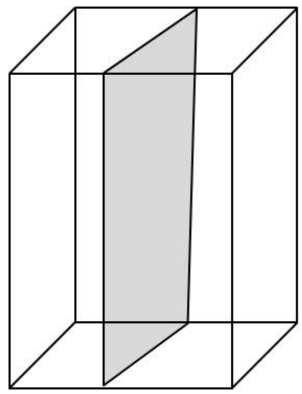
Take $D = 6$:

$$l_c^2 = (l_p^{(6)})^4/l_p^2$$
$$l_c = (l_p^{(6)})^2/l_p = 10^{-3}\text{cm}$$

At least not laughable.

But people don't talk in 10^{-3} cm. They talk in microns. 1 μm = 10^{-6} cm = 10^{-3} mm

So $D = 6$ gets us $l_c = 10\mu$m. Note: light's $\lambda \approx 0.5\mu$m.



String theory says we live in a D-brane. We and photons and electrons and all particles are open string attached to the D-brane. Can't see out.

Graviton is closed strings hanging out outisde the D-brane.

5 years ago, asked experimenters how far had tested gravity. Answer: $\approx$ cm.

2006 (hep-ph/0611184 Kapner et al) $R \geq 44\mu$m. So possible that $D = 6$!