MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department
Earth, Atmospheric, and Planetary Sciences Department

Astronomy 8.282J-12.402J April 5, 2006

Problem Set 8

Due: Wednesday, April 12 (in lecture)

Reading: Zeilik & Gregory: Chapters 16 & 17.

Reminder: Quiz #2 will be given out on Wednesday, April 19th, as a take-home exam. It will be due back in lecture on Friday, April 21st.

Problem 1

"Constructing the Galactic Rotation Curve"

The Table below gives the maximum radial velocity, $v_{\rm rad,max}$, that is observed for neutral hydrogen in the Galaxy as a function of galactic longitude, ℓ. Use the equation:

$$v_{\rm rad,max} = v_{\rm rot} - v_\odot \sin\ell$$

to construct the rotation curve of our Galaxy. Plot $v_{\rm rot}$ as a function of $\sin\ell$. (Note that $\sin\ell = R/R_\odot$, where R and $R_\odot$ are the radial distances from the galactic center of an arbitrary point and the position of the Sun, respectively.)

ℓ (degrees):	15	20	25	30	35	40	45	50
$v_{\rm rad,max}$ (km/s):	147	145	128	123	106	96	82	81
ℓ (degrees):	55	60	65	70	75	80	85	90
$v_{\rm rad,max}$ (km/s):	67	58	45	34	27	22	16	14

For the purpose of constructing this graph, assume that $v_\odot = 225$ km/s.

Problem 2

"Oort Constants"

Given the definitions of the Oort A and B constants:

$$A \equiv -\left(\frac{d\omega}{dR}\right)_0 \left(\frac{R_0}{2}\right),$$
$$B \equiv A - \omega_0$$

a. Show that A/B would equal -3 if the mass of our Galaxy were almost entirely concentrated in the center (i.e., the case where $\omega \propto R^{-3/2}$).

b. Show that $A/B = -1$ for the case where our Galaxy is assumed to have a "flat" rotation curve (i.e., $v_{\rm rot}$ = constant; $\omega \propto R^{-1}$)

Problem 3 **-Optional**

"Kinematic Distances"

Once the rotation curve, $v_{\rm rot}(R) \simeq \omega(R) \times R$, of our Galaxy is determined, we can use radial velocity ($v_{\rm rad}$) measurements to determine "kinematic distances" to objects. Recall the formula:

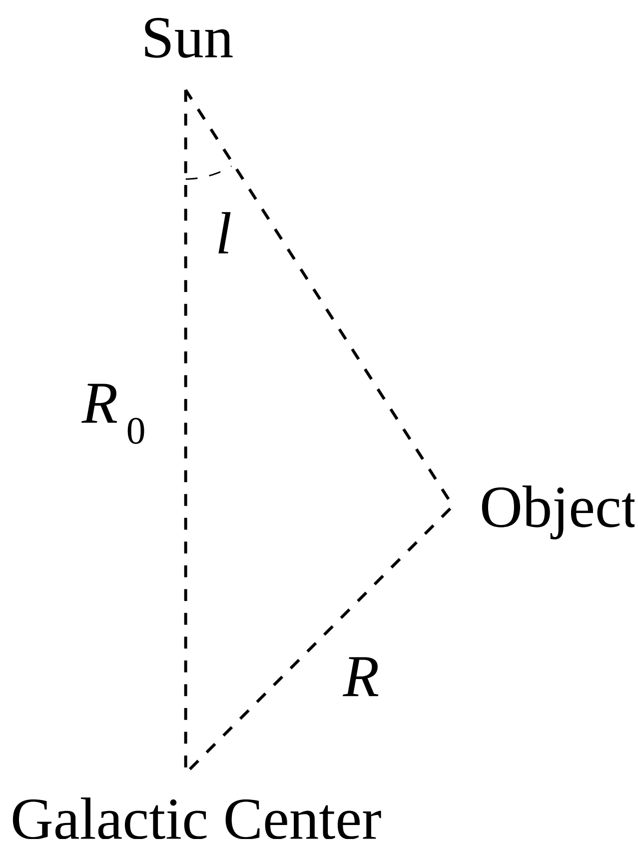
$$v_{\rm rad} = R_0(\omega - \omega_0) \sin \ell \,,$$

where R_0 is the distance from the Sun to the galactic center, and ω_0 is the Sun's angular velocity about the center of the Galaxy. (See the sketch below for the appropriate geometry.)

a. Assume that our Galaxy has a "flat" rotation curve (i.e., $v_{\rm rot}(R)$ = constant $\equiv v_0$), and that we know the values of R_0 and v_0. Derive a formula for R, the distance from an object to the galactic center, in terms of the known and observable quantities (R_0, v_0, ℓ, and $v_{\rm rad}$).

b. The kinematic distance d (from the Sun to the object) can then be derived from R, R_0, and ℓ. Find this relation.

c. A star at a galactic longitude $\ell = 20°$ is observed to have a radial velocity $v_{\rm rad}$ = 100 km/sec (away from the Earth). Use the expressions found in parts (a) and (b) to find the kinematic distance to the star. [Take R_0 = 8.5 kpc and v_0 = 225 km/sec.]



Problem 4

"Simplified Model of a Star"

Consider the following (somewhat unphysical) model of a star that is composed of an incompressible fluid – one in which the density ρ is independent of the pressure exerted on it. (Such an approximation is actually much better suited for constructing a model of a planet.)

a. Use the equation of hydrostatic equilibrium, $dP(r)/dr = -g(r)\rho$, and Newton's theorem that the local acceleration of gravity inside of a spherical distribution is $GM(r)/r^2$ [where $M(r)$ is the mass enclosed within radius r] to derive the pressure as a function of radius within the star. Assume a total stellar mass, M, a radius, R, and a uniform (constant) density $\rho = 3M/(4\pi R^3)$. Express your answer for $P(r)$ in terms of M, R, and G.

b. Sketch $P(r)$. Note that the radius of the stellar surface, R, is defined by $P(R) = 0$.

c. Evaluate your expression for the pressure at the center of a star [$P(r = 0)$] for the case where $M = 1M_\odot$ and $R = 1R_\odot$. Express your answer in units of the atmospheric pressure on Earth (10^6 dynes cm^{-2} = 10^5 Newtons m^{-2}).

As an interesting application of your expression for $P(R)$, you can use it to compute the pressure at the center of the Earth. Take $M_\oplus \simeq 6 \times 10^{27}$ g and $R_\oplus \simeq 6378$ km. The pressure at the center of the Earth is given in the texts as 3.9×10^{12} dynes cm^{-2} or 3.9×10^{11} Newtons m^{-2}. Your answer should agree to within a factor of $\sim$2.

d. Now find the temperature at the center of the model star, T_c. Utilize the fact that the "fluid" within a star actually obeys the ideal gas law to a high degree of accuracy:

$$P = nkT \,,$$

where P is the pressure, T the temperature, n the particle number density, and k is Boltzmann's constant. (Note that this is hardly consistent with the assumption of an incompressible fluid that was made above, but we shall proceed anyway.) Use the ideal gas law and the result of part (c) to find T_c. Note that $n = \rho/m$, where m is the average weight of a gas particle, and take $m \simeq 10^{-24}$ g.

e. In more accurate models of the Sun, the central temperature turns out to be higher than the value you found in part (d) because the Sun does not have a uniform density and is, in fact, centrally condensed. The actual central density in the Sun is approximately 150 grams cm^{-3}, again reflecting the high degree of central concentration. Assume that most of the energy generation via the nuclear burning of hydrogen occurs in the central region which contains about 20% of the total mass of the Sun. The total power generated by the Sun is $L_\odot = 4 \times 10^{33}$ ergs/sec. Use the following expression for nuclear energy production per gram of matter to estimate what the temperature near the center of the Sun, T_c, must be in order to produce the observed power output, $L_\odot$:

$$\varepsilon = 4.4 \times 10^5 \rho \exp\left(-\frac{3381}{T_c^{1/3}}\right) \text{ erg g}^{-1}\text{ s}^{-1}$$

(In evaluating this expression, assume that the central 20% of the mass of the Sun has a uniform temperature T_c and a uniform density $\rho = 150$ g cm^{-3}.)

The following parts are **optional**:

• Compute the gravitational potential energy, V, of our uniform density stellar model. Express your answer in terms of M, R, and G. [Hint: the potential energy lost in adding a shell of mass δM to an existing sphere of mass $M(r)$ is $\delta V = GM(r)\delta M/r$.]

• Evaluate V for $M = 1M_\odot$ and $R = 1R_\odot$.

• Find the Kelvin-Helmholtz timescale, $\tau_{\rm KH}$ (the time for a star to radiate away half of its stored energy) for the model star.

$$\tau_{\rm KH} = \frac{E_{\rm star}}{L}$$

where $E_{\rm star} = |V|/2$ (from the Virial theorem) and L is the luminosity of the star (use $L = L_\odot = 3.9 \times 10^{33}$ ergs s^{-1}). Express your answer in years.

Problem 5

"Fueling the Sun" **Optional**

Zeilik & Gregory; Problem 1, Chapter 16, page 330.

Problem 6

"Main-Sequence Lifetimes"

Zeilik & Gregory; Problem 6, Chapter 16, page 330.

Take the nuclear burning efficiency of the p-p chain to be 0.007. That is, for every mass of hydrogen, m, that is burned to helium, the energy generated is 0.007 mc^2.

Problem 7

"Nuclear Binding Energies"

Use the following table of Atomic Mass Excesses[1] (expressed in energy units of MeV) to compute how much energy is liberated in each of the following reactions from the p-p and CNO chains.

a. $\mathrm{H} + \mathrm{D} \rightarrow \mathrm{He}^3 + \gamma$
b. $\mathrm{He}^3 + \mathrm{He}^3 \rightarrow \mathrm{He}^4 + 2\mathrm{H}^1$
c. $\mathrm{N}^{15} + \mathrm{H}^1 \rightarrow \mathrm{O}^{16} + \gamma$
d. $\mathrm{He}^4 + \mathrm{He}^4 + \mathrm{He}^4 \rightarrow \mathrm{C}^{12} + \gamma$
e. $\mathrm{He}^4 + \mathrm{C}^{12} \rightarrow \mathrm{O}^{16} + \gamma$

[Hint: Add up the values of the mass excesses, $A - M$, for the nuclei on the left-hand-side of the reaction and subtract the sum of the mass excesses for the nuclei on the right-hand-side. The mass of a gamma ray (γ) is zero.]

[1] from "Principles of Stellar Evolution and Nucleosynthesis", by Donald D. Clayton; McGraw-Hill Book Company

Table 4-1 Atomic mass excesses†

Z	*Element*	*A*	*M − A, Mev*
0	*n*	1	8.07144
1	H	1	7.28899
	D	2	13.13591
	T	3	14.94995
	H	4	28.22000
		5	31.09000
2	He	3	14.93134
		4	2.42475
		5	11.45400
		6	17.59820
		7	26.03000
		8	32.00000
3	Li	5	11.67900
		6	14.08840
		7	14.90730
		8	20.94620
		9	24.96500
4	Be	6	18.37560
		7	15.76890
		8	4.94420
		9	11.35050
		10	12.60700
		11	20.18100
5	B	7	27.99000
		8	22.92310
		9	12.41860
		10	12.05220
		11	8.66768
		12	13.37020
		13	16.56160
6	C	9	28.99000
		10	15.65800
		11	10.64840
		12	0
		13	3.12460
		14	3.01982
		15	9.87320
7	N	12	17.36400
		13	5.34520
		14	2.86373
		15	0.10040
		16	5.68510
		17	7.87100
8	O	14	8.00800
		15	2.85990
		16	−4.73655
		17	−0.80770
		18	−0.78243
		19	3.33270
		20	3.79900
9	F	16	10.90400
		17	1.95190
		18	0.87240
		19	−1.48600
		20	−0.01190
		21	−0.04600
10	Ne	18	5.31930
		19	1.75200
		20	−7.04150
		21	−5.72990
		22	−8.02490
		23	−5.14830
		24	−5.94900
11	Na	20	8.28000
		21	−2.18500
		22	−5.18220
		23	−9.52830
		24	−8.41840
		25	−9.35600
		26	−7.69000
12	Mg	22	−0.14000
		23	−5.47240
		24	−13.93330
		25	−13.19070
		26	−16.21420
		27	−14.58260
		28	−15.02000
13	Al	24	0.1000
		25	−8.9310
		26	−12.2108
		27	−17.1961
		28	−16.8554
		29	−18.2180
		30	−17.1500
14	Si	26	−7.1320
		27	−12.3860
		28	−21.4899
		29	−21.8936
		30	−24.4394
		31	−22.9620
		32	−24.2000
15	P	28	−7.6600
		29	−16.9450
		30	−20.1970
		31	−24.4376
		32	−24.3027